Final exam in

Digital Communications (ETT051)

on October 24, 2013, 14–19.

Department of Electrical and Information Technology
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.

  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the textbook, add a reference in Your solution.

  Determine the bandwidth of the lowpass filter in this case
- If any data is lacking, make reasonable presumptions.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) Assume a conventional M-ary QAM system. Also assume that $g(t) = g_{hcs}(t)$ with duration $4T_s/5$. "If the bit rate is 800 kbps and $M = 64$ then the width of the mainlobe is 0.5 MHz."

b) "16-QAM is better than both bandpass 16-PAM and 16-FSK."

c) Assume a conventional M-ary QAM system that uses $g(t) = g_{tri}(t)$ with duration $3T_s/4$, a conventional AWGN channel, and ML receiver. "If $M = 256$ and $\mathcal{E}_b/N_0$ is 25.826 dB then $P_s \approx 4 \cdot 10^{-10}$."

d) Assume a conventional M-ary baseband PAM system. Also assume that $g(t) = g_{rec}(t)$ with duration T_s. "If M is increased than the bandwidth decreases."

e) "In case of long-range communication band-pass 256-PAM is a suitable method, especially for the uplink in a mobile to base station application."

(10 points)

Problem 2: Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver.

a) It is given that $z_1(t) = g_{rec}(t)$ with amplitude A and duration $T = T_b/2$.

It is also given that A^2/N_0 is 70 dB, and that the bit error probability must not be larger than 10^{-6}.

i) Here $z_0(t) = 0$. Which bit rates can then be used?

ii) Here $z_0(t) = -z_1(t)$. Which bit rates can then be used?

b) Consider the two signal alternatives: $z_1(t) = g_{rec}(t)$ with amplitude A and duration $T = T_b$, and the signal alternative $z_0(t) = g_{rec}(t)$ with amplitude A and duration $T = T_b/2$.

Calculate the bit error probability for this case if $\mathcal{E}_b/N_0$ is 18 dB.

Also calculate the energy-efficiency for this case compared to binary orthogonal signals.

(10 points)

Problem 3:

a) Consider the pulse $g_{rec}(t)$, where $g_{rec}(t)$ has duration $T = 1 \cdot 10^{-6}$ (s) and amplitude A.

The pulse above is the input signal to a 3-ray channel (multi-path). The parameters of the 3-ray channel are:

$\alpha_1 = 0.4$, $\tau_1 = 0$
$\alpha_2 = 0.2$, $\tau_2 = 0.25 \cdot 10^{-6}$ (s)
$\alpha_3 = -0.1$, $\tau_3 = 0.75 \cdot 10^{-6}$ (s)

i) Determine, and sketch, the output pulse from the 3-ray channel.

ii) Describe communication consequences that is introduced by the multi-path channel above.

b) Assume that the transmitted signal alternatives $\{s_\ell(t)\}_{\ell=0}^{M-1}$ are conventional baseband M-ary PAM signals. Also assume that a hcs-pulse with amplitude A and duration $T = T_s$ is used. It is also given that the width of the main-lobe equals 4 MHz.

An important requirement in many practical applications is that the average transmitted power, here denoted P_{tr}, should be a constant value regardless of which value of M is used.

This means that the value of the amplitude A of the hcs-pulse changes as M changes.

Therefore, let A_M denote this amplitude when M signal alternatives are used.

So, using M=2 and the amplitude A_2, yields the same average transmitted power as using M=4 and the amplitude A_4, as well as using M=8 and the amplitude A_8, and so on.

Calculate the ratios:

i) A_4/A_2.

ii) A_8/A_2.

iii) A_{64}/A_2.

What are your conclusions.

(10 points)

Problem 4:

a) Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 400 kHz, 800 kHz and 1200 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where α is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 20$ MHz, $f_2 = 20.7$ MHz, and $f_3 = 21.9$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 20.2$ MHz and $f_B = 21.2$ MHz.

Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 21.1$ MHz.

iii) The choice of $f_4 = 21.1$ MHz is not a correct choice but it can occurr due to a malfunction in the receiver. Determine the bandwidth of the lowpass filter if instead $f_4 = 20.7$ MHz, which is an example of a correctly chosen frequency.

b) i) Explain the connection between the union bound and Gray-coding.

ii) Assume a 16-ary PAM communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver. The pulse-shape used is $g_{rc}(t)$, with duration T and amplitude A. It is given that A^2T/N_0 equals 24.171 dB.

However, the 16 possible (non-conventional) amplitudes used here are:
0,1,5,9,13,17,21,25,29,33,37,41,45,49,53,57.

It is known that the communication link is very good with a large value $\mathcal{E}_b/N_0$.

Determine the value of the union bound in this case.

(10 points)

Problem 5: Here equally likely M-PSK signals are used. The pulse-shape is $g_{rec}(t)$ with amplitude A and duration $T = T_s$. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is also given that the symbol error probability in this case can be upper bounded by $2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$. It is also given that the width of the mainlobe equals 2 MHz, and that the system adaptivly selects the largest possible M.

It is a requirement that the bit error probability must not exceed the value $2 \cdot 10^{-8}$.

i) Determine requirements on the ratio $\bar{P}_z/N_0$ if the bit rate 5 Mbps is used.
ii) At a certain communication range it has been found that the ratio $\bar{P}_z/N_0 = 2.75 \cdot 10^8$. Determine the highest bit rate that can be used in this case.

(10 points)