Exercises

Digital Signal Processing

Exercise Problems with Solution

Mikael Swartling, Nedelko Grbic, and Bengt Mandersson

Seyedezahra Chamideh, Navya Sri Garigapati, Johan Isaksson,
Guoda Tian, and Jan Eric Larsson

Autumn 2019

Department of Electrical and Information Technology
Lund University

Introduction

Uppgift 1.1

Which of the following are periodic and determine the time period if it is periodic.

a) $\cos(0.01\pi n)$

b) $\cos(\pi \frac{30}{105} n)$

c) $\cos(3\pi n)$

d) $\sin(3n)$

e) $\sin(\pi \frac{62n}{10})$.

Uppgift 1.5

The analog signal $x_a(t)$ is $x_a(t) = 3\sin(100\pi t)$.

a) Sktech $x_a(t)$ for $0\,\text{ms} \leq t \leq 30\,\text{ms}$.

b) Sample with $F_s = 300\,\text{sample/s}$. Determine $x(n) = x_a(nT)$ where $T = 1/F_s$. Determine frequency f of $x(n)$ and show that $x(n)$ is periodic.

c) Sketch $x(n)$. What is the period and ? what does it corresponds to in ms?

d) Determine the minimum F_s so that when $x_a(t)$ is sampled max of $x(n)$ assumes value 3 at $0 \leq n < N$ where N is the time period of $x(n)$.

Uppgift 1.7

An analog signal contains frequencies up to $10\,\text{kHz}$.

a) Which sampling frequency can be used if we want to be able to reconstruct the signal frequency?

b) Assume the sampling frequency $F_s = 8\,\text{kHz}$. What happens to the frequency component $F_1 = 5\,\text{kHz}$?

c) Assume the sampling frequency $F_s = 8\,\text{kHz}$. What happens to the frequency component $F_1 = 9\,\text{kHz}$?

Uppgift 1.8

An analog electrocardiogram (EKG) contains frequencies up to $100\,\text{Hz}$.

a) What is the Nyquist rate for the siganl?

b) Which is the highest frequency component that can be uniquely represented by the sampling frequency $F_s = 250\,\text{Hz}$?

Comment: The highest frequency component of the signal is called Nyquist frequency and the dual frequency is called Nyquist rate. Thus the sampling frequency must be selected higher than the Nyquist rate to avoid folding distortion.

Uppgift 1.11

To show the effect of folding we do the following. We sample the signal $x(t) = 3\cos(100\pi t) + 2\sin(250\pi t)$ with sample frequency $F_s = 200\,\text{Hz}$ without having any analog filter before sampling. We then listen ot the signal with sample frequency$F_s = 1000\,\text{Hz}$ (ideal reconstruction). How does the signal look after the reconstruction?

Matlab: When recording with sound card in PC, the analog signal is automatically filtered with an analog filter switching frequency $F_s/2$ befpre sampling to avoid folding distortion. If we want to illustrate the example above, we can for example. compile with $F_s = 1000\,\text{Hz}$ which sets the limit frequency in the sound card filter to $500\,\text{Hz}$. Now retains only fifth sample and have then reduced the sampling rate to $200\,\text{Hz}$ and this corresponds to that we assembled with $200\,\text{Hz}$ but with the analog filter set at $500\,\text{Hz}$.

Uppgift 1.13

The discrete signal $x(n) = 6.35\cos((\pi/10)n)$ is quantized with the resolution

a) $\Delta = 0.1$, and

b) $\Delta = 0.02$.

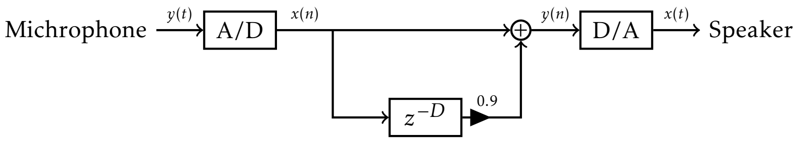
How many bits are needed in the A/D-converter?

Uppgift 1.14

Determine bit rate (bitar/s) and resolution if a seismic signal with dynamic 1 volt is sampled with $F_s = 20\,\text{Hz}$ with an 8-bit A/D-converter. Which maximum frequency can be represented in the digital signal?

Uppgift E1.1

MATLAB: To achieve echo effect, we use the following link (z^{-D} delayes the singal by D sample).



