# Final exam in

# Digital Communications (ETT051)

# on October 21, 2011, 14–19.

Department of Electrical and Information Technology
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

## Good Luck!

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**Problem 1:** Determine for each of the five statements below if it is true or false.
***Observe! As usual, motivations to your answers should be given.***

**a)** Assume a conventional M-ary PSK system. Also assume that $g(t) = g_{tri}(t)$ with duration $2T_s/5$. "If the bit rate is 600 kbps and $M = 8$ then the width of the mainlobe is 2 MHz."

**b)** "8-PSK is worse than 16-QAM"

**c)** Assume a conventional M-ary PSK system that uses $g(t) = g_{rc}(t)$ with duration $T_s$, a conventional AWGN channel, and ML receiver. "If $M = 16$ and $\mathcal{E}_b/N_0$ is 20.72 dB then $P_s \approx 2 \cdot 10^{-9}$."

**d)** "If $M = 2$ and $d^2 = 1.5$, then the signal alternatives are both antipodal and orthogonal."

**e)** "With uncoded equally likely signal alternatives the bit error probability is always larger than $1.5 \cdot 10^{-22}$ if $\mathcal{E}_b/N_0$ is 12 dB. "

(10 points)

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**Problem 2:** Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver. It is required that if $\mathcal{E}_b/N_0$ is 17.9 dB then $P_b = 8.54 \cdot 10^{-6}$.

a)

i) Determine $P_b$ if instead $\mathcal{E}_b/N_0$ is 22.845 dB.

ii) Determine also the energy efficiency compared with if orthogonal signal alternatives were used instead.

b)

Consider the two signal alternatives: $z_0(t) = g_{rec}(t)$ with amplitude $A$ and duration $T = T_b/2$, and the signal alternative $z_1(t) = g_{rec}(t)$ with amplitude $A/3$ and duration $T = T_b/2$.

Will these two signal alternatives satiesfy the requirement on the bit error probability above?

(10 points)

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**Problem 3:** Assume that the received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are conventional M-ary QAM signals. Also assume that a hcs pulse with amplitude $A$ and duration $T = T_s/4$ is used. It is given that here the communication bandwidth $W$ is $W = W_{99} = 4$ MHz. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is also given that the symbol error probability in this case can be upper bounded by $4Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$.

It is a requirement that the bit error probability must not exceed the value $4 \cdot 10^{-10}$.

i) Determine requirements on the ratio $\bar{P}_z/N_0$ if the bit rate is $R_b \approx 3.39$ Mbps.
ii) At a certain communication range it has been found that the ratio $\bar{P}_z/N_0 = 2 \cdot 10^8$. Determine the highest bit rate that can be used in this case.
iii) Describe differences and similarities between 16-ary PSK and 16-ary QAM.

(10 points)

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**Problem 4:**

**a)** Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 400 kHz, 200 kHz and 600 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where $\alpha$ is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 14$ MHz, $f_2 = 14.4$ MHz, and $f_3 = 14.9$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 14.25$ MHz and $f_B = 14.55$ MHz.

***Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.***

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 14.5$ MHz.

iii) The choice of $f_4 = 14.5$ MHz is not a correct choice but it can occurr due to a malfunction in the receiver. Determine the correct choice of $f_4$ if the receiver should recover the information in $u_2(t)$.

**b)** Explain in detail why $x[i]$ is so important in the analysis of ISI.

(10 points)

**Problem 5:**

Assume that conventional 8-ary PAM signals are sent from the transmitter. Here, a sent signal alternative has the form $A_l g_{rec}(t)$, where $g_{rec}(t)$ has duration $T = 0.8 \cdot 10^{-6}$ (s) and amplitude $A$.

The sent signal alternative above is the input signal to a 3-ray channel (multi-path). The parameters of the 3-ray channel are:

$\alpha_1 = 1/10$, $\tau_1 = 0$
$\alpha_2 = -1/20$, $\tau_2 = 0.4 \cdot 10^{-6}$ (s)
$\alpha_3 = 1/40$, $\tau_3 = 1.2 \cdot 10^{-6}$ (s)

a) Which bit rates do you recommend?

b) Assume that the input signal alternative is $3g_{rec}(t)$. Determine and make a detailed sketch of the output signal alternative from the 3-ray channel.

c) Determine the squared Euclidean distances $D^2_{min}$, $D^2_1$, ... , $D^2_{max}$ that are used to calculate the union bound.

(10 points)