EITG05 – Digital Communications

Lecture 8

Chapter 3: Carrier modulation techniques

Bandpass signals, digital and analog modulation

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From last lecture: Non-coherent receivers

- With phase-shift keying (PSK) the message $m[n]$ at time nT_s is put into the phase θ_n of the transmit signal

$$s(t) = g(t)\,\sqrt{2E}\,\cos(2\pi f_c t + \theta_n)\;, \quad nT_s \le t \le (n+1)T_s$$

- The channel introduces some attenuation α, some additive noise $N(t)$ and also some phase offset ν into the received signal

$$r(t) = \alpha\, g(t)\,\sqrt{2E}\,\cos(2\pi f_c t + \theta_n + \nu) + N(t)$$

- **Challenge:** the optimal receiver needs to know α and ν
- In some applications an accurate estimation of ν is infeasible (cost, complexity, size)
- **Non-coherent receivers:** receiver structures that can work well without knowledge of the exact phase offset

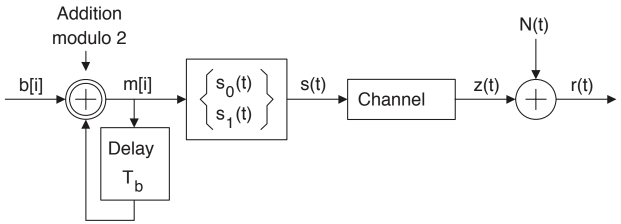
How can we modify our PSK transmission accordingly?

Differential Phase Shift Keying

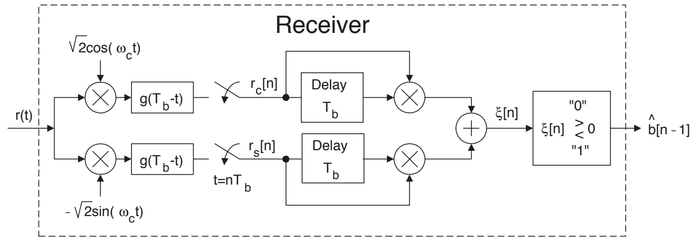
- With differential PSK, the message $m[n] = m_\ell$ is mapped to the phase according to

$$\theta_n = \theta_{n-1} + \frac{2\pi\ell}{M} \qquad \ell = 0, \ldots, M-1$$

- The transmitted phase θ_n depends on both θ_{n-1} and $m[n]$
- This differential encoding introduces memory and the transmitted signal alternatives become dependent
- **Example 5.25:** binary DPSK



Differential Phase Shift Keying ($M = 2$)



- The receiver uses no phase offset ν in the carrier waveforms
- Without noise, the decision variable is

$$\begin{aligned}\xi[n] &= r_c[n]\,r_c[n-1] + r_s[n]\,r_s[n-1]\\ &= A\cos(\theta_{n-1}+\nu)\,A\cos(\theta_{n-2}+\nu) + A\sin(\theta_{n-1}+\nu)\,A\sin(\theta_{n-2}+\nu)\\ &= A^2\cos(\theta_{n-1}-\theta_{n-2}) \quad \Rightarrow \text{ independent of } \nu\end{aligned}$$

- Note: non-coherent reception increases variance of noise

Chapter 3: Carrier modulation techniques

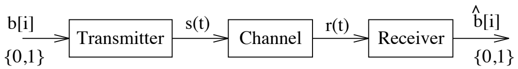
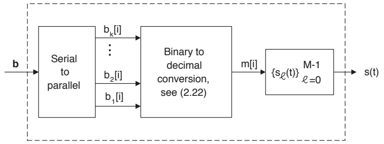


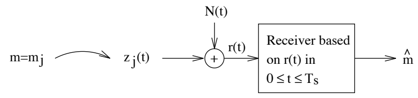
Figure 4.1: A digital communication system.