Simulate this in MATLAB and listen to the effect.

```
>> Fs = 10000;
>> t = (0:2*Fs-1)/Fs;           % generate 2s long time vector, FS=1/10000
>> x = chirp(t, 300, 2, 800);   % generate the sweep sine between 300 and 800 Hz
>> z = zeros(1, 500);           % generate 500 zeros
>> x1 = [x, z];
>> x2 = [z, 0.9*x];
>> y = x1+x2;                   % generera echo
```

Listen to the signal.

```
>> soundsc(x, Fs)
>> soundsc(y, Fs)
```

Select new `x` (collected numbers and listen to the echo effect).

Uppgift New-1

Write down the Euler's formula for $\cos(2\pi n)$, $\sin(2\pi n)$ and their derivatives.

Uppgift New-2

Find the sum of the following geometric series.

a) $\sum_{n=0}^{N} 2^n$

b) $\sum_{n=0}^{\infty} 0.5^n$

Uppgift New-3

Find the Nyquist rate for the following signals.

a) $x(t) = 3\cos(150\pi t)$

b) $x(t) = 3\cos(150\pi t) + 2\sin(400\pi t)$

c) $x(t) = 3\sin(100\pi t)\cos(250\pi t)$

Uppgift New-4

Express the following signals in terms of Impulse signal.

a) $x(n) = nu(n)$

b) $x(n) = u(n+1)$

b) $x(n) = u(n-1)$

Uppgift New-5

Which of the following are causal.

a) $x(n) = u(n+1)$

b) $x(n) = u(n-1)$

Uppgift New-6

Find the following signals for the given x(n)= $\{1, 2, 1, -3, 5, 8\}$ for n= $\{0, 1, 2, 3, 4, 5\}$

a) $x(n+1)$

b) $x(n-1)$

c) $x(2n)$

d) $x(0.5n)$

Uppgift New-7

Determine whether the following are Energy or power signal.

a) $(n) = u(n), n \geq 0$

b) $x(n) = nu(n), n \geq 0$

c) $x(n) = (0.5)^n u(n)$

Uppgift New-8

Identify the even or odd signals below.

a) $x(n) = sin(n)$

b) $x(n) = cos(n)$

b) $x(n) = cot(n)$

Uppgift New-9

Examine the linearity and time invariance of the following signals.

a) $y(n) = x(n-1)$

a) $y(n) = x(n^2)$

a) $y(n) = x^2(n)$

Uppgift New-10

The analog signal $x_a(t) = 3\cos(500\pi t) + 2\sin(1000\pi t)$. Determine the following.

a) What is the Nyquist rate.

b) How does the reconstructed signal will be with $F_s = 400\,\text{Hz}$, $F_s = 600\,\text{Hz}$,$F_s = 1100\,\text{Hz}$. Please comment on your results.

Impulse response and Convolution, chapter 2

Uppgift 2.1

A time discrete signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3} & -3 \leq n \leq -1 \\ 1 & 0 \leq n \leq 3 \\ 0 & n < -3 \text{ and } n > 3 \end{cases} \tag{1}$$

a) Sketch the signal $x(n)$.

b) Sketch the following alternative signal.

  i) First fold (mirroring around origin) $x(n)$ and then delay the resulting signal by 4 sampel.

  ii) First delay $x(n)$ by 4 sampel and then fold the resulting sample.

c) Sketch the signal $x(-n+4)$.

d) compare the results in part (b) and (c).

e) Express the signal $x(n)$ in term of $\delta(n)$ and $u(n)$.

Uppgift 2.7

Examine the following system with respect to the properties below

1) Static or dynamic.
2) Linear or nonlinear.
3) Time invariant or time varying.
4) Causal or non-causal. .
5) Stable or unstable.

a) $y(n) = \cos(x(n))$.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$.

c) $y(n) = x(n)\cos(\omega_0 n)$.

e) $y(n) = \mathrm{trun}(x(n))$, denotes the integer part of $x(n)$ obtained by rounding.

h) $y(n) = x(n)u(n)$.

j) $y(n) = x(2n)$.

n) Ideal sampling system with input $x_a(t)$ and output $x(n) = x_a(t)$ where $t = nT$ and $-\infty < n < \infty$.

Uppgift 2.13

a) Show that $\sum_n y(n) = \sum_k x(k) \sum_l h(l)$ where $y(n) = x(n) * h(n)$.

b) Show the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h .

Uppgift 2.4

adding this problem for stability
check if the discrete time systems below are BIBO stable

1) $y(n) = y(n-1) + x(n)$ for $n > 0$ is at rest [i.e., y(-1)=0] check if the system is BIBO stable.

4) $y(n) = x(n) + x(n-1) - x(n-4)$.

