EITF75 Systems and Signals

Lecture 4
More about the z-transform;
Poles and zeros

Fredrik Rusek

EITF75, z-transform

Analyzing a general difference equation (at rest)

Expression for general difference equation (at rest)

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

EITF75, z-transform

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Solution for general difference equation (at rest)

EITF75, z-transform

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$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

Solution for general difference equation (at rest)

Step 1:

Change y(n-k) to $z^{-k}Y(z)$

Step 2:

Change x(n-k) to $z^{-k}X(z)$

EITF75, z-transform

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Expression for general difference equation (at rest)

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

Solution for general difference equation (at rest)

$$Y(z)+\sum_{k=1}^{N} a_k z^{-k} Y(z)=\sum_{k=0}^{M} b_k z^{-k} X(z)$$

Step 1:

Change y(n-k) to $z^{-k} Y(z)$

Step 2:

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Step 1:

Change y(n-k) to z^{-k} Y(z)

Step 2:

Change x(n-k) to z^{-k} X(z)

Step 3:

Express Y(z) as H(z)X(z)

EITF75, z-transform

Analyzing a general difference equation (at rest)

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Solution for general difference equation (at rest)

$$Y(z) + \sum_{k=1}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}} \cdot X(z)$$

Step 1:

Change $y(n-k)$ to $z^{-k} Y(z)$

Step 2:

Change $x(n-k)$ to $z^{-k} X(z)$

Step 3:

Express $Y(z)$ as $H(z)X(z)$

EITF75, z-transform

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$H(z)$

Step 1:

Change y(n-k) to z^{-k} Y(z)

Step 2:

Change x(n-k) to z^{-k} X(z)

Step 3:

Express Y(z) as H(z)X(z)

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$$Y(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}} \cdot X(z)$$

$$H(z)$$

Step 4:

Find the roots of the denominator and nominator of H(z). Roots should be in terms of z, not z^{-1}

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Analyzing a general difference equation (at rest)

Expression for general difference equation

$$y(n)+\sum_{k=1}^{N}a_k y(n-k)=\sum_{k=0}^{M}b_k x(n-k)$$

Step 4:

Find the roots of the denominator and nominator of H(z). Roots should be in terms of z, not z^{-1}

Solution for general difference equation (at rest)

$$Y(z)+\sum_{k=1}^{N}a_k z^{-k}Y(z)=\sum_{k=0}^{M}b_k z^{-k}X(z)$$

$$Y(z)=\frac{b_0+b_1z^{-1}+\cdots+b_Mz^{-M}}{1+a_1z^{-1}+\cdots+a_Nz^{-N}}\cdot X(z)$$

$$Y(z)=\frac{z^{-M}}{z^{-N}}\,\frac{b_0z^M+b_1z^{M-1}+\cdots+b_M}{z^N+a_1z^{N-1}+\cdots+a_N}X(z)$$

$$H(z)$$

EITF75, z-transform

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Solution for general difference equation (at rest)

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$$Y(z)=\frac{b_0+b_1z^{-1}+\cdots+b_Mz^{-M}}{1+a_1z^{-1}+\cdots+a_Nz^{-N}}\cdot X(z)$$

Step 4:

Find the roots of the denominator and nominator of H(z). Roots should be in terms of z, not z^{-1}

$$Y(z)=\frac{z^{-M}}{z^{-N}}\frac{b_0z^{M}+b_1z^{M-1}+\cdots+b_M}{z^{N}+a_1z^{N-1}+\cdots+a_N}X(z)$$

$$=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$$

$H(z)$

Any difference equation at rest can be written on this form

EITF75, z-transform

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Expression for general difference equation

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

Solution for general difference equation (at rest)

$$Y(z)+\sum_{k=1}^{N} a_k z^{-k} Y(z)=\sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z)=\frac{b_0+b_1 z^{-1}+\cdots+b_M z^{-M}}{1+a_1 z^{-1}+\cdots+a_N z^{-N}}\cdot X(z)$$

$H(z)$

Step 4:

Find the roots of the denominator and nominator of H(z). Roots should be in terms of z, not z^{-1}

$$Y(z)=\frac{z^{-M}}{z^{-N}}\frac{b_0 z^M+b_1 z^{M-1}+\cdots+b_M}{z^N+a_1 z^{N-1}+\cdots+a_N}X(z)$$

zeros

$$=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$$

poles

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Expression for general difference equation

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

$$Y(z)=\frac{z^{-M}}{z^{-N}}\underbrace{\frac{b_0 z^M+b_1 z^{M-1}+\cdots+b_M}{z^N+a_1 z^{N-1}+\cdots+a_N}}_{H(z)}X(z)$$

$$=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$$

zeros

poles

Example

$$H(z)=\frac{B(z)}{A(z)}=\frac{z^{-1}-z^{-2}}{1-1.27z^{-1}+0.81z^{-2}}$$

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Expression for general difference equation

