Kösystem 28 maj 2018

Uppgift 1

a)

$$\begin{tikzcd} 0 \arrow[r, bend left, "10"] & 1 \arrow[l, bend left, "5"] \arrow[r, bend left, "10"] & 2 \arrow[l, bend left, "10"] \arrow[r, bend left, "10"] & 3 \arrow[l, bend left, "20"] \end{tikzcd} \Rightarrow \begin{cases} p_0 = \frac{1}{6} \\ p_1 = \frac{2}{6} \\ p_2 = \frac{2}{6} \\ p_3 = \frac{1}{6} \end{cases}$$

$$\boxed{P(\text{spärr}) = p_3 = \frac{1}{6}}$$

b)

$$\lambda_{serv} = 5 \cdot p_1 + 10 \cdot (p_2 + p_3) = \frac{40}{6} \ s^{-1} =$$

$$= \frac{40}{6} \cdot 3600 \ h^{-1} = \boxed{24\,000 \ h^{-1}}$$

c)

$$E(N) = 1 \cdot p_1 + 2 \cdot p_2 + 3 \cdot p_3 = \frac{9}{6}$$

$$\lambda_{eff} = 10 \cdot (1 - p_3) = \frac{50}{6}$$

$$E(T) = \frac{E(N)}{\lambda_{eff}} = \frac{9}{50} = \boxed{0{,}18 \ s}$$

d)

$$\frac{\lambda_{serv}}{\lambda_{eff}} = \boxed{\frac{4}{5} = 0{,}8}$$

e) Markovkedjan blir:

$$\begin{tikzcd} 0 & 1 \arrow[l, "5"] & 2 \arrow[l, "10"] & 3 \arrow[l, "20"] \end{tikzcd}$$

Medeltiden från 3 till 0 är $\frac{1}{20} + \frac{1}{10} + \frac{1}{5} = \boxed{0{,}35 \ s}$

Uppgift 2

a)

$$\begin{tikzcd} 0 \arrow[r, bend left, "20"] & 1 \arrow[l, bend left, "5"] \arrow[r, bend left, "15"] & 2 \arrow[l, bend left, "5"] \arrow[r, bend left, "10"] & 3 \arrow[l, bend left, "5"] \arrow[r, bend left, "5"] & 4 \arrow[l, bend left, "5"] \end{tikzcd} \Rightarrow \begin{cases} p_0 = 1/65 \\ p_1 = 4/65 \\ p_2 = 12/65 \\ p_3 = 24/65 \\ p_4 = 24/65 \end{cases}$$

$$E(N) = \sum_{i=1}^{4} i \cdot p_i = \boxed{\frac{196}{65} \approx 3{,}0}$$

b) $$\lambda_{eff} = 20p_0 + 15p_1 + 10p_2 + 5p_3 = \frac{320}{65}$$

$$E(T) = \frac{E(N)}{\lambda_{eff}} = \boxed{\frac{196}{320} = \frac{49}{80} \approx 0{,}61 \text{ s}}$$

c) Ankomstintensitet $= 20$ i tillstånd $0 \Rightarrow \boxed{E(\text{Idle}) = \frac{1}{20} \text{ s}}$

d) $$\frac{E(\text{Idle})}{E(\text{Busy}) + E(\text{Idle})} = p_0 \Rightarrow E(\text{busy}) = \frac{1-p_0}{p_0} E(\text{Idle}) =$$

$$= \frac{64/65}{1/65} \cdot \frac{1}{20} = \boxed{\frac{64}{20} = 3{,}2 \text{ s}}$$

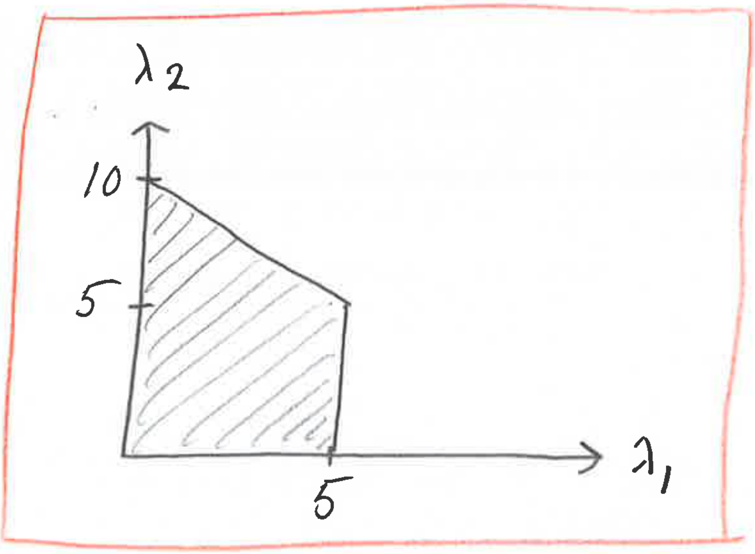
e) Fler buffertplatser än kunder $\Rightarrow P(\text{spärr}) = 0$