Uppgift 2.16

Calculate convolution $y(n) = x(n) * h(n)$ for following signals.

1) $x(n) = \{\ 1 \quad 1 \quad 1 \quad 1 \quad 1\ \}$ and $h(n) = \{\ 1 \quad 2 \quad 4\ \}$.

2) $x(n) = \{\ 1 \quad 2 \quad -1\ \}$ and $h(n) = x(n)$.

3) $x(n) = \{\ 1 \quad 2 \quad 3 \quad 4 \quad 5\ \}$ and $h(n) = \{\ 1\ \}$.

4) $x(n) = \{\ 1 \quad 1 \quad \underline{0} \quad 1 \quad 1\ \}$ and $h(n) = \{\ 1 \quad -2 \quad -3 \quad \underline{4}\ \}$.

5) $x(n) = 0.5^n u(n)$ and $h(n) = 0.25^n u(n)$.

Uppgift 2.17

Calculate convolution $y(n) = x(n) * h(n)$ for following signals.

a) $x(n) = \{\ \underline{1} \quad 1 \quad 1 \quad 1\ \}$ och $h(n) = \{\ \underline{6} \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad 0\ \}$

b) $x(n) = \{\ \underline{1} \quad 1 \quad 1 \quad 1\ \}$ och $h(n) = \{\ 6 \quad 5 \quad 4 \quad \underline{3} \quad 2 \quad 1 \quad 0 \quad 0\ \}$

c) $x(n) = \{\ \underline{0} \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1\ \}$ och $h(n) = \{\ 1 \quad 1 \quad 0 \quad 0 \quad \underline{0}\ \}$

d) $x(n) = \{\ \underline{0} \quad 0 \quad 1 \quad 1 \quad 1 \quad 1\ \}$ och $h(n) = \{\ 1 \quad 1 \quad \underline{0}\ \}$

MATLAB: Lös ovanstående faltningar i MATLAB (conv.m).

Uppgift 2.21

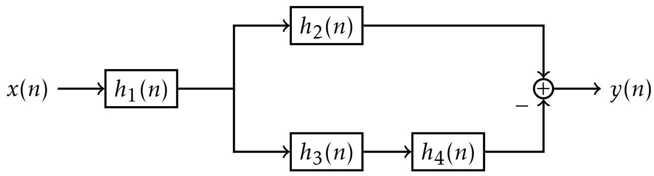
Calculate convolution $y(n) = x(n) * h(n)$ for following signals.

a) $x(n) = a^n u(n)$ och $h(n) = b^n u(n)$ för både $a \neq b$ och $a = b$.

b) $x(n) = \left\{ 1 \quad 2 \quad \underline{1} \quad 1 \right\}$ och $h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$.

Uppgift 2.35

Consider the interconnection of LTI systems as shown in the following Figure.



a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$,$h_3(n)$ and $h_4(n)$

b) Determine $h(n)$ when $h_1(n) = \left\{ 0.5 \quad 0.25 \quad 0.5 \right\}$, $h_2(n) = h_3(n) = (n+1)u(n)$ and $h_4(n) = \delta(n-2)$.

c) Determine the response of the system in part (b) if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$.

d) check if the systems is BIBO stable?

Uppgift 2.61

Compute the correlation sequence $r_{xx}(l)$ and cross correlation function $r_{xy}(l)$ for the following signal sequences.

$$x(n) = 1 \quad \text{for } n_0 - N \leq n \leq n_0 + N, \text{ noll otherwise, and} \tag{2}$$

$$y(n) = 1 \quad \text{for } -N \leq n \leq N, \text{ noll otherwise.} \tag{3}$$

Uppgift 2.62

Calculate the autocorrelation sequences $r_{xx}(l)$ of the following signals.

a) $x(n) = \left\{ 1 \quad 2 \quad 1 \quad 1 \right\}$.

b) $y(n) = \left\{ 1 \quad 1 \quad 2 \quad 1 \right\}$.

What is your conclusion ?

MATLAB: Lös a) och b) ovan i MATLAB.

Uppgift 2.64

An audio signal generated by a loudspeaker is reflected at two different walls with the reflection coefficients r_1 and r_2. The signal is recorded by a microphone near the speaker and after sampling discrete time signal is $x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$ where k_1 and k_2 are the delays of the echoes. Determine and sketch the autocorrelation function $r_{xx}(l)$ for $x(n)$.