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

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zeros

$$=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$$

poles

Example

$$H(z)=\frac{B(z)}{A(z)}=\frac{z^{-1}-z^{-2}}{1-1.27z^{-1}+0.81z^{-2}}$$

Convention: This is the notation for the transfer function H(z)

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Expression for general difference equation

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

$$Y(z) = \boxed{\frac{z^{-M}}{z^{-N}} \frac{b_0 z^M + b_1 z^{M-1} + \cdots + b_M}{z^N + a_1 z^{N-1} + \cdots + a_N}} X(z)$$

$$= b_0 \cdot \frac{z^{-M}}{z^{-N}} \cdot \frac{(z - z_1) \cdots (z - z_M)}{(z - p_1) \cdots (z - p_N)} \cdot X(z)$$

zeros

poles

Example

$$H(z) = \frac{B(z)}{A(z)} = \frac{z^{-1} - z^{-2}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

$$= \frac{z-1}{z^2 - 1.27z + 0.81}$$

On the above form

EITF75, z-transform

Expression for general difference equation

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$

$Y($

Step 4:

Find the roots of the denominator and nominator of H(z). Roots should be in terms of z, not z^{-1}

Example

$$H(z)=\frac{B(z)}{A(z)}=\frac{z^{-1}-z^{-2}}{1-1.27z^{-1}+0.81z^{-2}}$$

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zeros

$$= b_0 \cdot \frac{z^{-M}}{z^{-N}} \cdot \frac{(z - z_1) \cdots (z - z_M)}{(z - p_1) \cdots (z - p_N)} \cdot X(z)$$

poles

Example

$$H(z) = \frac{B(z)}{A(z)} = \frac{z^{-1} - z^{-2}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

$$= \frac{z - 1}{z^2 - 1.27z + 0.81}$$

poles

$$z = \frac{1.27}{2} \pm \sqrt{\left(\frac{1.27}{2}\right)^2 - 0.81}$$

$$= 0.9e^{\pm j\frac{\pi}{4}}$$

A complex conjugated pair: FPE type II

EITF75, z-transform

Expression for general difference equation

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

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zeros

$$= b_0 \cdot \frac{z^{-M}}{z^{-N}} \cdot \frac{(z - z_1)\cdots(z - z_M)}{(z - p_1)\cdots(z - p_N)} \cdot X(z)$$

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Example

$$H(z) = \frac{B(z)}{A(z)} = \frac{z^{-1} - z^{-2}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

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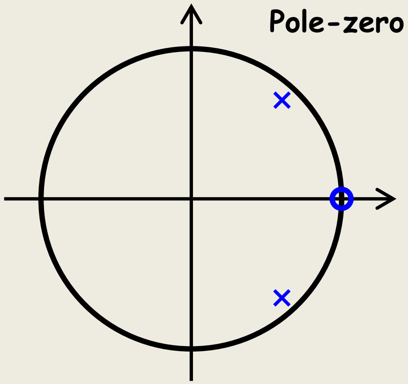
poles $z = \frac{1.27}{2} \pm \sqrt{\left(\frac{1.27}{2}\right)^2 - 0.81}$

$= 0.9e^{\pm j\frac{\pi}{4}}$

zeros $z = 1$

EITF75, z-transform



Filter response	When...
$H(z) = 0$	z is a zero.
$H(z) = \infty$	z is a pole.
$H(z) \approx 0$	z is close to a zero.
$H(z) \gg 1$	z is close to a pole.

Useful to show behavior of a filter. More details once we reach Fourier transforms

Example

$$H(z) = \frac{B(z)}{A(z)} = \frac{z^{-1} - z^{-2}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

