LUND UNIVERSITY
Electrical and Information Technology

Exam in
Digital Communications, EITG05

August 21, 2019

- During this exam you are allowed to use a calculator, the compendium, a printout of the lecture slides, and Tefyma (or equivalent).
- Please use a new sheet of paper for each solution. Write your anonymized assessment code + a personal identifier on each paper.
- Solutions should clearly show the line of reasoning and follow the methods presented in the course. If you use results from the compendium or lecture slides, please add a reference in your solution.
- If any data is lacking, make reasonable assumptions.

Good luck!

Problem 1

Determine for each of the five statements below if it is true or false.
Give a motivation for each of your answers.

(a) Consider M-ary QAM signaling with a rectangular pulse shape $g(t) = g_{rec}(t)$ of amplitude A and duration $T = T_s$.

"If M is increased, then the same bit rate can be achieved with a smaller bandwidth."

(b) *"2-PAM signaling with $A_0 = 0$ and $A_1 = 1$ has better energy efficiency d^2_{min} than 2-PAM signaling with $A_0 = -1$ and $A_1 = 1$".*

(c) Assume a conventional M-ary baseband PAM system. Also assume that $g(t) = g_{hcs}(t)$ with duration $3T_s/4$.

"If the bit rate is 400 kbps and $M = 32$ then the width of the mainlobe is 320 kHz."

(d) Consider the union bound

$$P_s \leqslant c\,Q\left(\sqrt{\frac{D^2_{\min}}{2N_0}}\right) + c_1\,Q\left(\sqrt{\frac{D^2_1}{2N_0}}\right) + \cdots + c_x Q\left(\sqrt{\frac{D^2_{\max}}{2N_0}}\right) .$$

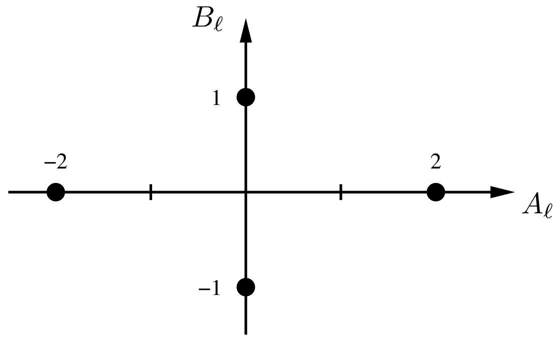
"For 4-PSK signaling, there are three distinct terms in this bound, i.e., x+1=3."

(e) *"Differential PSK (DPSK) performs better than PSK if the phase offset is unknown and the signal-to-noise ratio is large."*

(10p)

Problem 2

Consider a transmission using the following 4-QAM signal constellation:



A rectangular pulse shape $g_{rec}(t)$ of duration $T = T_s$ and amplitude $A = 1$ is used.

(a) Choose some Gray mapping to assign bits to the different signal alternatives.

(b) For the carrier frequency $f_c = 2/T_s$ and the message sequence

$$\mathbf{m} = (m[0]\ m[1]\ m[2]\ m[3]\ m[4]) = (2\ 1\ 0\ 3\ 1)\ ,$$

draw the transmit signal $s(t)$ within the time interval $0 \leqslant t \leqslant 5T_s$.

(c) Determine the average energy per symbol $\bar{E}_s$ and the minimum Euclidean distance D^2_{min} for this constellation. All signal alternatives are transmitted with equal probability.

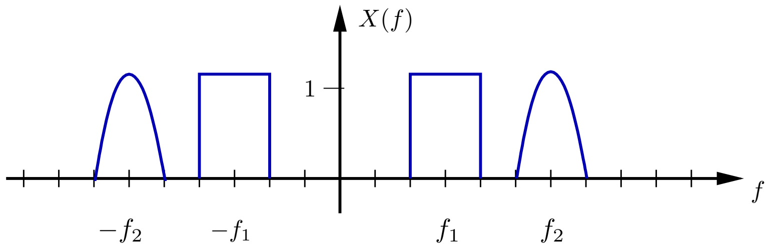
(d) For comparison, determine $\bar{E}_s$ and D^2_{min} for a conventional 4-PSK constellation. Which of the two constellations is more energy-efficient?

Remark: Parts (c) and (d) can be solved independently from (a) or (b).