Z Transform, Chapter 3

Uppgift N3.1

The following first order difference equation is given:

$$y(n) + y(n-1) = x(n) \tag{4}$$

where

$$x(n) = 5u(n) \tag{5}$$

a) z-transform the equation to express it in the z-domain

b) Perform an inverse z-transform to express the original equation without $y(n-1)$ and $x(n)$

Uppgift N3.2

The following second order difference equation is given:

$$3y(n) - y(n-1) - y(n-2) = x(n) \tag{6}$$

a) Express the equation on the form $Y(z) = H(z)X(z)$ and determine $H(z)$

b) Determine $h(n)$

Uppgift 3.1

Determine the z-transform for

a) **Like next one but that starts at n = 0, ask about ROC?**

a) $x(n) = \left\{ \begin{array}{cccccccc} 3 & 0 & 0 & 0 & 0 & \underline{6} & 1 & -4 \end{array} \right\}$

b) **Like next one but starts at n => 0**

b) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 5 \\ 0 & n \leq 4 \end{cases}$

Uppgift 3.2

Determine the z-transform for the signals below, and sketch their pole-zero patterns. **Fix letters?**

a) $x(n) = (1+n)u(n)$.

c) $x(n) = (-1)^n \cdot 2^{-n} \cdot u(n)$.

f) **Like next one, but no phase shift (or scaling?)**

f) $x(n) = Ar^n \cos(\omega_0 n + \phi)u(n) \quad 0 < r < 1$.

h) $x(n) = \left(\frac{1}{2}\right)^n \cdot [u(n) - u(n-10)]$.

Uppgift 3.8

How to reach the convolution expression seems very ad hoc in these problems Use the convolution property of the z-transform to determine the following z-transforms:

a) Determine $Y(z)$ expressed in $X(z)$ for $y(n) = \sum_{k=-\infty}^{n} x(k)$.

b) Determine $X(z)$ for $x(n) = (n+1)u(n)$. Hint: show that $x(n) = u(n) * u(n)$.

Uppgift 3.9

The z-transform $X(z)$ of the real signal $x(n)$ includes a pair of complex-conjugate zeros and a pair of complex-conjugate poles. What happens to these pairs if we multiply $x(n)$ by $e^{j\omega_0 n}$?
Hint: Use the scaling theorem in the z-domain.

Uppgift 3.14

Determine the inverse z-transform of $x(n)$, $x(n)$ is causal, of the following signals.

a) $X(z) = \dfrac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$

b) $X(z) = \dfrac{1}{1-z^{-1}+0.5z^{-2}}$

c) $X(z) = \dfrac{z^{-6}+z^{-7}}{1-z^{-1}}$

d) $X(z) = \dfrac{1+2z^{-2}}{1+z^{-2}}$

g) $X(z) = \dfrac{1+2z^{-1}+z^{-2}}{1+4z^{-1}+4z^{-2}}$

MATLAB: Plot some of these functions for $z = \mathrm{e}^{\mathrm{j}\omega}$ using MATLAB.

Uppgift 3.16

Determine the convolution of $x_1(n)$ and $x_2(n)$ below using the z-transform.

a) $x_1(n) = \left(\frac{1}{4}\right)^n \cdot u(n-1)$ and $x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right] u(n)$.

c) $x_1(n) = 0.5^n u(n)$ and $x_2(n) = \cos(\pi n) u(n)$.

Uppgift 3.14

Compute the inverse Z transform $x(n)$, $x(n)$ causal, of the following signals.

a) $X(z) = \dfrac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$

b) $X(z) = \dfrac{1}{1-z^{-1}+0.5z^{-2}}$

c) $X(z) = \dfrac{z^{-6}+z^{-7}}{1-z^{-1}}$

d) $X(z) = \dfrac{1+2z^{-2}}{1+z^{-2}}$

g) $X(z) = \dfrac{1+2z^{-1}+z^{-2}}{1+4z^{-1}+4z^{-2}}$

MATLAB: Plot som of these functions for $z = \mathrm{e}^{\mathrm{j}\omega}$ using MATLAB.

Uppgift 3.16

Compute the convolution between $x_1(n)$ and $x_2(n)$ below using the Z transform.

a) $x_1(n) = \left(\frac{1}{4}\right)^n \cdot u(n-1)$ och $x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right] u(n)$.

c) $x_1(n) = 0.5^n u(n)$ och $x_2(n) = \cos(\pi n) u(n)$.

Uppgift 3.35

Compute the output signal $y(n) = x(n) * h(n)$ for

a) $h(n) = \left(\frac{1}{3}\right)^n u(n)$ och $x(n) = (1/2)^n \cdot \cos(\pi/3n) u(n)$.

d) $y(n) = 0.5x(n) - 0.5x(n-1)$ och $x(n) = 10\cos(\pi/2\ n)\ u(n)$.