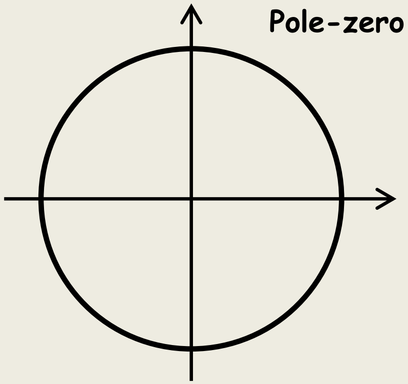
$$= \frac{z-1}{z^2 - 1.27z + 0.81}$$

poles $\quad z = \frac{1.27}{2} \pm \sqrt{\left(\frac{1.27}{2}\right)^2 - 0.81}$

$$= 0.9e^{\pm j\frac{\pi}{4}}$$

zeros $\quad z = 1$

EITF75, z-transform



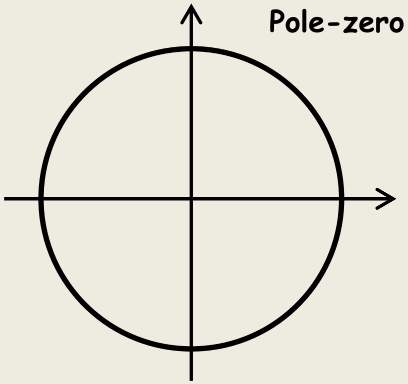
Pole-zero plot

Example

$$H(z) = 1 - z^{-1}$$

FIR filter

EITF75, z-transform

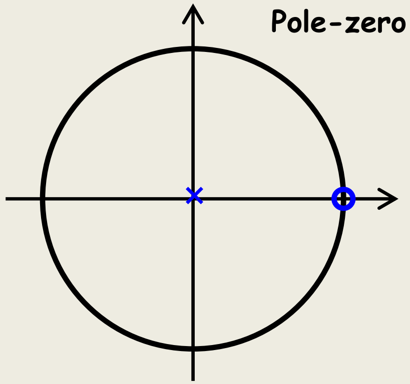


Example

$$H(z) = 1 - z^{-1}$$

$$= \frac{z-1}{z}$$

EITF75, z-transform



Pole-zero plot

The 2-tap FIR filter has 1 zero and 1 pole at z=0

Example

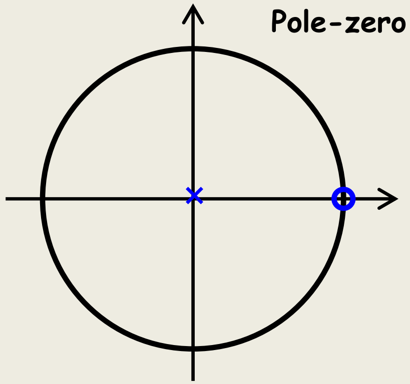
$$H(z) = 1 - z^{-1}$$

$$= \frac{z-1}{z}$$

poles $z = 0$

zeros $z = 1$

EITF75, z-transform



Pole-zero plot

The 2-tap FIR filter has 1 zero and 1 pole at z=0

Homework: Give general rule for L-tap FIR filter

Example

$$H(z) = 1 - z^{-1}$$

$$= \frac{z-1}{z}$$

poles $z = 0$

zeros $z = 1$

EITF75, z-transform

Status check

EITF75, z-transform

Status check

- We have the difference equation $\boxed{y(n)+\sum_{k=1}^{N}a_k y(n-k)=\sum_{k=0}^{M}b_k x(n-k)}$ at rest

EITF75, z-transform

Status check

- We have the difference equation $$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$$ at rest
- We can find the transform of y(n) as $$Y(z)=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$$

EITF75, z-transform

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- Assume that $\boxed{X(z)=\frac{N(z)}{Q(z)}}$

EITF75, z-transform

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- We have the difference equation $y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$ at rest
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- Assume that $X(z)=\frac{N(z)}{Q(z)}$
- We get that $Y(z)=\frac{\text{Some polynomial in } z}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$

$q_1,\dots,q_L$ poles of Q(z)

EITF75, z-transform

Status check

- We have the difference equation $y(n)+\sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$ at rest
- We can find the transform of y(n) as $Y(z) = b_0 \cdot \frac{z^{-M}}{z^{-N}} \cdot \frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)} \cdot X(z)$
- Assume that $X(z) = \frac{N(z)}{Q(z)}$
- We get that $Y(z) = \frac{\text{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$
- Partial fraction expansion $Y(z) = \sum_{k=1}^{N} \frac{A_k}{1-z^{-1}p_k} + \sum_{k=1}^{L} \frac{Q_k}{1-z^{-1}q_k}$

Assuming all poles are real and distinct

EITF75, z-transform

Status check

- We have the difference equation $y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$ at rest
- We can find the transform of y(n) as $Y(z) = b_0 \cdot \frac{z^{-M}}{z^{-N}} \cdot \frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)} \cdot X(z)$
- Assume that $X(z) = \frac{N(z)}{Q(z)}$
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NOTE the form of the desired PFE

EITF75, z-transform

Status check

- We have the difference equation $y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{M} b_k x(n-k)$ at rest
- We can find the transform of y(n) as $Y(z)=b_0\cdot\frac{z^{-M}}{z^{-N}}\cdot\frac{(z-z_1)\cdots(z-z_M)}{(z-p_1)\cdots(z-p_N)}\cdot X(z)$
- Assume that $X(z)=\frac{N(z)}{Q(z)}$
- We get that $Y(z)=\frac{\text{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$
- Partial fraction expansion $Y(z)=\sum_{k=1}^{N}\frac{A_k}{1-z^{-1}p_k}+\sum_{k=1}^{L}\frac{Q_k}{1-z^{-1}q_k}$
- Invert $y(n)=\sum_{k=1}^{N}A_k(p_k)^n u(n)+\sum_{k=1}^{L}Q_k(q_k)^n u(n)$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1)$$

$$x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z)$$

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1)$$

$$x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z)$$

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z) \qquad X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{1}{z - \frac{1}{2}} \frac{z}{z - \frac{1}{3}}$$

zero $z = 0$

poles $z = \frac{1}{3}$ $z = \frac{1}{2}$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z) \qquad X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{1}{z - \frac{1}{2}} \frac{z}{z - \frac{1}{3}} \overset{\text{PFE}}{=} \frac{A_1}{z - \frac{1}{2}} + \frac{Q_1}{z - \frac{1}{3}}$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z) \qquad X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}\frac{1}{1-\frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{1}{z-\frac{1}{2}}\frac{z}{z-\frac{1}{3}} \overset{\text{PFE}}{=} \frac{A_1}{z-\frac{1}{2}} + \frac{Q_1}{z-\frac{1}{3}}$$

