Answers to exam in Digital Communications (ETT051) on October 29, 2014, 14-19.

1.

a) True since $W_{lobe} = \frac{4}{T} = \frac{16}{3T_s} = \frac{16}{15} R_b = 202.7$ kHz.

b) True since $\mathcal{E}$ is the same and

$$d^2_{min,16-QAM} = 0.8 > d^2_{min,16-PSK} = 0.3$$

c) True since

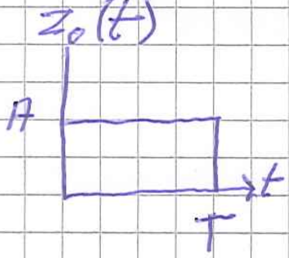
$$P_s \approx c\,Q\left(\sqrt{d^2_{min}\frac{\mathcal{E}_b}{N_0}}\right) = \frac{14}{8} Q\left(\sqrt{\frac{18}{63} \cdot 47.915}\right) = \frac{14}{8} Q(3.7) \approx \overbrace{1.89 \cdot 10^{-4}}$$

d) False since if $P_0 \neq P_1$ then the MAP receiver is better than the ML receiver.

e) False since

$$E_b = \frac{A^2 T}{3}, \quad P = \frac{1}{T_b}\frac{A^2 T}{3} = \frac{3A^2}{10} \neq \frac{3A^2}{5}$$

2.



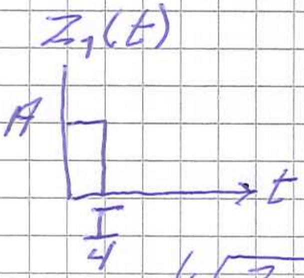
$$\frac{A^2T}{N_0} = 30.42$$

i)

$$z_1(t) = 2z_0(t) \Rightarrow D^2 = A^2T$$

$$P_b = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q(3.9) = 4.81\cdot10^{-5}$$

ii)



$$\Rightarrow D^2 = A^2\frac{3T}{4}$$

$$P_b = Q\left(\sqrt{\frac{A^2 3T}{2N_0 4}}\right) = Q(3.378) \approx 3.37\cdot10^{-4}$$

iii)

Pair i): $E_b = \frac{A^2T + 4A^2T}{2} = \frac{5A^2T}{2} \Rightarrow d_i^2 = 1/5$

Pair ii): $E_b = \frac{A^2T + A^2T/4}{2} = \frac{5}{8}A^2T \Rightarrow d_{ii}^2 = 3/5$

Pair ii) is 4.77 dB better than pair i)

3

a)

$$T + 4\cdot10^{-6} \le T_s = 10\cdot10^{-6} \Rightarrow T \le 6\cdot10^{-6}$$

Test $g_{rc}(t)$: $W_{99.9} = \frac{3.46}{T} \le 10^{6} \Rightarrow T \ge 3.46\cdot10^{-6}$

So, $g_{rc}(t)$ is suitable if $3.46\mu s \le T \le 6\mu s$

b)

i) Please see the compendium.

ii) $$Z_i(t) = A_i\, g(t) \Rightarrow D^2_{i,j} = (A_i - A_j)^2 E_g$$

i	A_i
0	2
1	1
2	0
3	-1

$D^2_{0,1} = E_g$, $D^2_{0,2} = 4E_g$, $D^2_{0,3} = 9E_g$

$D^2_{1,2} = E_g$, $D^2_{1,3} = 4E_g$, $D^2_{2,3} = E_g$

$\therefore$ $D^2_{min} = E_g$, $D^2_1 = 4E_g$, $D^2_2 = 9E_g$

$$\mathcal{E}_b = \frac{3E_g}{4}, \quad \mathcal{E}_b/N_0 = 13.5$$

$\Rightarrow d^2_{min} = 2/3$, $d^2_1 = 8/3$, $d^2_2 = 6$

$c_0 = (1+2+2+1)/4 = 1.5$, $c_1 = (1+1+1+1)/4 = 1$

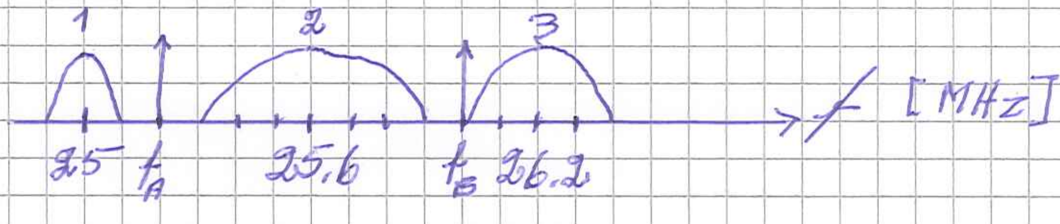
$c_2 = (1+0+0+1)/4 = 0.5$

union bound $= 1.5\,Q(3) + Q(6) + 0.5\,Q(9) \approx 2.03\cdot10^{-3}$

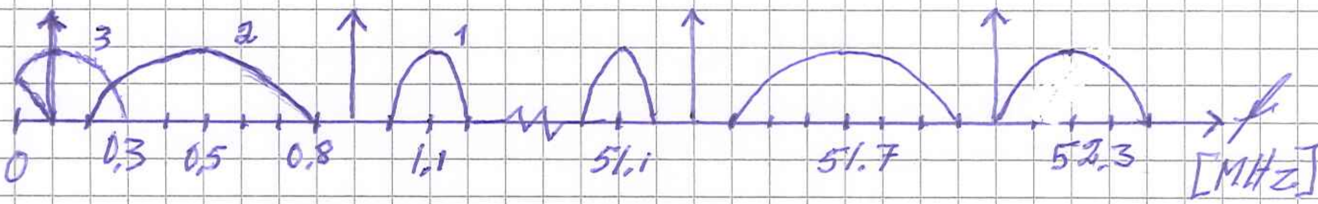
4.

a)

i) The frequency content in $r(t)$ is symmetric around $f=0$.



ii) The frequency content in $y(t)$ is symmetric around $f=0$.



iii) $W_{filter} = 200\text{ kHz}$

b)

i) Please see the compendium.

ii) Let us first correlate $r(t)$ with $g(t)$ and denote the result with C, i.e.

$$C = \int_0^{T_s} r(t)g(t)\,dt$$

z_i is then obtained by

$$z_i = A_i C - E_i/2 \;,\; i=0,1,\ldots,M-1$$

5.

$\rho \geq 4.5$, $T = T_s$, $\frac{\mathcal{E}_b}{N_0}$ close to 35.5 dB

Let us test with $g_{ncs}(t)$. $\Rightarrow W_{qq} = \frac{2.36}{T_s}$

$$\rho = \frac{R_b}{W_{qq}} = \frac{k}{2.36} \geq 4.5 \Rightarrow k \geq 10.62$$

Let us consider 4096-QAM:

$$P_s \leq 4Q\left(\sqrt{d_{min}^2 \frac{\mathcal{E}_b}{N_0}}\right) \leq 7.596 \cdot 10^{-8} = 4 \cdot Q(5.5)$$

$$\frac{3\log_2(M)}{M-1} \frac{\mathcal{E}_b}{N_0} \geq 5.5^2$$

$$\Rightarrow \frac{\mathcal{E}_b}{N_0} \geq 3.441 \cdot 10^3 \quad (\approx 35.37\,dB)$$

$\therefore$ $g_{ncs}(t)$ and 4096-QAM

satisfies the requirements.