Uppgift 3.40

The input and output signal for an LTI system are given by $x(n) = 0.5^n u(n) - 0.25(0.5)^{n-1} \cdot u(n-1)$ och $y(n) = (\frac{1}{3})^n u(n)$.

a) Compute the impulse response $h(n)$ and the system function $H(z)$.

b) Comnpute the difference equation.

c) Computte a realization of minimal order.

d) Is the system stable?

Uppgift 3.49

Use the single-sided Z transform to compute $y(n)$ where $n \geq 0$ for the following cases.

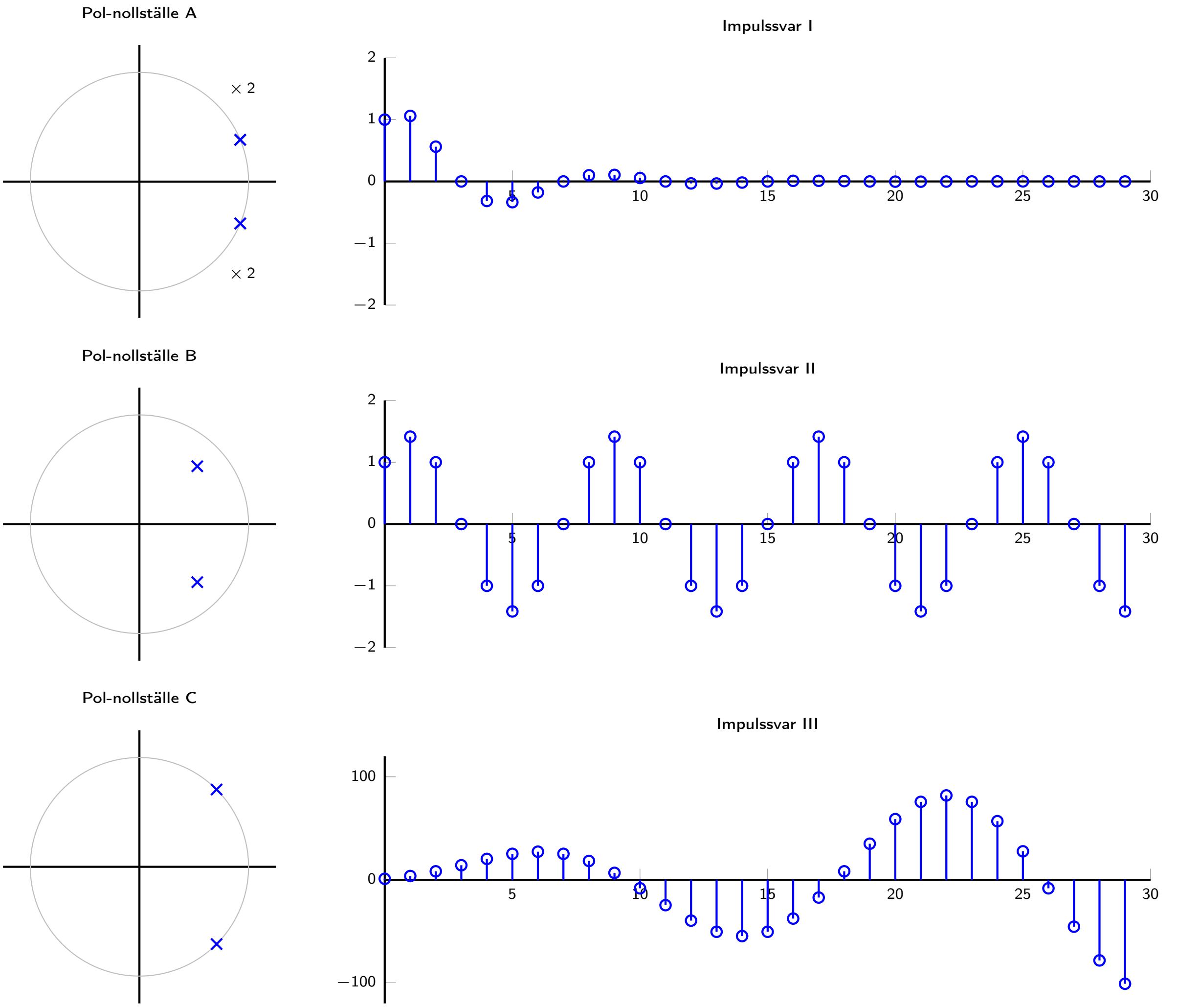
b) $y(n) - 1.5y(n-1) + 0.5y(n-2) = 0$, $y(-1) = 1$, $y(-2) = 0$.

c) $y(n) = 0.5y(n-1) + x(n)$, $x(n) = (1/3)^n u(n)$, $y(-1) = 1$.

d) $y(n) = 0.25(y(n-2) + x(n)$, $x(n) = u(n)$, $y(-1) = 0$, $y(-2) = 1$.

