Final exam in Digital Communications (ETT051) on October 26, 2012, 14–19.

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- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) Assume a conventional 64-ary QAM system. Also assume that $g(t) = g_{hcs}(t)$ with duration $4T_s/5$. "If the bit rate is 10 Mbps then the width of the mainlobe is 6.25 MHz."

b) "Baseband 4-PAM is as good as 16-QAM."

c) Assume a conventional M-ary QAM system that uses $g(t) = g_{tri}(t)$ with duration $T = 2T_s/3$, a conventional AWGN channel, and ML receiver. "If $M = 64$ and $\mathcal{E}_b/N_0$ is 19.72 dB then $P_s \approx 1.48 \cdot 10^{-10}$."

d) "With M-PSK, the energy of a signal alternative depends on the value of the phase."

e) " A MAP receiver is not the same as the minimum Euclidean distance receiver, and not the same as a ML-receiver."

(10 points)

Problem 2: Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver.

The pulse $g_{rec}(t)$ has amplitude A and duration $T = 3T_b/4$.

a) Assume that the two signal alternatives are: $z_0(t) = g_{rec}(t)$, and $z_1(t) = 4g_{rec}(t)$.
Calculate the bit error probability if A^2T_b/N_0 is 9.3651 dB.

b) Assume that the two signal alternatives are: $z_0(t) = g_{rec}(t)$, and $z_1(t) = -2g_{rec}(t)$.
Calculate the bit error probability if A^2T_b/N_0 is 9.3651 dB.

c) Calculate the difference in energy efficiency for the cases in a) and b).

(10 points)

Problem 3:

a) Assume the received equally likely signal alternatives $\{z_\ell(t)\}_{\ell=0}^{2}$. The signals are: $z_0(t) = g(t)$, $z_1(t) = g(t - T_s/4)/2$, and $z_2(t) = g(t - T_s/2)/3$. The pulse g(t), and its energy E_g, are known and given. The duration of the pulse $g(t)$ is $T = T_s/4$. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used.

Determine an expression of the union bound.

b) Assume M-ary PAM transmission. Explain possible consequences of a multi-path channel (i.e. N-ray channel) having 10 signal paths. The more consequences you explain, the better.

(10 points)

Problem 4:

a) Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 800 kHz, 400 kHz and 200 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where α is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 8$ MHz, $f_2 = 8.8$ MHz, and $f_3 = 9.2$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 7.6$ MHz and $f_B = 8.5$ MHz.

Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 8.9$ MHz.

iii) The choice of $f_4 = 8.9$ MHz is not a correct choice but it can occurr due to a malfunction in the receiver. Determine the correct choice of f_4 if the receiver should recover the information in $u_3(t)$.

b) Assume that the received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are conventional 16-ary QAM signals. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$.

In a specific application it is for practical reasons needed to first amplify the received signal $r(t)$. The signal after amplification is denoted $r_G(t)$, and $r_G(t) = Gr(t)$. $G = 100$.

i) Suggest, in detail, an ML-receiver that has $r_G(t)$ as input signal.

ii) Determine an expression that is close to the symbol error probability for the receiver in i), if the channel conditions are very good.

(10 points)

Problem 5:

Assume that the received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are conventional M-ary bandpass PAM signals. Also assume that a pulse with amplitude A and duration $T = T_s/2$ is used. It is given that the bandwidth $W_{99.9}$ should be used, and that the bandwidth efficiency is very close to 0.4335 bps/Hz. The system is adaptive, and selects M with respect to the received signal quality. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is also given that the symbol error probability in this case can be upper bounded by $2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$.

It is a requirement that the bit error probability must not exceed the value $2 \cdot 10^{-8}$.

i) Determine requirements on the ratio $T_s\bar{P}_z/N_0$.

ii) A person claims that as long as $\bar{P}_z/N_0$ is larger than $5.1R_b$, then bandpass M-PAM is a good choice. Determine if the person is correct or not.

(10 points)