What we have done so far:

Chapter 2:
From $b[i]$ and $m[i]$ to signals $s_\ell(t)$



Chapter 4:
From signals $z_j(t) + N(t)$ to $\hat{m}[i]$ and $\hat{b}[i]$



Now more on:

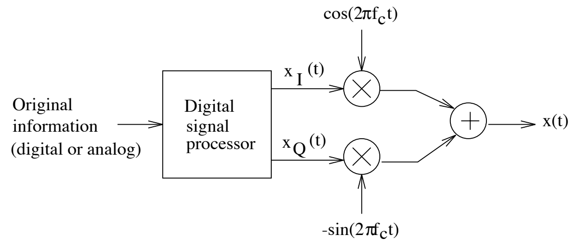
- properties of bandpass signals
- the channel: from $s(t)$ over $z(t)$ to $r(t)$
- efficient receivers for bandpass signals

Bandpass Signals

- A general bandpass signal can always be written as

$$x(t) = x_I(t)\ \cos(2\pi f_c t)\ -\ x_Q(t)\ \sin(2\pi f_c t)\ , \quad -\infty \le t \le \infty$$

- $x_I(t)$: inphase component $\qquad x_Q(t)$: quadrature component
- Corresponding transmitter structure:



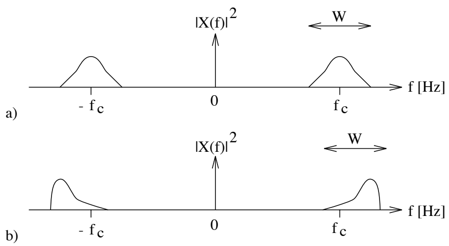
- The information is contained in the signals $x_I(t)$ and $x_Q(t)$ (for both analog or digital modulation)
- Not only wireless systems use carrier modulation

Spectrum of bandpass signals

- Computing the Fourier transform of $x(t)$ we get

$$X(f) = \frac{X_I(f+f_c) - j\,X_Q(f+f_c)}{2} + \frac{X_I(f-f_c) + j\,X_Q(f-f_c)}{2}$$

- Normally, $X_I(t)$ and $X_Q(t)$ have baseband characteristic, and f_c is much larger than their bandwidth
- The spectrum can be symmetric or non-symmetric around f_c



- **Remember:** real signals $x(t)$ always have even $|X(f)|$

DSB-SC Carrier Modulation

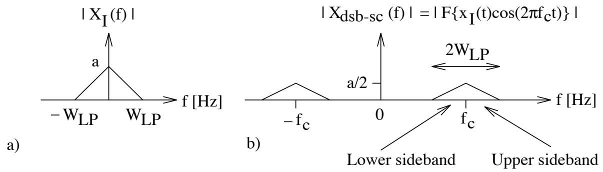
- Double sideband-suppressed (DSB-SC) carrier modulation is a special case of our general model
- In this case only $x_I(t)$ contains information and $x_Q(t) = 0$, i.e.,

$$x_{dsb-sc}(t) = x_I(t)\ \cos(2\pi f_c t)$$

- The Fourier transform then simplifies to

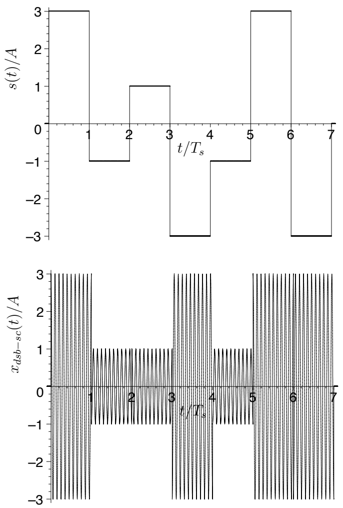
$$X(f) = \frac{X_I(f+f_c)}{2} + \frac{X_I(f-f_c)}{2}$$

- $X_I(f)$ is symmetric around $f = 0 \Rightarrow X_I(f)$ is symmetric around f_c

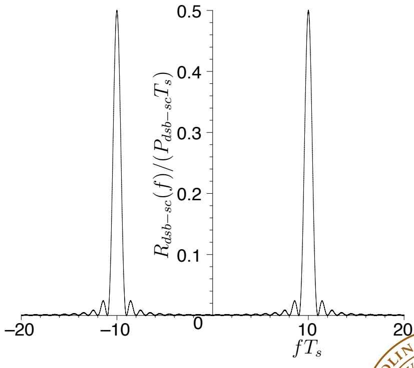


Where does the name come from?

Example 3.1: 4-ary PAM

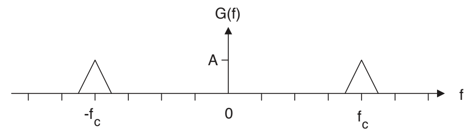


$$x_I(t) = s(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\, g_{rec}(t - nT_s)$$



How can we revert the frequency shift to f_c?