Handpåläggning

$$A_1 = 3$$

$$Q_1 = -2$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z) \qquad X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{1}{z - \frac{1}{2}} \frac{z}{z - \frac{1}{3}} = \frac{3}{z - \frac{1}{2}} - \frac{2}{z - \frac{1}{3}}$$

EITF75, z-transform

Example

$$y(n) - \frac{1}{2}y(n-1) = x(n-1) \qquad x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = z^{-1}X(z) \qquad X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

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$$= \frac{3z^{-1}}{1 - \frac{1}{2}z^{-1}} - \frac{2z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

Checkpoint: Why did the z^{-1} appear in the nominators?

EITF75, z-transform

Example

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EITF75, z-transform

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EITF75, z-transform

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Observation: No matter the input X(z), this part always remains (with different scaling)

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Will die out as n grows: TRANSIENT

$$y(n) = 3\left(\frac{1}{2}\right)^{n-1}u(n-1) - 2\left(\frac{1}{3}\right)^{n-1}u(n-1)$$

EITF75, z-transform

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EITF75, z-transform

Because the input vanishes

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EITF75, z-transform

This one does not

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Coefficients in PFE may change

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This signal is always present in the output.

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This signal does not vanish for large n: Steady-state solution

Will die out as n grows: TRANSIENT

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If the poles of H(z) are less than 1 in magnitude, then output vanishes at large n, whenever the input does.

EITF75, z-transform

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If the poles of H(z) are less than 1 in magnitude, then output vanishes at large n,whenever the input does.
Putting money on the bank and collecting interest cannot be a system with poles less than 1 in magnitude

EITF75, z-transform

What remains to be done?

EITF75, z-transform

What remains to be done?

We need to study how to invert a complex conjugated pair

EITF75, z-transform

Compute transforms of these

$$h(n) = r^n \cdot \sin(\omega n) u(n)$$

$$h(n) = r^n \cdot \cos(\omega n) u(n)$$

EITF75, z-transform

Compute transforms of these

$$h(n) = r^n \cdot \sin(\omega n) u(n)$$

$$= r^n \cdot \frac{1}{2\mathrm{j}} \cdot \left(\mathrm{e}^{\mathrm{j}\omega n} - \mathrm{e}^{-\mathrm{j}\omega n}\right) u(n)$$

$$h(n) = r^n \cdot \cos(\omega n) u(n)$$

$$= r^n \cdot \frac{1}{2} \cdot \left(\mathrm{e}^{\mathrm{j}\omega n} + \mathrm{e}^{-\mathrm{j}\omega n}\right) u(n)$$

EITF75, z-transform

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$$H(z) = \frac{1}{2j} \sum_n (re^{j\omega})^n \cdot u(n)$$

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Two Geometric sums

$$h(n) = r^n \cdot \cos(\omega n) u(n)$$

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Two Geometric sums

EITF75, z-transform

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EITF75, z-transform

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EITF75, z-transform

Important transforms

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Sufficient to carry out inversion of complex conjugated pairs

EITF75, z-transform

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Sufficient to carry out inversion of complex conjugated pairs

Homework: Prove that all 2nd order polynomials with complex roots can be written in the form $1-2r\cos(\omega)z^{-1}+r^2z^{-2}$

That means that the denominators of H(z) above encompass all complex conjugated pairs that exist

EITF75, z-transform

Example A quite messy one: You will need to look at it at home as well

Invert $$H(z) = z^{-1} \cdot \frac{1 - z^{-1}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

EITF75, z-transform

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Condition for roots being a complex conjugated pair?

$$h(n) = r^n \cdot \sin(\omega n)u(n)$$

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That this number...

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Condition for roots being a complex conjugated pair?

