Final exam in

Digital Communications (ETT051)

on October 24, 2016, 14–19.

Department of Electrical and Information Technology
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) Assume a conventional 32-ary PAM system. Also assume that $g(t) = g_{hcs}(t)$ with duration $T_s/4$. "If the width of the main lobe is 0.8 MHz then the bandwidth-efficiency is 5/6 bps/Hz."

b) "M^2-QAM is better than bandpass M-PAM"

c) Assume a conventional 256-ary QAM system that uses $g(t) = g_{tri}(t)$ with duration $3T_s/8$, a conventional AWGN channel, and ML receiver. "If $\mathcal{E}_b/N_0$ is 26.5216 dB then $P_s \approx 1.9 \cdot 10^{-10}$."

d) Assume the pulse $g_{rc}(t)$ with amplitude A and duration $3T_s/4$. "With M=2 and equally likely signal alternatives $s_0(t) = 2g_{rc}(t)$ and $s_1(t) = -3g_{rc}(t)$, the average signal power is $1.757A^2$."

e) "An M-PPM signal alternative will have a relatively narrow frequency content if T_s is fixed and if M is large."

(10 points)

Problem 2: Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver. In this problem two pairs of signal alternatives are considered.

All four signal alternatives below are zero outside the time interval $0 \leq t \leq T$.

Pair 1: $z_0(t) = A, 0 \leq t \leq T$, $z_1(t) = 4A, 0 \leq t \leq T$.

Pair 2: $z_0(t) = A, 0 \leq t \leq T$, $z_1(t) = -3A, 0 \leq t \leq 8T/15$, and $z_1(t) = 0, t > 8T/15$.

Furthermore, A^2/N_0 is 65 dB below.

i) For Pair 1 determine an expression of the bit error probability that contains the ratio A^2/N_0.

ii) For Pair 2 determine an expression of the bit error probability that contains the ratio A^2/N_0.

iii) Find the difference in energy-efficiency in dB between Pair 1 and Pair 2. Conclusions?

iv) Consider Pair 1, and assume here that $T = T_b$. Calculate the bit rate and the bit error probability if $\mathcal{E}_b/N_0$ is 12 dB.

(10 points)

Problem 3:

a) The communication channel is here a 3-ray channel. For the first signal path the multiplication parameter is 0.8 and the delay is zero. For the second signal path the multiplication parameter is -0.5 and the delay is $0.5 \cdot 10^{-6}$s. For the third signal path the multiplication parameter is 0.1 and the delay is $1.2 \cdot 10^{-6}$s.

i) Calculate and sketch the output signal from the 3-ray channel if the input signal is $g_{rec}(t)$ with amplitude A and duration $1.5 \cdot 10^{-6}$s, and if nothing else is sent.

ii) Assume here that the input signal to the 3-ray channel is a sequence of conventional and equally likely 16-PAM signal alternatives where the $g_{hcs}(t)$ pulse is used. It is also known that the mainlobe-based bandwidth equals 6 MHz.

Determine suitable bit rates.

b)

i) Calculate the value of the complete union bound for a general 16-ary signal constellation, if it is assumed that the communication link has a very small ("close to zero") signal-to-noise ratio $\mathcal{E}_b/N_0$.

What are your conclusions concerning the union bound at very small signal-to-noise ratios?

ii) Explain in detail why the discrete-time pulse x[i] is of interest in the context of ISI.

(10 points)

Problem 4:

a) Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 600 kHz, 200 kHz and 400 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where α is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 53$ MHz, $f_2 = 53.5$ MHz, and $f_3 = 54$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 53.1$ MHz and $f_B = 53.3$ MHz.

Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 53.2$ MHz.

iii) The choice of $f_4 = 53.2$ MHz is not a correct choice but it can occurr due to a mal-function in the receiver. Determine the frequency f_4 if the desired user is user 2.

b) Consider a conventional communication link with AWGN (with power spectral density $N_0/2$). The ML receiver is assumed to be used.

Consider the received 4-ary signal constellation below where,

$z_0(t) = g_{rc}(t)$, $z_1(t) = 2g_{rc}(t)$, $z_2(t) = 4g_{rc}(t)$, and $z_3(t) = 7g_{rc}(t)$.

The signals are equally likely.

Determine all the normalized squared Euclidean distances, and the corresponding coefficients, in the complete union bound.

How much are these values changed if the $g_{hcs}(t)$ pulse is used instead of the $g_{rc}(t)$ pulse?

Compared with 4-ary conventional PAM, what are the main differences when using the 4-ary signal constellation above?

(10 points)

Problem 5:

Here equally likely M-PSK signals are used. The pulse-shape is $g_{tri}(t)$ with amplitude A and duration $T = 3T_s/4$. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is also given that the symbol error probability in this case can be upper bounded by
$2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$. It is also given that the width of the mainlobe equals 3 MHz, and that the system adaptivly selects the most suitable M at each $\bar{P}_z/N_0$.

It is a requirement that the bit error probability must not exceed the value $2 \cdot 10^{-6}$.

i) Determine the bandwidth efficiency if the ratio $\bar{P}_z/N_0 = 3 \cdot 10^7$. For which values of $\bar{P}_z/N_0$ will this bandwidth-efficiency be used?

ii) At a certain communication range it has been found that the ratio $\bar{P}_z/N_0 = 6 \cdot 10^8$. Estimate the symbol error probability in this case.

iii) If only "M-PSK" is changed to "bandpass M-PAM", in the problem formulation above, would that imply a higher bandwidth efficiency in i)?

(10 points)