Answers to exam in Digital Communications, ETT051, October 28, 2015.

1

a) True since $W_{lobe} = \frac{4}{T} = \frac{8}{k} R_b$

$R_b = \frac{10}{8} \cdot 2 \cdot 10^6 = 2.5$ Mbps

b) False since $\rho_{16QAM} = 4 \cdot \rho_{BPSK}$ and

$\rho_{8PSK} = 3 \rho_{BPSK}$

c) False since $P_s \approx 2\, Q\left(\sqrt{d_{min}^2\, \mathcal{E}_b/N_0}\right)$

where $d_{min}^2 = 8 \sin^2(\pi/16) = 0.30448$

$P_s \approx 2 \cdot Q(5.3) \approx 1.158 \cdot 10^{-7}$

d) Assuming equally likely signals, the statement is False. The power spectral density results, and the symbol error probability results, that we have used in this course do not depend on how the k bits are mapped to the $M = 2^k$ signals.

e) False since $P_s \approx 10^{-18}$ implies roughly 10^{18} simulated symbols per symbol error. To get statistical significance several symbol errors should be simulated. Hence, the simulation will take a very long time, at each $\mathcal{E}_b/N_0$.

$\therefore$ NOT practical nor efficient.

2

i) $D^2 = \frac{9A^2T}{4}$, $\frac{A^2}{N_0} = 1.9953\cdot10^8$

$$\frac{D^2}{2N_0} = \frac{9A^2}{8N_0}\,0.6\,T_b = 5.1993^2$$

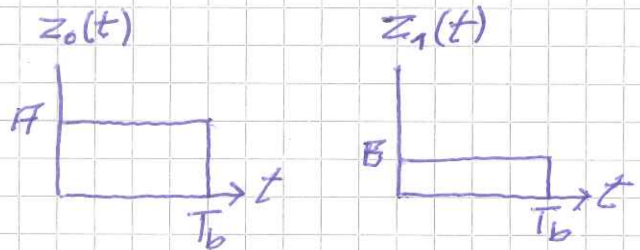
$\Rightarrow R_b = 4.9822$ Mbps

ii) $$E_b = \frac{A^2T + A^2T/4}{2} = \frac{5A^2T}{8}$$

$$\Rightarrow d^2 = \frac{D^2}{2E_b} = 9/5$$

i) is $10\log(9/5) = 2.5527$ dB better.

iii) Additional info to this part:
d^2_{min} in this part should not be smaller than in i).



$$\frac{D^2}{2N_0} = \frac{(A-B)^2T_b}{2N_0} = 5.1993^2$$

$$R_b = \frac{A^2}{N_0}\frac{(1-B/A)^2}{2\cdot 5.1993^2}$$

There are several possible answers:

e.g. $B = -A \Rightarrow R_b = 2\frac{A^2}{N_0}\cdot\frac{1}{27.033} = 14.762$ Mbps $(d^2_{0,1} = 2)$

$B = -2A \Rightarrow R_b = \frac{9}{2}\frac{A^2}{N_0}\cdot\frac{1}{27.033} = 33.214$ Mbps $(d^2_{0,1} = 9/5)$

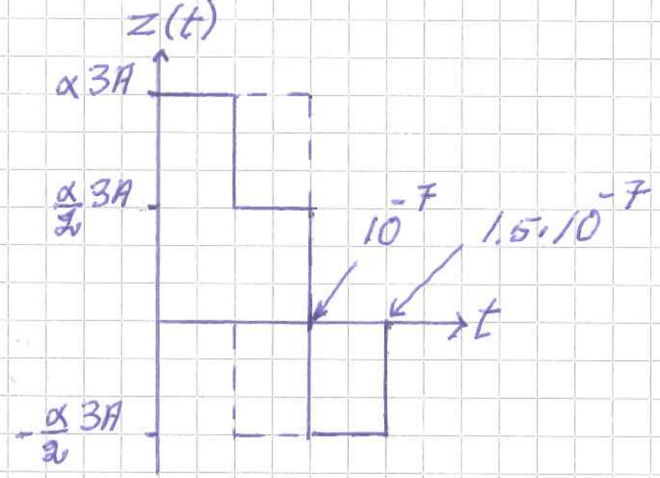
3

a)