Condition satisfied

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$$h(n) = r^n \cdot \cos(\omega n)u(n)$$

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The problem is harder than just identifying one of the below transforms

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$$h(n) = r^n \cdot \cos(\omega n)u(n)$$

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$$H(z) = \frac{1 - r\cos(\omega)z^{-1}}{1 - 2r\cos(\omega)z^{-1} + r^2z^{-2}}$$

EITF75, z-transform

Example A quite messy one: You will need to look at it at home as well

Invert $$H(z) = z^{-1} \cdot \frac{1 - z^{-1}}{1 - 1.27z^{-1} + 0.81z^{-2}}$$

The problem is harder than just identifying one of the below transforms

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EITF75, z-transform

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Of this form

$h(n) = r^n \cdot \sin(\omega n)u(n)$

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EITF75, z-transform

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"A scaling away" from being on this form

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EITF75, z-transform

Example A quite messy one: You will need to look at it at home as well

$$h(n) = r^{n-1} \cdot \left[\cos(\omega_0 \cdot (n-1)) + \frac{r\cos(\omega_0) - 1}{r\sin(\omega_0)} \cdot \sin(\omega_0 \cdot (n-1))\right] \cdot u(n-1)$$

$$= 0.9^{n-1} \cdot \left[\cos\left(\frac{\pi}{4} \cdot (n-1)\right) - 0.57\sin\left(\frac{\pi}{4} \cdot (n-1)\right)\right] \cdot u(n-1)$$

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EITF75, z-transform

Summary

A complex conjugated pair of poles is inverted to a damped/amplified sine/cosine

$$h(n) = r^n \cdot \sin(\omega n)u(n) \qquad h(n) = r^n \cdot \cos(\omega n)u(n)$$

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EITF75, z-transform

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A complex conjugated pair of poles is inverted to a damped/amplified sine/cosine

If the pole is outside the unit circle, r>1, the oscillation is amplified over time

If it is inside, r<1, it is attenuated

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Thus, a stable (=asymptotically vanishing) sequence has all poles inside the unit circle

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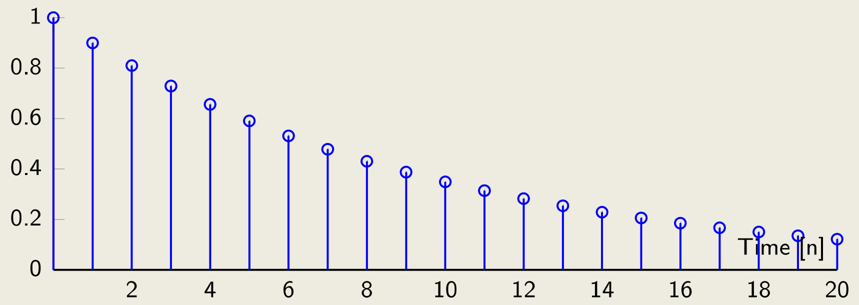
For real poles, same story: pole outside the unit circle gave unstable time-sequence

Thus, a stable (=asymptotically vanishing) sequence has all poles inside the unit circle

For a system to be BIBO stable, the transfer function must have all poles inside the unit circle

EITF75, z-transform

Draw the pole-zero plot



EITF75, z-transform

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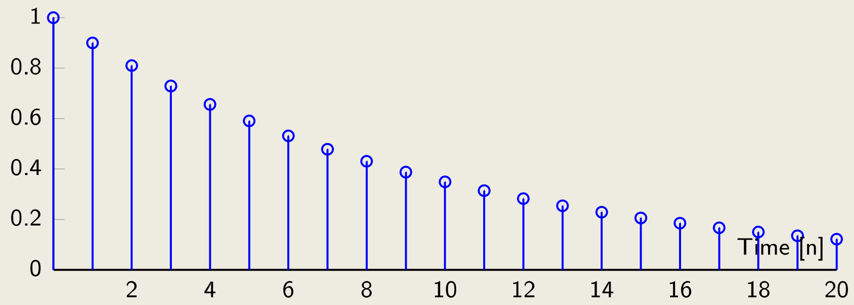
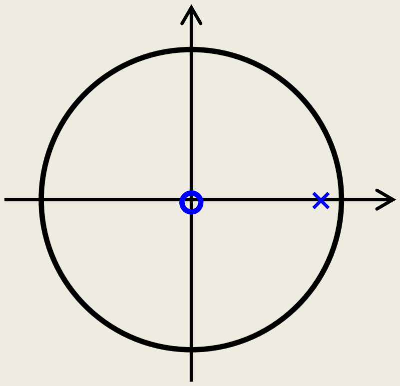
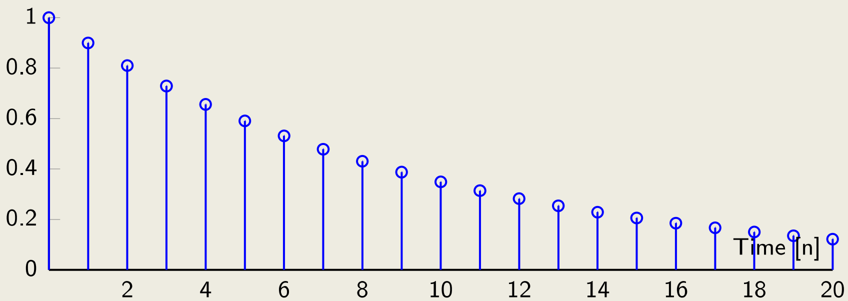


