Answers & Hints to the exam in the course Digital Communications (ETT051), October 20, 2009, 08-13.

Problem 1.

a) FALSE, see on-off signal pairs.

b) FALSE, since $W_{lobe} = 3/T = 640$ kbps.

c) TRUE, since $d_{min}^2 = 2sin^2(\pi/64)log_2(64) \approx 2.89 \cdot 10^{-2}$ and $P_s \approx 2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0}) = 2Q(5.63) \approx 2 \cdot 10^{-8}$.

d) TRUE, since QPSK has twice as high bandwidth efficiency as BPSK, and both methods have the same d_{min}^2.

e) FALSE, see M-FSK.

Problem 2.

$P_b = Q(\sqrt{D^2/(2N_0)}) \leq 10^{-12} = Q(7.0345)$. $2/T = 1.6 \cdot 10^6$.

It is found that $D_{z_0,z_1}^2 = \alpha^2 E_g = \alpha^2 A^2 T$.

So, $D^2/(2N_0) = \alpha^2 A^2 T/(2N_0) \geq 7.0345^2$, which leads to that $A^2/N_0 \geq 7.92 \cdot 10^7/\alpha^2$.

$\mathcal{E}_b = (E_{z_0} + E_{z_1})/2 = \alpha^2 E_g 5/2$

$d^2 = \alpha^2 E_g/(\alpha^2 E_g 5) = 0.2$ (Independent of pulse shape.)

Therefore, the energy efficiency is 10 dB worse than antipodal signal alternatives.

$d^2 \mathcal{E}_b/N_0 = d^2 P_z/(R_b N_0) \geq 7.0345^2$ leads to that $P_z/N_0 \geq 1.48 \cdot 10^8$ (Independent of pulse shape.)

If $g_{hcs}(t)$ with amplitude A and duration T_1 is used:

The same requirement on the ratio P_z/N_0 as above.

$D_{z_0,z_1}^2 = \alpha^2 E_g = \alpha^2 A^2 T_1/2$, and $3/T_1 = 1.6 \cdot 10^6$ leads to that $A^2/N_0 \geq 7.0345^2 \cdot 1.6 \cdot 10^6 \cdot 4/(3\alpha^2) = 1.06 \cdot 10^8/\alpha^2$.

Problem 3.

$d_{min}^2 \mathcal{E}_b/N_0 = 6log_2(M)/(M^2-1) \cdot P_z/(R_b N_0) = 6/(M^2-1) \cdot T_s P_z/N_0$.

If $M = 8$ it is given that $6/63 \cdot T_s P_z/N_0 = 169.03^2$, which implies that $T_s P_z/N_0 = 3 \cdot 10^5$.

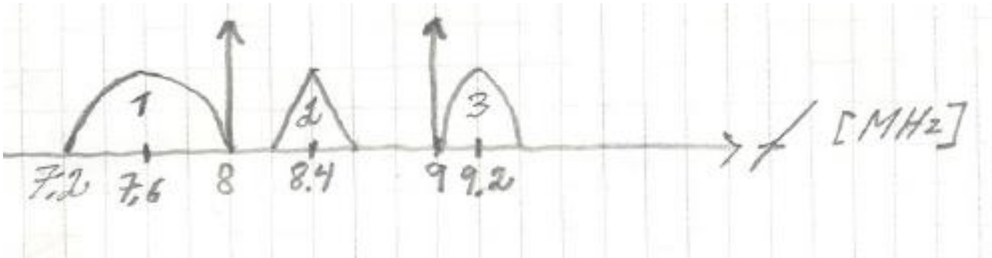
The bit error probability requirement leads to that: $6/(M^2-1) \cdot T_s P_z/N_0 \geq 6.706^2$ from which we obtain that $M \leq 200.07$. Hence, $M = 128$ and $R_b = 7/T_s$.

M-QAM has higher d_{min}^2 than M-PAM.

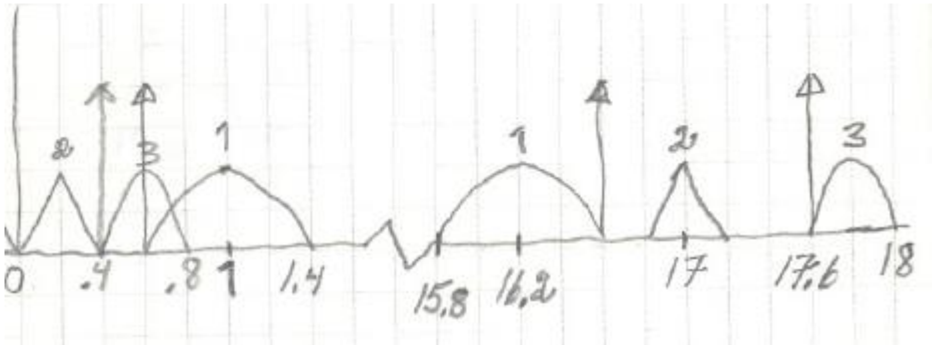
Problem 4.

a)

r(t): (Symmetry around f=0)



y(t): (Symmetry around f=0)



The choice of 8.6 MHz in the receiver is a bad choice. It should be 7.6, 8.4 or 9.2 (MHz), then the corresponding user signal will be found around the frequency f=0.

b) See the compendium.

Problem 5.

a)

$D_{0,1}^2 = 4E$. All the other squared Euclidean distances $= 2E = D_{min}^2$.
$c = ((2+3+3+2)/4 = 2.5$, and $c_1 = (1+0+0+1)/4 = 1/2$.
Hence, the union bound $= 2.5Q(\sqrt{E/N_0}) + 0.5Q(\sqrt{2E/N_0})$.
At high signal-to-noise ratios the union bound is very good and it is then approximatly equal to $cQ(\sqrt{D_{min}^2/(2N_0)})$. At low signal-to-noise ratios the union bound can be much larger than the true P_s, making it less useful.

b)

We can use the correlation receiver in figure 4.10 in the compendium with:
$z_1(t) - z_0(t) = 2\alpha g_{rec}(t) - \alpha g_{rec}(t) = \alpha g_{rec}(t)$.
Integration time $= T = 2/(1.6 \cdot 10^6)$,
and decision threshold $= (E_{z_1} - E_{z_0})/2 = 3\alpha^2 A^2 T/2$.