Uppgift E3.1

Combine each pole zero plot with the corresponding impulse response. How is the impulse response affected by the distance beween the pole and the unit circle? How is the impulse response affected by double poles?



Uppgift E3.2

A second-order, linear, and time-invariant system is defined by

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{7}$$

a) Compute the output signal $y(n)$ when the input signal is $x(n) = 3\sin\left(\frac{\pi}{2}n\right)u(n)$ where

$$y(-1) = 1/3 \tag{8}$$

and

$$b_0 = 0 \quad b_1 = 1/5 \quad b_2 = 0 \tag{9}$$

$$a_1 = 1/2 \quad a_2 = 0 \tag{10}$$

b) Let the input signal be $x(n) = \sin(2\pi f_0 n)$ för $-\infty < n < \infty$. The coefficients are chosen so that the output becomes $y(n) = 0$ för $-\infty < n < \infty$. This occurs when

$$b_0 = b_1 = b_2 = 2 \tag{11}$$

and

$$a_1 = -\sqrt{2} \quad a_2 = 1/4 \tag{12}$$

What is the frequency f_0 in the input signal?

c) Again, the input signal is $x(n) = \sin(2\pi f_0 n)$, $-\infty < n < \infty$. When the coefficients are

$$b_0 = 1 \quad b_1 = 1 \quad b_2 = 1/4 \tag{13}$$

and

$$a_1 = -1 \quad a_2 = 1 \tag{14}$$

the output signal becomes $y(n) = A_0 \sin(2\pi f_0 n + \theta_0) + A_1 \sin(2\pi f_1 n + \theta_1)$ för $-\infty < n < \infty$. What is the frequency f_1?

Uppgift E3.3

An LTI system is described by the difference equation

$$y(n) - y(n-1) + \frac{3}{16}\, y(n-2) = x(n) \tag{15}$$

Compute the output signal $y(n)$ when

$$x(n) = \left(\frac{1}{2}\right)^n \cdot u(n) + \sin\left(2\pi \frac{1}{4} n\right) \quad -\infty < n < \infty \tag{16}$$

Uppgift E3.4

A system is described by the difference equation $y(n) - y(n-1) + 0.5y(n-2) = x(n)$. The initial values are $y(-1) = 0$ and $y(-2) = 2$, and the input signal is $x(n) = u(n)$. When $n \geq 0$ compute

a) the zero input solution, $y_{zi}(n)$.

b) the zero state solution, $y_{zs}(n)$.

c) the transient solution, $y_{tr}(n)$.

d) the stationary solution, $y_{ss}(n)$.

Uppgift E3.5

A system is described by the difference equation

$$y(n) - \frac{1}{4}\, y(n-1) = x(n) \tag{17}$$

Compute the output signal $y(n)$ when

$$x(n) = \begin{cases} \sin\left(2\pi \frac{1}{4}\, n\right) & n < 0 \\ 0 & n \geq 0 \end{cases} \tag{18}$$

The Fourier Transform, LTI Systems, and Sampling, Chapter 4, 5, and 6

Uppgift 1

The transfer function of the system is given as

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})}$$

Find the output signal $y(n)$ if the input signal is given by

$$x(n) = u(n-1) + sin(2\pi \frac{1}{4} n + \frac{\pi}{4})$$

Uppgift 4.8

Two discrete signals $s_k(n)$ and $s_l(n)$ are orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} s_k(n)s_l^*(n) = \begin{cases} A_k & k = l \\ 0 & k \neq l \end{cases} \tag{19}$$

If $A_k = 1$ the signal are orthonormal.

a) Show the relation

$$\sum_{n=0}^{N-1} \mathrm{e}^{\mathrm{j}2\pi\frac{kn}{N}} = \begin{cases} N & k = 0 \quad k = \pm N \quad k = \pm 2N \quad \dots \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

b) Illustrate a) for $N = 6$ by drawing $s_k(n) = \mathrm{e}^{\mathrm{j}2\pi\frac{kn}{N}}$ for $k = 1,2,3,4,5,6$ and $n = 0,1,2,3,4,5$.

c) Show that harmonic signals $s(n) = \mathrm{e}^{\mathrm{j}2\pi\frac{kn}{N}}$ arer orthogonal over any interval with length N.

