Answers & Hints to exam in the course Digital Communications (ETT051), October 23, 2008, 14-19.

Problem 1.

a)

True since $W_{lobe} = 5R_b/k = 1/2$ MHz.

b)

True since $W_{99} = 2.36R_b/k = R_b/3.39$.

c)

True since $P_s \approx 4Q(\sqrt{d^2_{min}\mathcal{E}_b/N_0}) = 4Q(\sqrt{(24/255)448.906}) = 4Q(6.5)$.

d)

False since they have the same bandwidth efficiency.

e)

True since $Q(\sqrt{2 \cdot 46.77}) = Q(9.67) > 1.5 \cdot 10^{-22}$.

Problem 2.

$P_b = Q(1.6) = Q(\sqrt{d^2 \cdot 4.7543})$, so $d^2 = 0.5385$. At 17.174 dB we get instead that $P_b = Q(5.3) = 5.79 \cdot 10^{-8}$. This choice of signal alternatives is $10log_{10}(2/0.5385) = 5.7$ dB worse than antipodal signals.

$\mathcal{E}_b = (A^2T_b + A^2xT_b)/2$ and $D^2 = A^2T_b(1-x)$. Hence, $d^2 = D^2/(2\mathcal{E}_b) = (1-x)/(1+x) = 0.5385$ which gives $x = 0.3$.

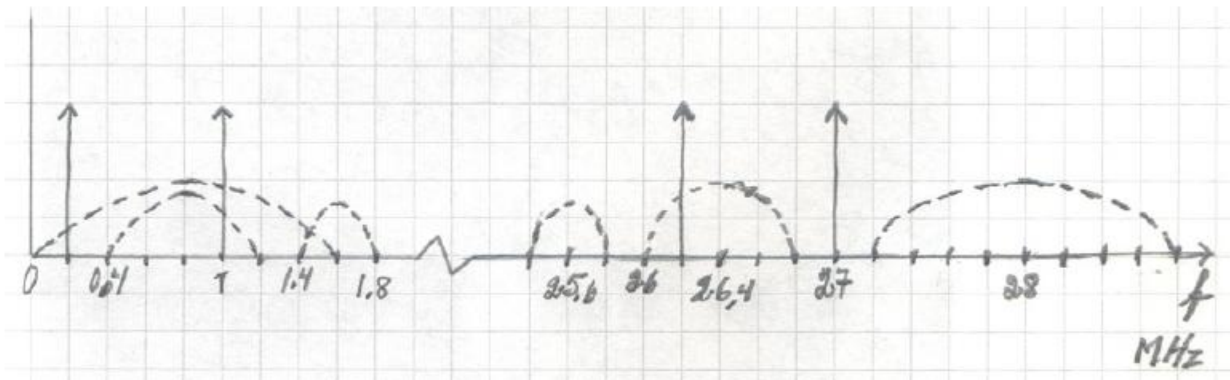
Problem 3.

i)+ii): Since $W_{lobe} = 3/T$ we find that $T_s = 5 \cdot 10^{-6}$. Furthermore, the error probability requirement implies that $d^2_{\text{min}}\mathcal{E}_b/N_0 \geq 36$, which for M-ary PSK also can be formulated as, $2T_s sin^2(\pi/M)P_z/N_0 \geq 36$.
Hence, for $M = 8$ we obtain that $P_z/N_0 \geq 2.46 \cdot 10^7$.
For general M, $P_z/N_0 \geq 3.6 \cdot 10^6/sin^2(\pi/M)$, and it is seen that the energy efficiency is decreased as M is increased.
iii) See the compendium.

Problem 4.

a)

i)+ii): The figure below shows a sketch of the different components that contribute to the frequency content in $y(t)$ (symmetry around $f = 0$).



iii) If $f_4 = 14.4$ MHz then only the signal $u_3(t)$ will be present at the output of the low-pass filter, provided that the bandwidth of the low-pass filter is 800 kHz.

b)

i) $D_{0,1}^2 = 16E_g$, $D_{0,2}^2 = 64E_g$, $D_{1,2}^2 = 16E_g$. So, $D_{min}^2 = 16E_g$ and $D_1^2 = 4D_{min}^2$.
The energies in the signal alternatives are $E_0 = 16E_g$, $E_1 = 64E_g$ and $E_2 = 144E_g$, implying that $E_z = 224E_g/3$.
We therefore have that $D_{min}^2/(2N_0) = (3/28)(E_z/N_0)$ and $D_1^2/(2N_0) = (3/7)(E_z/N_0)$.
The union bound is: $(4/3)Q(\sqrt{(3/28)(E_z/N_0)}) + (2/3)Q(\sqrt{(3/7)(E_z/N_0)})$.
ii) If instead, e.g., the amplitudes are $(-\sqrt{112}, 0, \sqrt{112})$ then E_z is the same, but the Euclidean distances are significantly increased which will decrease the union bound.

Problem 5.

i) The ML receiver correlates the received noisy signal $r(t)$ with the signal $(z_1(t) - z_0(t))$, and the result is compared with the threshold $(E_{z_1} - E_{z_0})/2$. The signal $z_1(t) = -z_0(t - 0.4T_b)$ implying that the threshold is 0 since $E_{z_1} = E_{z_0}$.

The signal $z_0(t)$ is equal to:
αA within the time interval $0 < t < 0.2T_b$,
$(\alpha + \beta)A$ within the time interval $0.2T_b < t < 0.4T_b$,
βA within the time interval $0.4T_b < t < 0.6T_b$,
and equal to 0 otherwise.

ii) It is found that:
$D^2_{z_0,z_1} = (\alpha^2 A^2 T_b/5)(1 + 3(1 + \beta/\alpha)^2 + (\beta/\alpha)^2)$
$\mathcal{E}_b = E_{z_0} = (\alpha^2 A^2 T_b/5)(1 + (1 + \beta/\alpha)^2 + (\beta/\alpha)^2)$
$\beta = 0$: $P_b = Q(\sqrt{d^2_{z_0,z_1} 9.12}) = Q(\sqrt{9.12}) = Q(3.02)$
$\beta = -\alpha/2$: $P_b = Q(\sqrt{(2/3) \cdot 9.12}) = Q(2.47)$
$\beta = \alpha/2$: $P_b = Q(\sqrt{(8/7) \cdot 9.12}) = Q(3.23)$