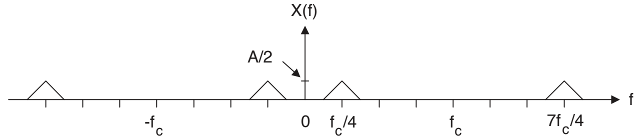
Hint: check Example 2.19 (p. 68)



Find the frequency content of

$$x(t) = g(t)\cos(2\pi f_0 t)\ , \quad f_0 = 3f_c/4$$

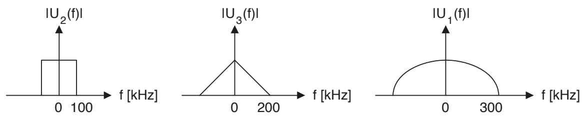
Solution:
If we apply (2.157) using $G(f)$ above, we obtain the frequency content in $x(t)$ as

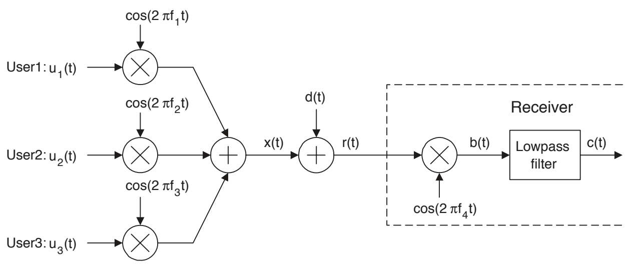


How should we choose f_0 to get the baseband signal back?

Problem 3.9

In the three-user (digital) communication system below, the frequency content in the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are,





It is known that the individual carrier frequencies are: $f_1 = 3.5$ MHz, $f_2 = 4.0$ MHz, $f_3 = 3$ MHz. The disturbance $d(t)$ is $d(t) = \cos(2\pi 2 f_d t)$ where $f_d = 1.7$ MHz.
Only frequencies up to 100 kHz pass the lowpass filter.

Envelope and Phase

- A frequency shift corresponds to a multiplication with $e^{j2\pi f_c t}$
- For connecting this to the cosine and sine function we use

$$e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$

- The general bandpass signal can then be written in terms of a frequency shifted version of a complex signal $x_I(t) + jx_Q(t)$

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$
$$= Re\left\{\left(x_I(t) + jx_Q(t)\right)e^{j2\pi f_c t}\right\}$$

- Expressing $x_I(t) + jx_Q(t)$ in terms of magnitude and phase we get

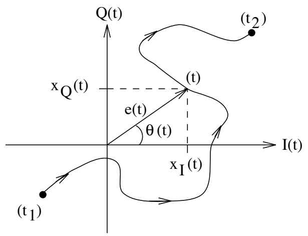
$$x(t) = e(t)\cos(2\pi f_c t + \theta(t))\ , \quad -\infty \le t \le \infty$$

with

$$e(t) = \sqrt{x_I^2(t) + x_Q^2(t)} \ge 0$$
$$x_I(t) = e(t)\cos(\theta(t))$$
$$x_Q(t) = e(t)\sin(\theta(t))$$

I-Q Diagram

- In the representation
$$x(t) = x_I(t)\cos(2\pi f_c t) \;-\; x_Q(t)\sin(2\pi f_c t)$$
the information is contained in the inphase component $x_I(t)$ and quadrature component $x_Q(t)$
- In the representation
$$x(t) = e(t)\cos(2\pi f_c t + \theta(t))\,, \quad -\infty \le t \le \infty$$
the information is contained in the envelope $e(t)$ and instantaneous phase $\theta(t)$



connection: I-Q diagram

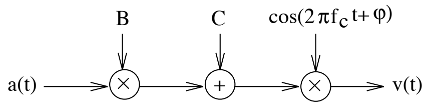
Analog Information Transmission

- Suppose that the information signal is an analog waveform $a(t)$
Examples: music, speech, video
- If we use digital modulation, the waveform $a(t)$ is first converted to a binary sequence $b[i]$, which then is mapped to signals $s_\ell(t)$
- In case of analog modulation, the waveform $a(t)$ is used directly to modulate the carrier signal
- Let $v(t)$ denote the bandpass signal of an analog transmitter

$$\begin{aligned} v(t) &= v_I(t)\cos(2\pi f_c t) \;-\; v_Q(t)\sin(2\pi f_c t)\,, \quad -\infty \le t \le \infty \\ &= e(t)\cos\big(2\pi f_c t + \theta(t)\big) \end{aligned}$$