Uppgift 3

a) Villkoren:

$$\begin{cases} \lambda_1 < 5 \\ \lambda_2 < 10 \\ 0{,}5\,\lambda_1 < 10 \\ 0{,}5\,\lambda_1 + \lambda_2 < 10 \\ \lambda_1 + \lambda_2 < 10 \end{cases}$$

ger



b) $E(N_1) = \dfrac{\lambda_1}{\mu_1 - \lambda_1} = \dfrac{4}{5-4} = 4$

$E(N_2) = \dfrac{5}{10-5} = 1$

$E(N_3) = \dfrac{2}{10-2} = \dfrac{1}{4}$

$E(N_4) = \dfrac{7}{10-7} = \dfrac{7}{3}$

$E(N_5) = \dfrac{9}{10-9} = 9$

$E(T) = \dfrac{\sum E(N_i)}{\lambda_1 + \lambda_2} \approx \boxed{1{,}84\ s}$

Uppgift 3 forts.

c) Två vägar $1 \to 4 \to 5$ och $2 \to 4 \to 5$

$$E(T_{145}) = \frac{N_1}{\lambda_1} + \frac{N_4}{\lambda_4} + \frac{N_5}{\lambda_5} = \frac{7}{3}$$

$$E(T_{245}) = \frac{N_2}{\lambda_2} + \frac{N_4}{\lambda_4} + \frac{N_5}{\lambda_5} = \frac{23}{15}$$

$$\lambda_{145} = 2$$
$$\lambda_{245} = 5$$

Medeltiden blir $= \dfrac{\lambda_{145}}{\lambda_{145}+\lambda_{245}} \cdot E(T_{145}) +$

$$+ \frac{\lambda_{245}}{\lambda_{145}+\lambda_{245}} \cdot E(T_{245}) = \boxed{\frac{37}{21} \approx 1{,}76 \text{ s}}$$

d) T_b = tid i buffert, T_s = tid i betjäning

$$E(T_b) = E(T) - E(T_s) =$$

$$= E(T) - \frac{\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5}{\lambda_1 + \lambda_2} \approx \boxed{1{,}50 \text{ s}}$$

Uppgift 4

a)
$$\begin{cases} \lambda_1 = 10 + 0{,}75\,\lambda_4 \\ \lambda_2 = \lambda_1 + \lambda_3 \\ \lambda_3 = 0{,}5\,\lambda_2 \\ \lambda_4 = 0{,}5\,\lambda_2 \end{cases} \Rightarrow \begin{cases} \lambda_1 = 40 \\ \lambda_2 = 80 \\ \lambda_3 = 40 \\ \lambda_4 = 40 \end{cases}$$

Använder man att $E(N) = \dfrac{\lambda}{\mu - \lambda}$ så får man

$$\boxed{\begin{cases} E(N_1) = 4 \\ E(N_2) = 2 \\ E(N_3) = 2 \\ E(N_4) = 2 \end{cases}}$$

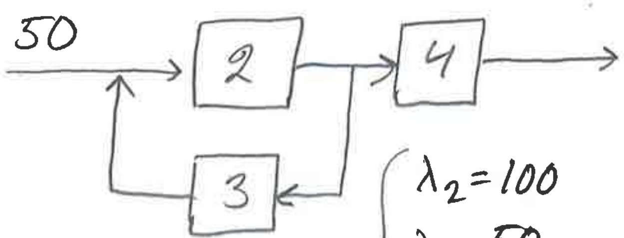
b)
$$E(T_2) = \frac{E(N_2)}{\lambda_2} = \frac{2}{80} = \frac{1}{40}$$

Totala tiden i nod 2 $= \dfrac{\lambda_2}{\lambda} \cdot E(T_2) = \boxed{\dfrac{80}{10} \cdot \dfrac{1}{40} = 0{,}2\,s}$

c) $E(\text{antal besök i nod } k) = \dfrac{\lambda_k}{\lambda} \Rightarrow$

$$E(\text{antal betjäningar}) = \sum_{i=1}^{4} \frac{\lambda_i}{\lambda} = 20$$

d) Nod 1 överbelastas och blir en källa med utintensitet $50\ s^{-1}$. Nätet ser ut så här:



$$\begin{cases} \lambda_2 = 100 \\ \lambda_3 = 50 \\ \lambda_4 = 50 \end{cases} \Rightarrow \boxed{\begin{aligned} E(N_2) &= \frac{100}{120-100} = 5 \\ E(N_3) &= \frac{50}{60-50} = 5 \\ E(N_4) &= E(N_3) = 5 \end{aligned}}$$