Let $s(t)$ denote input and $z(t)$ output, then

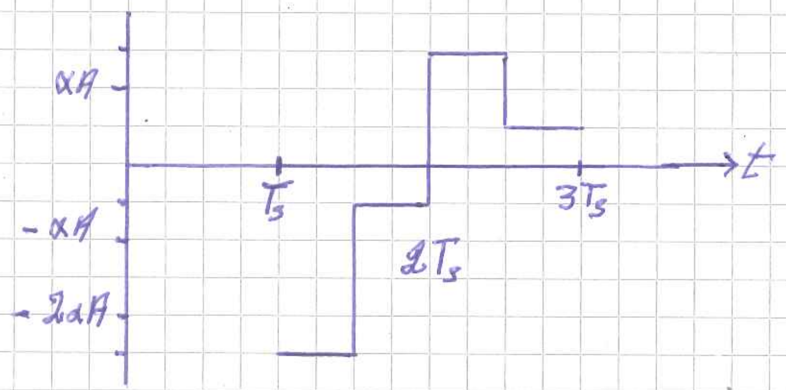
$$z(t) = \alpha s(t) - \frac{\alpha}{2} s\left(t - \frac{10^{-7}}{2}\right).$$

i) $s(t) = 3g(t)$



ii) $z(t)$ in $T_s \le t \le 3T_s$ means that only the (first) three pulses need to be considered.

It is found that:



There is signal overlap.

Increasing T_s to $1.5 \cdot 10^{-7}$ removes the overlap.

3

b)

i) $\rho = \frac{R_b}{W}$ is small when M is large

$d_{min}^2 = \log_2(M)$ is high — " —

$\therefore$ $\begin{cases} \text{Low bit rate applications (to keep } W \text{ small!)} \\ \text{Low } \mathcal{E}_b/N_0 \text{ applications} \end{cases}$

ii) All signal alternatives are orthogonal.

Assume $E_0 = E_1 = E$ and $E_2 = E_3 = E/4$

$D_{01}^2 = 2E$ $\quad D_{12}^2 = 5E/4$ $\quad D_{23}^2 = E/2$

$D_{02}^2 = 5E/4$ $\quad D_{13}^2 = 5E/4$

$D_{03}^2 = 5E/4$

$D_{min}^2 = E/2$, $D_1^2 = 5E/4$, $D_2^2 = 2E$

$\mathcal{E}_b = 5E/16$

$\Rightarrow d_{min}^2 = 0.8$, $d_1^2 = 2$, $d_2^2 = 3.2$

$c_0 = \frac{1}{4}(0+0+1+1) = 1/2$

$c_1 = \frac{1}{4}(2+2+2+2) = 2$

$c_2 = \frac{1}{4}(1+1+0+0) = 1/2$

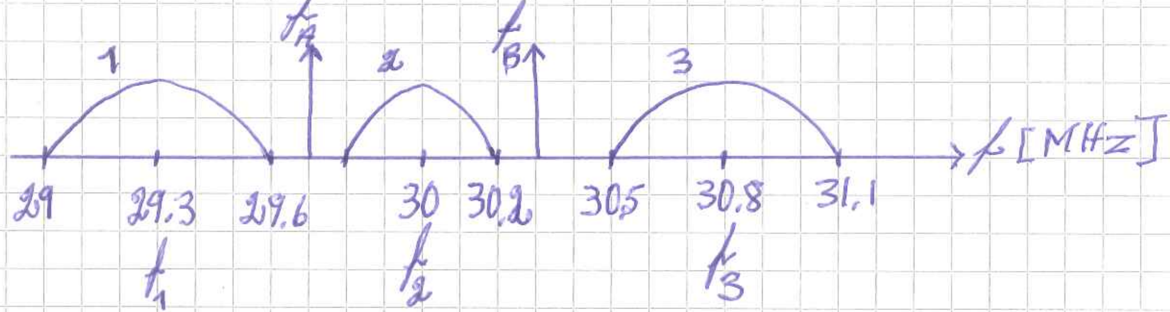
union bound $= 0.5\, Q\left(\sqrt{0.8\, \mathcal{E}_b/N_0}\right) + 2\, Q\left(\sqrt{2\, \mathcal{E}_b/N_0}\right) +$
$+ 0.5\, Q\left(\sqrt{3.2\, \mathcal{E}_b/N_0}\right)$

At larger $\mathcal{E}_b/N_0$: $P_s \approx c \cdot Q\left(\sqrt{d_{min}^2\, \mathcal{E}_b/N_0}\right)$

d_{min}^2 is also an energy-efficiency parameter.