- **Amplitude modulation (AM):**
the waveform $a(t)$ modulates the envelope $e(t)$ only
- **Frequency modulation (FM):**
here $a(t)$ modulates the instantaneous phase $\theta(t)$ only

Amplitude Modulation (AM)



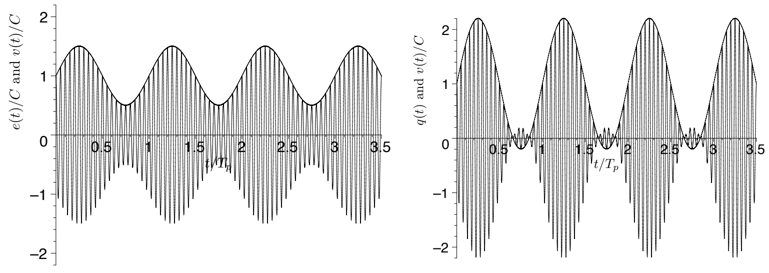
- The AM signal is the sum of a DSB-SC signal and carrier wave

$$\begin{aligned} v(t) &= \big(a(t)B + C\big)\cos(2\pi f_c t + \varphi) \\ &= a(t)B\cos(2\pi f_c t + \varphi) \;+\; C\cos(2\pi f_c t + \varphi) \end{aligned}$$

- Let us introduce the modulation index
$$m = \frac{B\,a_{max}}{C} \le 1\,, \quad \text{where} \;\; a_{max} = \max|a(t)|$$
- Using the normalized signal $a_n(t) = a(t)/a_{max}$ we can write
$$v(t) = \big(1 + m\,a_n(t)\big)\,C\cos(2\pi f_c t + \varphi)$$

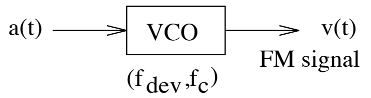
Example: AM signal

$$e(t)/C = 1 + m\,a(t)\,,\; a_n(t) = \sin(2\pi f_p t)\,,\; f_p = 1/Tp$$



- $m = 0.5 < 1$:
the information signal $a_n(t)$ is contained in the envelope $e(t)$
- $m = 1.2 > 1$: (right picture)
overmodulation: the baseband signal $q(t) = (1 + 1.2\,a_n(t))$ is no longer equal to $e(t)$

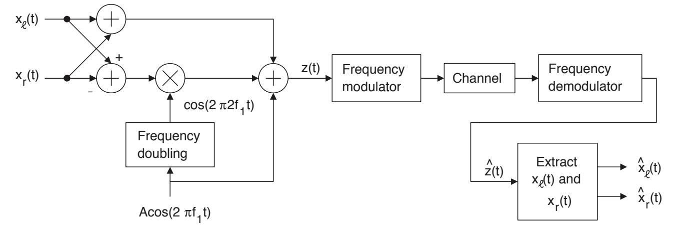
Frequency Modulation (FM)



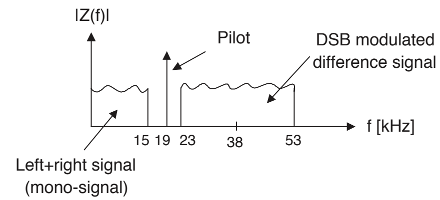
- With FM modulation, the transmitted signal
$$v(t) = \sqrt{2P}\cos(2\pi f_c t + \theta(t))$$
is generated by a voltage controlled oscillator (VCO)
- The information carrying signal $a(t)$ is related to the phase $\theta(t)$ by
$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_{dev}\cdot a(t)$$
- The signal $a(t)$ hence modulates the instantaneous frequency
$$f_{ins}(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + f_{dev}\,a(t)$$
- FM modulation is a non-linear operation, hard to analyze

Example 3.13: FM stereo

A possible block-diagram of conventional analog FM stereo is shown below.



