Answers to exam in Digital Communications
ETTO51, October 24, 2016.

1.

a) True since,

$$W = \frac{W_{tube}}{2} = \frac{3}{2T} = \frac{12}{2T_s} = \frac{12}{10} R_b \Rightarrow \rho = \frac{R_b}{W} = \frac{5}{6} \text{ bps/Hz.}$$

b) True since,

M^2-QAM and bandpass M-PAM have the same $d^2_{min} = \frac{6\log_2(M)}{M^2-1}$, but

$$\rho_{M^2\text{-QAM}} = 2\log_2(M)\rho_{BPSK} > \rho_{M\text{-PAM}_{bp}} = \log_2(M)\rho_{BPSK}$$

c) False since,

$$P_s \approx 3.75\, Q\left(\sqrt{d^2_{min} \frac{\mathcal{E}_b}{N_0}}\right) = 3.75 \cdot Q(6.5) =$$
$$= 1.5 \cdot 10^{-10} \neq 1.9 \cdot 10^{-10}$$

d) False since,

$$\bar{P} = R_b \bar{E}_b = R_b \frac{4E_g + 9E_g}{2} = R_b \frac{13}{2} \cdot \frac{3}{8} A^2 \frac{3T_b}{8} =$$
$$= \frac{117}{64} A^2 = 1.8281 A^2 \neq 1.757 A^2.$$

e) False since,

the duration T of a PPM signal is at most $\frac{T_s}{M}$. So, if M is large then T is small, and this implies a broad (large) frequency content.

2.

i) $D^2 = \int_0^T (4A-A)^2 dt = 9A^2 T \Rightarrow P_b = Q\left(\sqrt{\frac{9A^2T}{2N_0}}\right)$

ii) $D^2 = \int_0^T (z_0(t) - z_1(t))^2 dt = \int_0^{8T/15} (A+3A)^2 dt + \int_{8T/15}^{T} A^2 dt =$

$= 16A^2 \frac{8T}{15} + A^2 \frac{7T}{15} = 9A^2 T \Rightarrow P_b = Q\left(\sqrt{\frac{9A^2T}{2N_0}}\right)$

iii)

Pair 1: $\mathcal{E}_b = \frac{A^2T + 16A^2T}{2} \Rightarrow d^2 = \frac{9A^2T}{17A^2T} = 9/17$

Pair 2: $\mathcal{E}_b = \frac{A^2T + 9A^2 8T/15}{2} = A^2T \frac{29}{10} \Rightarrow d^2 = 45/29$

Pair 2 is $10\log_{10} \frac{45/29}{9/17} = 4.67$ dB better.

iv)

$\frac{\mathcal{E}_b}{N_0} = 15.8489 = \frac{17}{2}\frac{A^2 T_b}{N_0}$

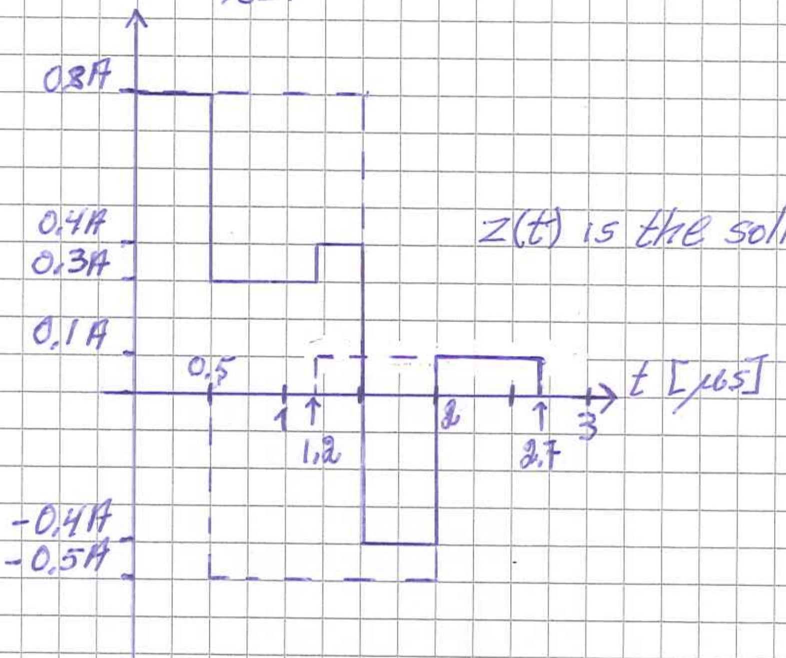
$\Rightarrow R_b = \frac{17}{2} \cdot \frac{3.16227 \cdot 10^6}{15.8489} = 1.696$ Mbps

$P_b = Q\left(\sqrt{\frac{9}{17} \cdot 15.8489}\right) = Q(2.897) \approx 1.87 \cdot 10^{-3}$

3

a)

i) $z(t) = \sum_{i=1}^{3} \alpha_i \, g_{rec}(t - \tau_i)$



z(t) is the solid line.

ii)

$$W = \frac{W_{lobe}}{2} = \frac{3}{2T} = 6 \cdot 10^{6} \Rightarrow T = 0.25 \mu s$$

$$T_s = 4T_b \geq T + 1.2 \cdot 10^{-6} = 1.45 \mu s \quad \text{(no overlap)}$$

$$\Rightarrow R_b \leq \frac{4}{1.45} \text{ Mbps} = 2.7586 \text{ Mbps}$$

b)

i) union bound $= c\, Q\left(\sqrt{d_{min}^2 \, \mathcal{E}_b/N_0}\right) + \ldots + c_{max} \, Q\left(\sqrt{d_{max}^2 \, \mathcal{E}_b/N_0}\right)$