Uppgift 4.9

Compute the Fourier transform for the following signals.

a) $x(n) = u(n) - u(n-6)$.

b) $x(n) = 2^n u(-n)$.

c) $x(n) = (0.25)^n u(n+4)$.

d) $x(n) = (\alpha^n \sin(\omega_0 n))\, u(n)$ för $|\alpha| < 1$.

g) $x(n) = \left\{ -2 \quad -1 \quad \underline{0} \quad 1 \quad 2 \right\}$.

Uppgift 4.10

Compute the time-domain signal for the Fourier transform

a) $X(\omega) = \begin{cases} 0 & 0 \leq \omega \leq \omega_0 \\ 1 & \omega_0 < \omega \leq \pi \end{cases}$

b) $X(\omega) = \cos^2 \omega$

The Fourier transforms are symmetric for negative frequencies so that the signals have real values.

Uppgift 4.12

Compute the time-domain signal for the Fourier transform

c) $X(\omega) = \begin{cases} 2 & \omega_c - W/2 \leq |\omega| \leq \omega_c + W/2 \\ 0 & \text{annars} \end{cases}$

Uppgift 4.14

The sequence $x(n)$ is given by $x(n) = \left\{ -1 \quad 2 \quad \underline{-3} \quad 2 \quad -1 \right\}$ and its Fourier transform is $X(\omega)$. Compute the following, without calculating $X(\omega)$ explicitly

a) $X(0)$

b) $\arg X(\omega)$

c) $\displaystyle\int_{-\pi}^{\pi} X(\omega)d\omega$

d) $X(\pi)$

e) $\displaystyle\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$

MATLAB: Compute $X(\omega)$ and plot $|X(\omega)|$ and $\arg X(\omega)$.

Uppgift 5.2

a) Compute and draw the Fourier transform $W_R(\omega)$ for a rectangular window

$$w_R(n) = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

b) Compute and draw the Fourier transform for a trinagular window $w_T(n)$

$$w_T(n) = \begin{cases} n & 0 \le n \le M/2 \\ M - n & M/2 < n \le M \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

by using the convolution of two rectangular windows as in a.

MATLAB: Plot $W_R(\omega)$ for $M = 2$, $M = 4$, and $M = 5$.

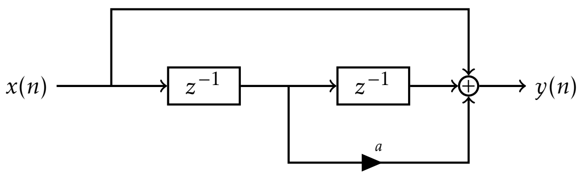
Uppgift 5.17

A time-discrete circuit is given by the figure with $a = -2\cos(\omega_0)$

a) Computer the impulse response $h(n)$.

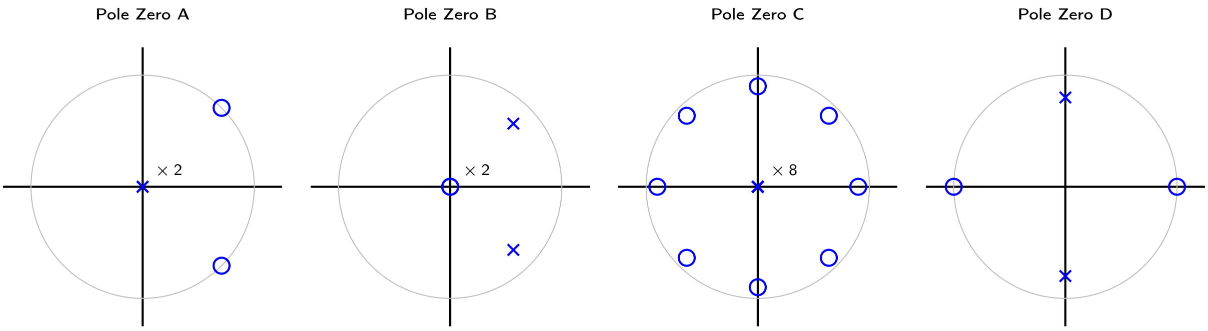
b) Draw $|H(\omega)|$ and $\arg H(\omega)$.

c) Compute $y(n)$ when $x(n) = 3\cos(\pi/3\, n + \pi/6)$ for $-\infty < n < \infty$ and $\omega_0 = \pi/2$.



Uppgift 5.25

Draw $|X(f)|$ corresponding to the following pole-zero configurations.