$x_\ell(t)$ and $x_r(t)$ denotes the left and the right audio-channel, respectively, and they are both bandlimited to 15 [kHz]. The frequency f_1 = 19 [kHz] (often referred to as a so-called pilot-tone).



Digital Information Transmission

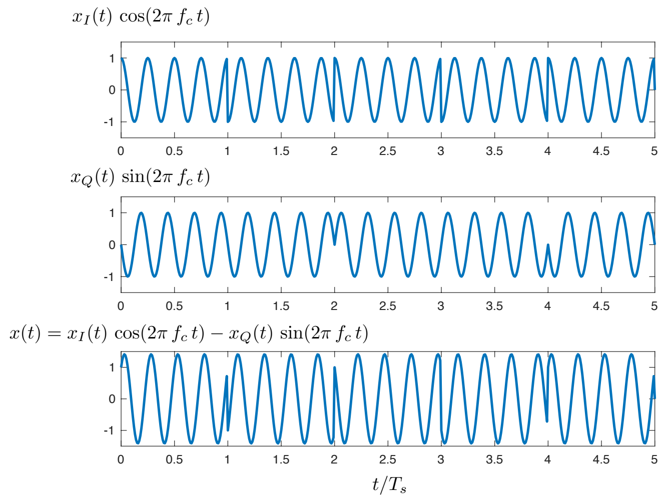
- In Chapter 2 the signal alternatives $s_\ell(t)$ could have arbitrary shape within the signaling interval $0 \le t \le T_s$
- The bandpass signal for digital modulation then has the form

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) \;-\; x_Q(t)\sin(2\pi f_c t) \\ &= \left(\sum_{n=-\infty}^{\infty} s_{m[n],I}(t - nT_s)\right)\cos(2\pi f_c t) \\ &\quad - \left(\sum_{n=-\infty}^{\infty} s_{m[n],Q}(t - nT_s)\right)\sin(2\pi f_c t) \end{aligned}$$

- In case of M-ary QAM we have
$$x_I(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\,g(t - nT_s)\ , \quad x_Q(t) = \sum_{n=-\infty}^{\infty} B_{m[n]}\,g(t - nT_s)$$
- Also M-ary FSK signals have bandpass characteristics

A simple Matlab example

How does a QPSK signal look like? Here is an example:



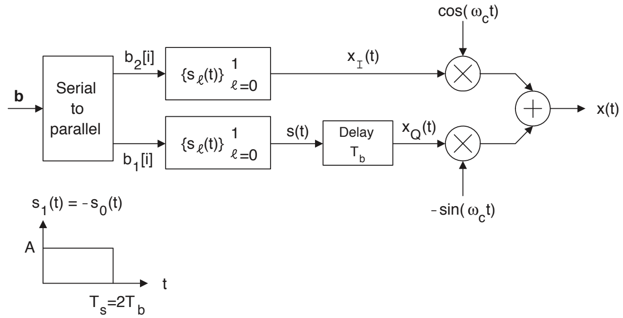
And how it was done:

```
% Example: QPSK signal

t=0:0.01:5;
fc=4;
pRec=ones(1,(length(t)-1)/5);
sI=zeros(1,length(t)); sQ=zeros(1,length(t));

dataI=[1 -1 1 -1 1];
indPulse=1:(length(t)-1)/5;
for i=1:length(dataI),
  sI(indPulse)=dataI(i)*pRec;
  indPulse=indPulse+length(indPulse);
end;

dataQ=[-1 -1 1 1 -1];
indPulse=1:(length(t)-1)/5;
for i=1:length(dataQ),
  sQ(indPulse)=dataQ(i)*pRec;
  indPulse=indPulse+length(indPulse);
end;

sCarI=cos(2*pi*t*fc); sCarQ=sin(2*pi*t*fc);

figure(1);
subplot(3,1,1); plot(t,sI.*sCarI);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');

subplot(3,1,2); plot(t,sQ.*sCarQ);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');

subplot(3,1,3); plot(t,sI.*sCarI - sQ.*sCarQ);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');
```