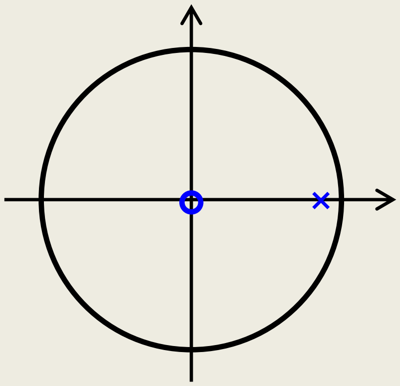


Stable -> pole inside
Non-oscillating -> not complex pair
Non-sign-alternating->positive pole

EITF75, z-transform

Draw the pole-zero plot





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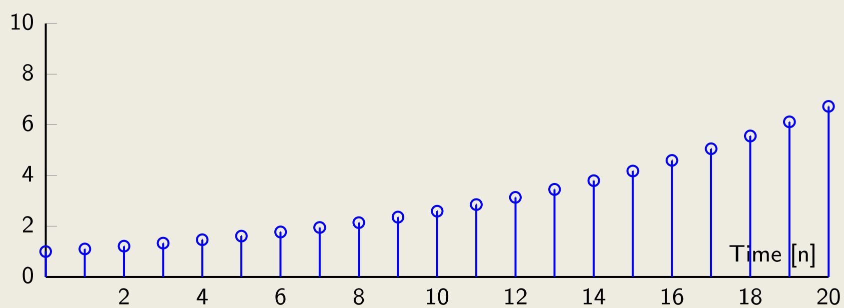
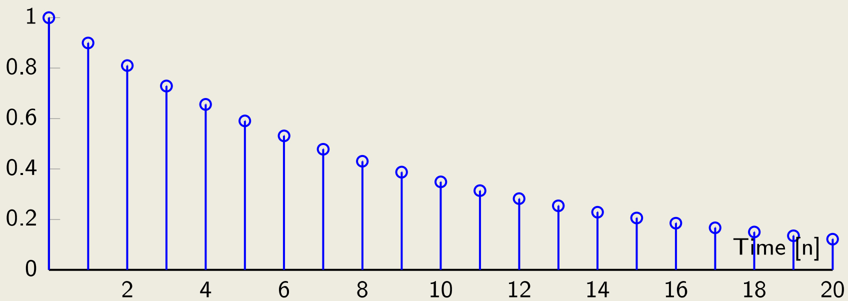
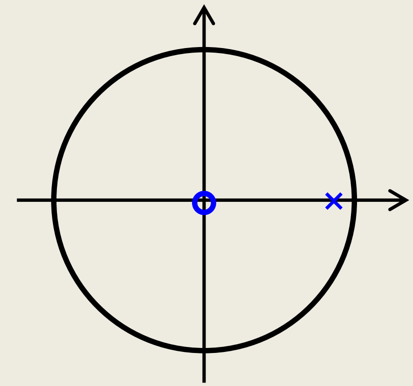


EITF75, z-transform

Draw the pole-zero plot





unStable -> pole outside
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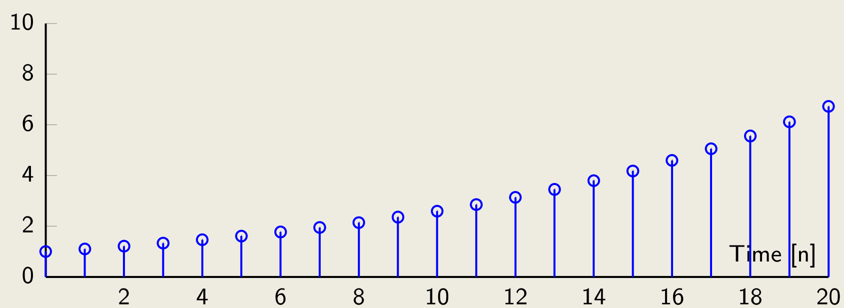
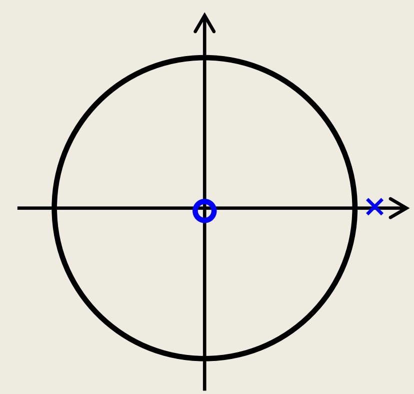
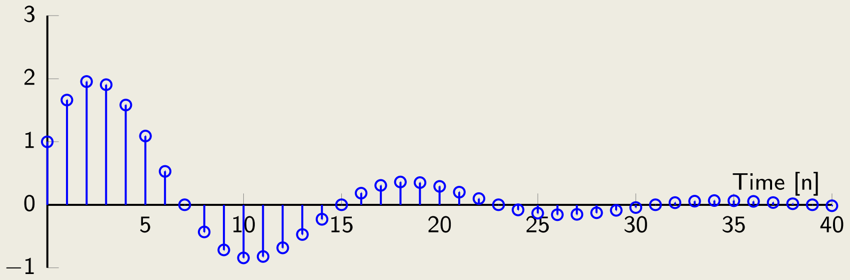
Non-sign-alternating->positive pole