(10p)

Problem 3

Two users transmit at carrier frequencies $f_1 = 150\,\mathrm{MHz}$ and $f_2 = 300\,\mathrm{MHz}$, respectively. The bandpass signal $x(t) = x_1(t) + x_2(t)$ has the following frequency spectrum $X(f)$:



Before the signal of user 2 is digitized at the receiver, $x(t)$ is converted to an intermediate frequency as follows:

$$y(t) = x(t) \cdot \cos\big(2\pi\, f_3\, t\big)\ .$$

(a) Draw the spectrum $Y(f)$ of the signal $y(t)$ if $f_3 = 250\,\mathrm{MHz}$ is chosen.

(b) Assume now that $f_3 = 350\,\mathrm{MHz}$ is chosen instead. Explain why both choices of f_3 are valid for the considered signal $x(t)$.

Consider now a 2-ray multi-path channel with impulse response

$$h(t) = \sum_{i=1}^{2} \alpha_i\, \delta(t - \tau_i)\ , \quad \text{where } \alpha_1 = 1, \alpha_2 = -0.5, \tau_1 = 0\,\mu\mathrm{s}, \tau_2 = 2\,\mu\mathrm{s}\ .$$

Binary antipodal signaling with rectangular pulse $g_{rec}(t)$ of amplitude A and duration $T = 3\,\mu\mathrm{s}$ is used for transmission over this channel.

(c) What is the smallest symbol time T_s for which no overlap of signal alternatives will occur after the channel?

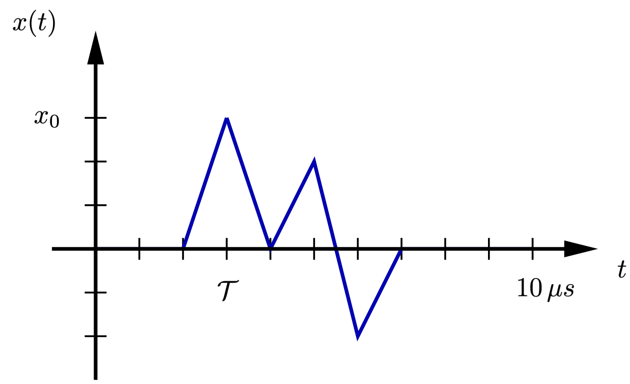
(d) Your task is to implement an ML receiver for the system by means of a matched filter. Draw the impulse response $v(t)$ of the matched filter.

Remark: Parts (c) and (d) can be solved independently from (a) and (b).

(10p)

Problem 4

Consider a communication system employing 8-PAM modulation with equally likely signal alternatives. The combination of the transmit pulse $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$ can be written as $x(t) = g(t) * h(t) * v(t)$. The signal is sampled in the receiver at time instants $\mathcal{T} + i\,T_s$, $i = 0, 1, 2, \ldots$.



(a) Assume that $T_s = 1\,\mu s$ and draw the discrete impulse response $x[i]$. Explain why ISI occurs in this case.

(b) For the case $T_s = 1\,\mu s$, give an example of an amplitude sequence $A[i]$ for which the worst case ISI occurs. Is there a risk for erroneous decisions without noise?

(c) Determine the largest bit rate R_b that can be achieved without ISI.

(d) Let us now assume that the bit rate determined in (c) is too small, and we have to tolerate some ISI. Which value of T_s do you choose? Explain why.

(10p)

Problem 5

Consider a communication system with binary antipodal baseband PAM signaling and a triangular pulse shape $g(t) = g_{tri}(t)$ with amplitude A and duration T. The signal alternatives are transmitted with equal probability and the system bandwidth W is measured by the main-lobe.

A requirement is that the bit error probability P_b satisfies $P_b \leqslant 10^{-7}$.

(a) Determine the maximum bandwidth efficiency ρ that can be achieved if the signal-to-noise ratio at the receiver is equal to $\mathcal{SNR}_r = 1.35$.

(b) Under the conditions in (a), the bit time T_b must be larger than the pulse duration T. What is the ratio T/T_b?

(c) Assume now that we want to hide the transmitted signal in the noise. In order to achieve this we need to reduce the pulse amplitude A such that $R(f) \leqslant N_0/2$ for all frequencies f.

How much do we need to reduce the ratio T/T_b in this case? Determine the corresponding value of $\mathcal{SNR}_r$ and a suitable choice of the amplitude A.

(10p)