script Ln 32 Col 30

Example 3.5: offset QPSK

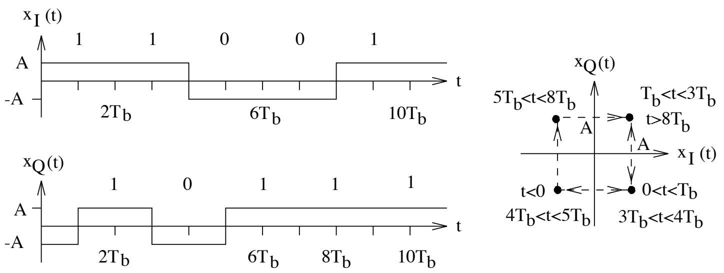
Below, two information carrying baseband signals $x_I(t)$ and $s(t)$ are first generated. Binary antipodal signaling with a rectangular pulse shape is used for both $x_I(t)$ and $s(t)$. The signal $x_Q(t)$ is a delayed version of $s(t)$, $x_Q(t) = s(t - T_b)$.



The information bit rate (in $\mathbf{b}$) is $R_b = 1/T_b$. Hence, the signaling rate in the quadrature components is $R_s = R_b/2$.

QPSK signal with delayed transmission of $x_Q(t)$

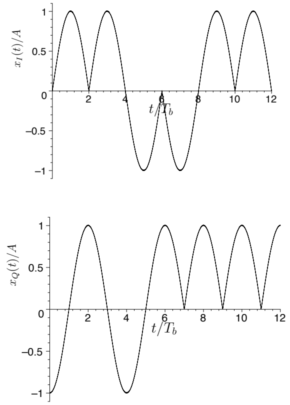
Example 3.5: offset QPSK



- **Special feature:** $x_I(t)$ and $x_Q(t)$ can never change at the same time
- it follows that the envelope does not pass the origin, i.e., $e(t) > 0$
- the variation of instantaneous power $\mathcal{P}(t) = e^2(t)/2$ is small, which allows more efficient power amplifiers

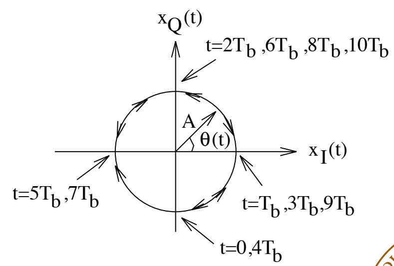
Example 3.6: constant envelope signaling

Change pulse shape: half cycle sinusoidal $g_{hcs}(t)$ instead of $g_{rec}(t)$



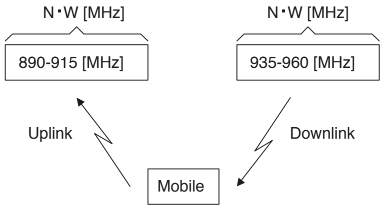
The squared envelope becomes

$$e^2(t) = x_I^2(t) + x_Q^2(t)$$
$$= A^2 \sin^2(\pi t/(2T_b)) + A^2 \cos^2(\pi t/(2T_b))$$
$$= A^2 \quad \Rightarrow \text{constant envelope } e(t) = A$$



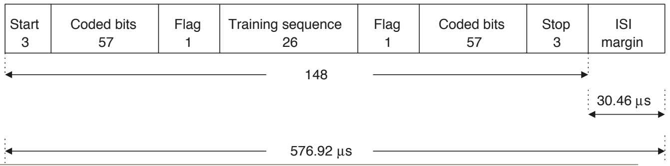
Continuous phase modulation (CPM) is used in GSM

Example 3.7: GSM



Each sub-band of W [Hz] *carries information from* X *users, which are time-multiplexed using* X *time-slots. The total number of speech-channels (or data-channels) in the uplink (and in the downlink) is* $N \cdot X$*.*

A specific user is allocated one of the N *sub-bands, and one of the* X *time-slots. A time-slot has duration 576.92* [μs]*, and 148 binary symbols are transmitted within this time, see the figure below.*



From 2G to 4G

- **GSM:** (Global System for Mobile Communications) based on combined time-division multiple access (TDMA) and frequency division multiple access (FDMA)
- **UMTS:** (Universal Mobile Telecommunications Service) based on wideband code division multiple access (W-CDMA) each user has an individual code, no TDMA or FDMA
- **LTE (advanced):** (Long Term Evolution) orthogonal frequency-division multiple access (OFDMA)

Multiple access:
refers to how different active users are separated