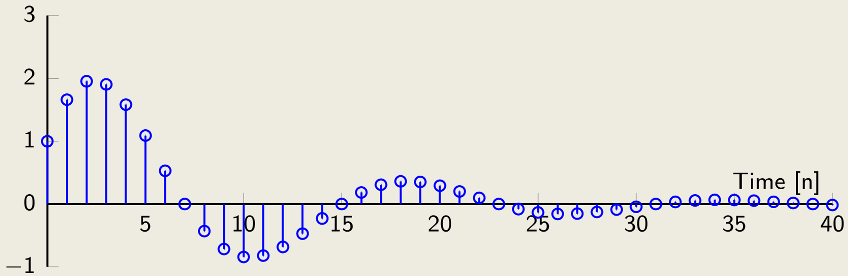


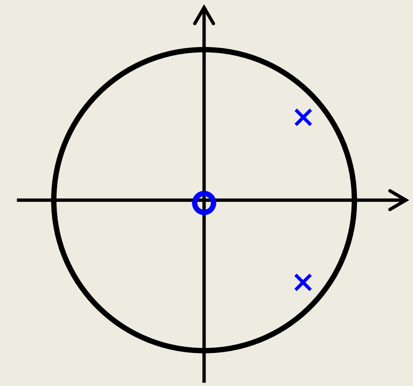


EITF75, z-transform



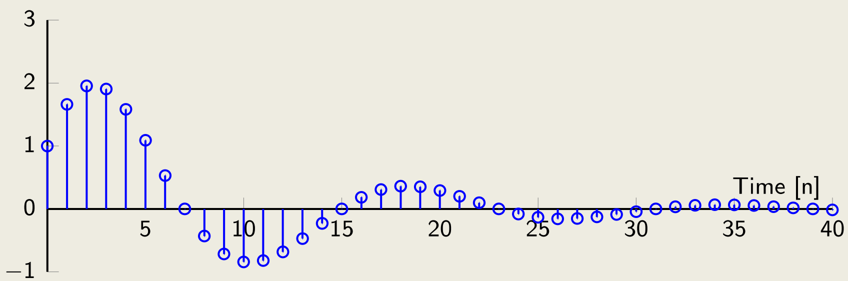
EITF75, z-transform

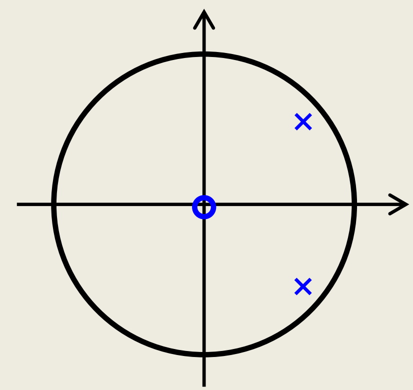




Stable -> poles inside
oscillating -> complex pair
Homework:why are poles in the right half plane?
2 zeros at z=0

EITF75, z-transform





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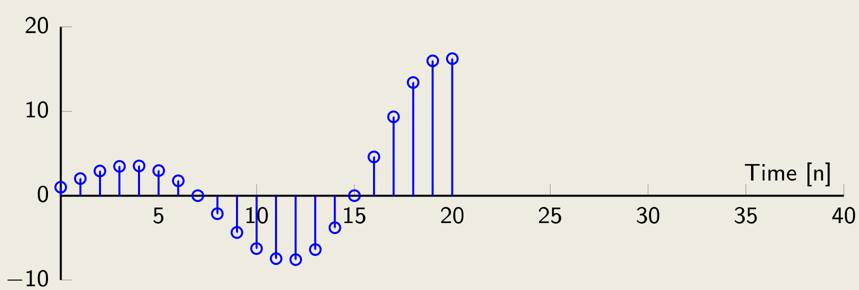
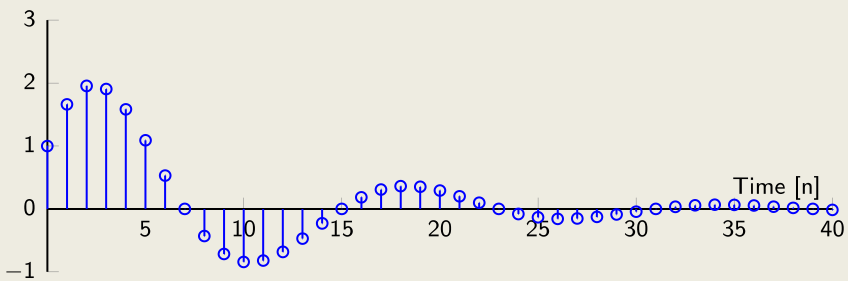
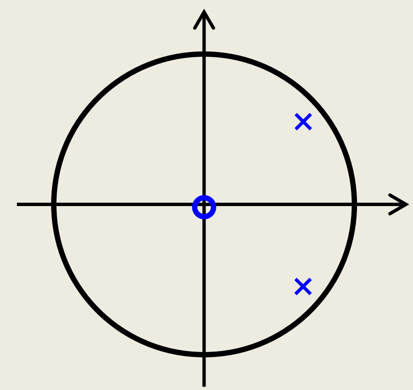