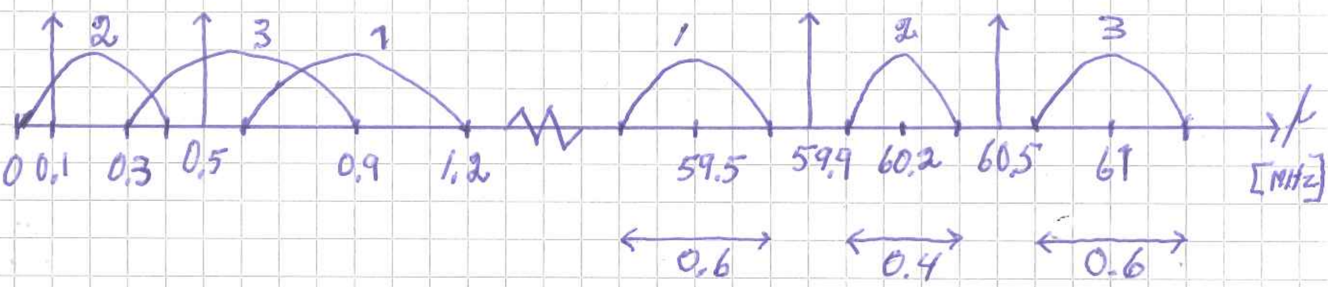
4.

a) i)



There is symmetri around $f = 0$.

ii)



There is symmetri around $f = 0$.

iii) Since $f_4 = f_3$, $W_{lp} = 300$ kHz.

b)

i) ISI_{wc} can be reduced by, e.g.,

- increasing T_s (e.g., $T_{s,new} = I \cdot T_s$, I = integer)
- decreasing M and keeping T_s fixed.

An additional possibility is to change $g(t)$ and $v(t)$ to try to obtain a better $x(t)$.

ii)

$$\rho = \frac{R_b}{W} = 2 \; , \; SNR_r = \rho \cdot \frac{\mathcal{E}_b}{N_0} = 14.501$$

$$\Rightarrow \frac{\mathcal{E}_b}{N_0} = 7.25 \quad (8.6\,\text{dB})$$

$P_b = 10^{-5}$ cannot be obtained for M-PSK at $\frac{\mathcal{E}_b}{N_0}$ 8.6 dB.

So, measurement errors have been made, i.e, errors in the test procedure and/or test equipment.

The person is correct.

5.

$W_{99.9} = \frac{c}{T_s} = 8 \cdot 10^6$, c will be found below.

$$\frac{P_Z}{N_0} = 2.2336 \cdot 10^9 \Rightarrow \text{new } M$$

$$P_b \leq P_s = 2\,Q\left(\sqrt{d_{min}^2 \frac{E_b}{N_0}}\right) \leq 2 \cdot 10^{-9}$$

$$d_{min}^2 \frac{E_b}{N_0} \geq \lambda = 5.9978^2 = 35.974$$

$$\frac{6 \log_2(M)}{M^2-1} \cdot \frac{P_Z}{R_b N_0} = \frac{6}{M^2-1} \frac{T_s P_Z}{N_0} \geq 35.974$$

$$\frac{P_Z}{N_0} \geq \frac{35.974 \cdot 8 \cdot 10^6 (M^2-1)}{6 \cdot c}$$

$$M^2-1 \leq \frac{2.2336 \cdot 10^9 \cdot 6c}{35.974 \cdot 8 \cdot 10^6} = \begin{cases} 290.58 \ (\text{rm}) \\ 255.19 \ (\text{hcs}) \\ 161.12 \ (\text{rc}) \end{cases}$$

So, $M^2-1 = 255 \Rightarrow M = 16$, $c = 5.48$ (hcs)

$$\Rightarrow T_s = \frac{5.48}{8 \cdot 10^6} = 0.685\ \mu s = k\,T_b$$

$$\Rightarrow R_b = \begin{cases} 5.839 \text{ Mbps}, & M = 16 \\ 4.3796 \text{ Mbps}, & M = 8 \end{cases}$$

When M is changed P_s is close to $2 \cdot 10^{-9}$, and P_s is then decreased if $\frac{P_Z}{N_0}$ increases, until M is changed again which increases P_s to close to $2 \cdot 10^{-9}$.