Uppgift 5

a) Man kan se detta som ett M/G/1-system där tiden för betjäning har täthetsfunktionen:

$$f(t) = 0{,}5\,\delta(t-0{,}02) + 0{,}5\,\delta(t-0{,}06)$$

$$E(x) = 0{,}5\cdot 0{,}02 + 0{,}5\cdot 0{,}06 = 0{,}04 \Rightarrow \rho = \lambda E(x) = 0{,}8$$

$$E(x^2) = 0{,}5\cdot 0{,}02^2 + 0{,}5\cdot 0{,}06^2 = 0{,}002$$

$$E(N) = \rho + \frac{\lambda^2 E(x^2)}{2(1-\rho)} = 0{,}8 + \frac{400\cdot 0{,}002}{2(1-0{,}8)} = \boxed{2{,}8}$$

b) E(tid i system för kund av typ A) =

= E(betjäningstid för kund av typ A) + E(tid i buffert) =

$$= 0{,}02 + \frac{E(N_q)}{\lambda} = 0{,}02 + \frac{E(N)-\rho}{\lambda} =$$

$$= 0{,}02 + \frac{2}{20} = \boxed{0{,}12\text{ s}}$$

c) $E(\text{Idle}) = \frac{1}{20}$ (dvs $\frac{1}{\lambda}$)

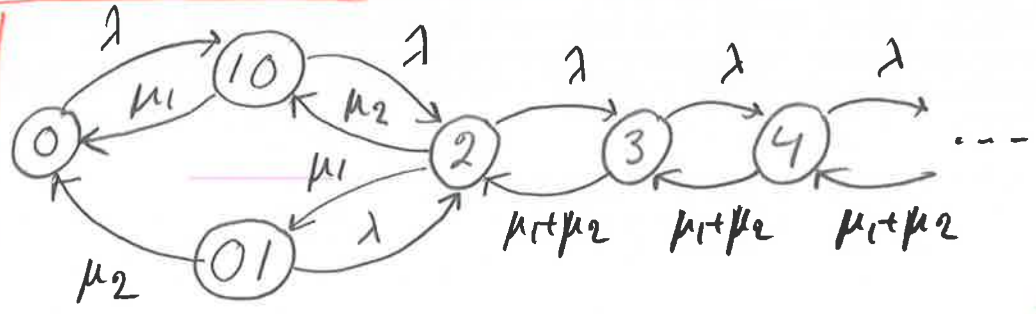
$$\frac{E(\text{Busy})}{E(\text{Busy}) + E(\text{Idle})} = \rho \Rightarrow E(\text{Busy}) = \frac{0{,}8}{1-0{,}8}\,E(\text{Idle}) =$$

$$= 4\cdot\frac{1}{20} = \boxed{0{,}2\text{ s}}$$

Uppgift 6

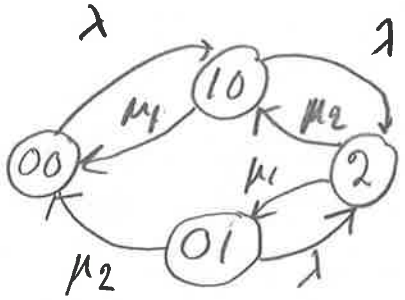
a) $\boxed{\lambda < \mu_1 + \mu_2}$

b)



c)

Markovkedjan blir



$$\Longrightarrow \begin{cases} p_{00} = 7/22 \\ p_{10} = 5/22 \\ p_{01} = 4/22 \\ p_2 = 6/22 \end{cases}$$

I' processor 1: $\mu_1 p_{10} + \mu_1 p_2 = 1$

I processor 2: $\mu_2 p_{01} + \mu_2 p_2 = \frac{10}{22} \approx 0{,}45$

d) $E(N) = 1 \cdot (p_{10} + p_{01}) + 2 \cdot p_2 = \frac{21}{22}$

$\lambda_{eff} = \lambda(1 - p_2) = \frac{32}{22}$

$$\boxed{E(T) = \frac{E(N)}{\lambda_{eff}} = \frac{21}{32} \approx 0{,}66}$$

Uppgift 6 forts.

e) $T_i = E$ (tiden innan bägge arbetar för första gången om man är i tillstånd i)

$$\begin{cases} T_{00} = \frac{1}{\lambda} + T_{10} \\ T_{10} = \frac{1}{\lambda+\mu_1} + \frac{\mu_1}{\lambda+\mu_1} T_{00} + \frac{\lambda}{\lambda+\mu_1} T_2 \\ T_2 = 0 \end{cases}$$

$\Longrightarrow$

$$\boxed{T_0 = \frac{3}{2} = 1{,}5 \text{ s}}$$