If $\mathcal{E}_b/N_0 \approx 0 \Rightarrow Q\left(\sqrt{d^2 \mathcal{E}_b/N_0}\right) \approx 0.5$

$\Rightarrow$ union bound $\approx (c + \ldots + c_{max}) 0.5 = \frac{M-1}{2} = 7.5 \gg 1$,

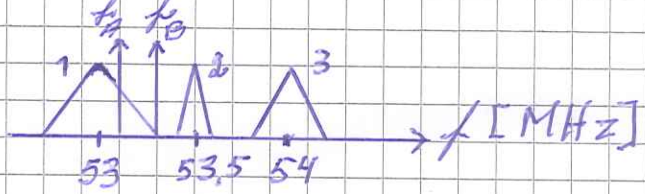
Hence, at such small values of $\mathcal{E}_b/N_0$ the union bound is not useful.

ii) Please see pages 440, 442 and 446.

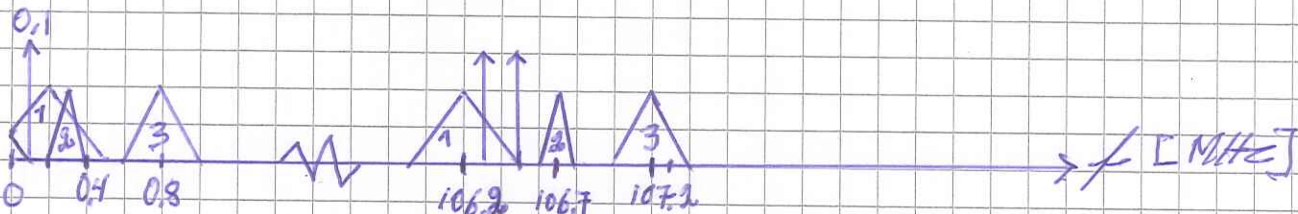
4

a)

i) Frequency content in $r(t)$ (symmetric around $f=0$):



ii) Frequency content in $y(t)$ (symmetric around $f=0$):



For clarity, overlapping contributions have not been added.

iii) $f_1 = f_2 = 53.5$ MHz since then user 2 is obtained around $f=0$ in $y(t)$.

b) $D_{min}^2 = D_{0,1}^2 = E_g$, $D_1^2 = D_{1,2}^2 = 4E_g$, $D_2^2 = D_{2,3}^2 = D_{0,2}^2 = 9E_g$

$D_3^2 = D_{1,3}^2 = 25E_g$, $D_4^2 = D_{0,3}^2 = 36E_g$

$c_{min} = (1+1)/4 = 1/2$, $c_1 = (1+1)/4 = 1/2$, $c_2 = (1+2+1)/4 = 1$,

$c_3 = (1+1)/4 = 1/2$, $c_4 = (1+1)/4 = 1/2$.

$E_b = (E_g + 4E_g + 16E_g + 49E_g)/8 = 70E_g/8$

$\Rightarrow d_{min}^2 = \frac{E_g}{70E_g/4} = \frac{4}{70}$, $d_1^2 = 4d_{min}^2$, $d_2^2 = 9d_{min}^2$, $d_3^2 = 25d_{min}^2$,

$d_4^2 = 36d_{min}^2$. c_i and d_i are independent of the pulse used.

The current system has a much smaller d_{min}^2 compared with conventional 4-PAM ($d_{min}^2 = 0.8$).

5. $W_{lobe} = \frac{4}{T} = \frac{16}{3T_s} = 3 \cdot 10^6 \Rightarrow T_s = \frac{16}{9} \cdot 10^{-6}$

$P_b \le P_s \le 2Q\left(\sqrt{d_{min}^2 \mathcal{E}_b/N_0}\right) \le 2 \cdot 10^{-6}$

$d_{min}^2 \frac{\mathcal{E}_b}{N_0} = 2\log_2(M)\sin^2(\pi/M)\frac{P_Z}{R_b N_0} \ge 4.7534^2$

$$\frac{P_Z}{N_0} \ge \frac{4.7534^2 \cdot 10^6}{16/9} \cdot \frac{1}{2\sin^2(\pi/M)} = \begin{cases} 12.71 \cdot 10^6, & M=4 \\ 43.39 \cdot 10^6, & M=8 \\ 167 \cdot 10^6, & M=16 \\ 661.5 \cdot 10^6, & M=32 \end{cases}$$

i) $\frac{P_Z}{N_0} = 30 \cdot 10^6 \Rightarrow M=4$, $\rho = \frac{k}{T_s W} = \frac{2}{16/3} = \frac{3}{8}$ bps/Hz in the interval $12.71 \cdot 10^6 \le \frac{P_Z}{N_0} < 43.39 \cdot 10^6$.

ii) $\frac{P_Z}{N_0} = 6 \cdot 10^8 \Rightarrow M=16$

$\Rightarrow P_s \le 2Q\left(\sqrt{d_{min}^2 \frac{P_Z}{R_b N_0}}\right) = 2Q(9.01) = 2.26 \cdot 10^{-19}$

iii) Bandpass M-PAM

$\Rightarrow \frac{6}{M^2-1} \cdot \frac{T_s P_Z}{N_0} \ge 4.7534^2$, $\frac{P_Z}{N_0} = 30 \cdot 10^6$

$\Rightarrow M < 3.89 \Rightarrow M=2 \Rightarrow \rho = 3/16$ bps/Hz

∴ No, in this case $\rho = 3/16$ which is smaller than was obtained in i) for QPSK.