EITF75, z-transform





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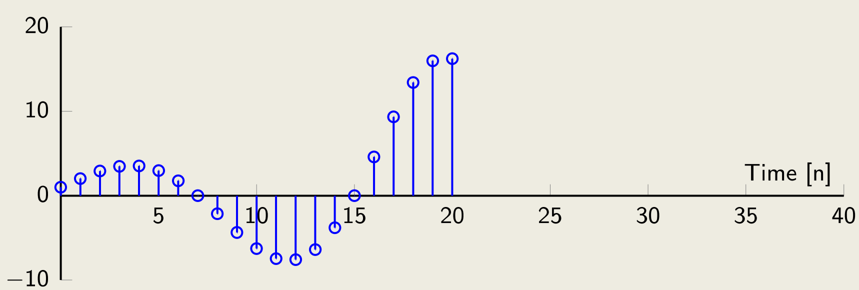

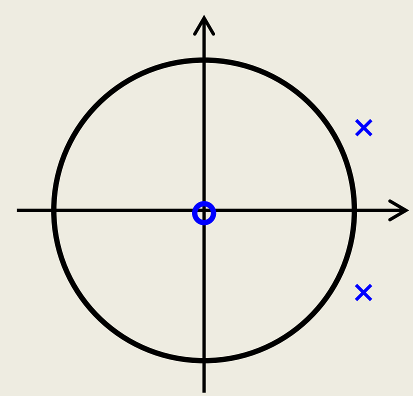

EITF75, z-transform

$$Y(z) = \frac{\textit{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$$

Some comments on *some polynomial in z*

EITF75, z-transform

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Some comments on *some polynomial in z*

Assume *some polynomial in z* = $z^3 + z^2 + 1$

EITF75, z-transform

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Assume *some polynomial in z* = $z^3 + z^2 + 1$

$$Y(z) = \frac{z^3 + z^2 + 1}{(z-1)(z-2)}$$

$p_1 = 1$

$q_1 = 2$

EITF75, z-transform

$$Y(z) = \frac{\textit{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$$

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Degree of nominator ≥ degree of nominator
Use polynomial division

EITF75, z-transform

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$$Y(z) = \frac{z^3 + z^2 + 1}{(z-1)(z-2)} = \frac{z^3 + z^2 + 1}{z^2 - 3z + 2} = \frac{z^3 - 3z^2 + 2z + z^2 + 1 + 3z^2 - 2z}{z^2 - 3z + 2}$$

I do it like this (use your own favorite method)

EITF75, z-transform

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$$= z + \frac{4z^2-2z+1}{z^2-3z+2}$$

EITF75, z-transform

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EITF75, z-transform

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$$= z + \frac{4z^2 - 2z + 1}{z^2 - 3z + 2} = z + \frac{4z^2 - 12z + 8 - 2z + 1 + 12z - 8}{z^2 - 3z + 2}$$

$$= z + 4 + \frac{10z - 7}{z^2 - 3z + 2} = z + 4 - \frac{3z^{-1}}{1 - z^{-1}} + \frac{13z^{-1}}{1 - 2z^{-1}}$$

EITF75, z-transform

$$Y(z) = \frac{\textit{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)\cdot(z-q_1)\cdots(z-q_L)}$$

Some comments on *some polynomial in z*

Assume *some polynomial in z* = $z^3 + z^2 + 1$

$$Y(z) = \frac{z^3 + z^2 + 1}{(z-1)(z-2)}$$

Invert

$$y(n) = \delta(n+1) + 4\delta(n) - 3u(n-1) + 13 \cdot 2^{n-1} \cdot u(n-1)$$

$$Y(z) = z + 4 + \frac{10z-7}{z^2-3z+2} = z + 4 - \frac{3z^{-1}}{1-z^{-1}} + \frac{13z^{-1}}{1-2z^{-1}}$$

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Degree to high

$$Y(z) = \frac{z^3 + z^2 + 1}{(z-1)(z-2)}$$

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EITF75, z-transform

$$Y(z) = \frac{\textit{Some polynomial in z}}{(z-p_1)\cdots(z-p_N)} \cdot \frac{}{(z-q_1)\cdots(z-q_L)}$$

Some comments on *some polynomial in z*

Lesson learnt: If input and filter are causal, the degree of the nominator of Y(z) cannot exceed the degree in the denominator

Assume *some polynomial in z* = $z^3 + z^2 + 1$

Degree to high

$$Y(z) = \frac{z^3 + z^2 + 1}{(z-1)(z-2)}$$

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