Uppgift 5.26

Design a filter that blocks the frequency $\omega_0 = \pi/4$ and compute the output signal when the input signal is $x(n) = \sin(\pi/4\, n)u(n)$ for $n = 0, 1, 2, 3, 4$.

Uppgift 5.35

A second-order system has a double pole at $p_{1,2} = 0.5$ and tywo zeros at $z_{1,2} = e^{\pm j3\pi/4}$. Computet the gain so that $|H(0)| = 1$.

Uppgift 5.39

Compute the 3 dB bandwidth for the filters where $0 < a < 1$.

$$H_1(z) = \frac{1-a}{1-az^{-1}} \tag{23}$$

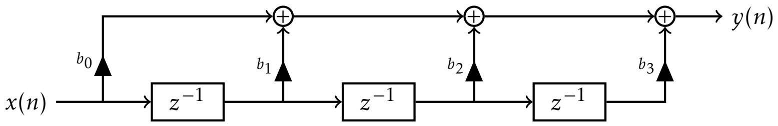
$$H_2(z) = \frac{1-a}{2} \cdot \frac{1+z^{-1}}{1-az^{-1}} \tag{24}$$

(25)

MATLAB: Plot the amplitude functions in MATLAB and estimate the bandwidths from the plots.

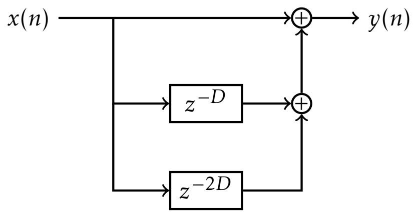
Uppgift E4.1

Design the circuit below so that the constant amplification is 1 and the frequencies $\omega = \pi/2$ and $\omega = \pi$ are blocked. Compute the amplitude and phase functions.



Uppgift E4.2

To create an echo effect, we use the circuit below.

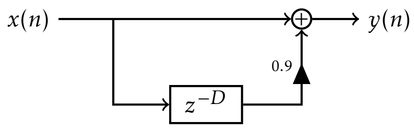


a) Compute the poles and zeros for $D = 500$.

b) Compute and draw the amplituce function $|H(f)|$.

Uppgift E4.3

To create an echo effect, we use the circuit below.



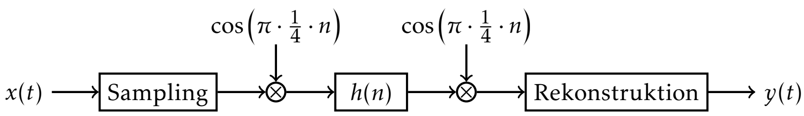
a) Compute the impulse response.

b) Compute the amplitude function $|H(f)|$.

c) Compute the poles and zeros for $D = 500$.

Uppgift E4.4

The input signal to the system below is $x(t) = \cos(2\pi 1000t) + \cos(2\pi 6000t)$. The impulse response is $h(n) = u(n) - u(n-8)$. The sampling rate is 8 kHz.



Compute the output signal $y(t)$ for ideal reconstruction.

MATLAB: Simulate the system in MATLAB. Use a sound signal.

Sampling

Uppgift E4.5

Assyume a continuous signal $x_a(t) = \mathrm{e}^{-10 \cdot t} \cdot u(t)$, t in seconds.

a) Compute the Fourier transform $X_a(F)$ and $|X_a(F)|^2$.

b) We sample the signal with $F_s = 100\,\text{Hz}$. Before the sampling, we filter it with an anti-aliasing filter. Assume that this is an ideal low pass filter with a cutoff frequency of $F_s/2 = 50\,\text{Hz}$. How much of the energy in the signal $x_a(t)$ is blocked by the filter?

c) The filtered signal is sampled with F_s. Compute the absolute value of the Fourier transform of the sampled signal $y(n)$. Also compute the Fourier transform of the sampled signal without the anti-aliasing filter.

Uppgift E4.6

Compute the output signal from the circuit below when the input signal is $x_a(t) = 2 \cdot \cos(2\pi \cdot F_0 t)$ då $F_0 = 600\,\text{Hz}$ and $F_s = 1000\,\text{Hz}$.

$$x(t) \longrightarrow \boxed{\text{A/D}} \xrightarrow{x(n)} \boxed{(-1)^n} \xrightarrow{z(n)} \boxed{\uparrow \times 2} \xrightarrow{y(n)} \boxed{\text{D/A}} \longrightarrow y(t)$$

The sign changer performs the operation$x_1(n) = (-1)^n x(n)$ and the collector

$$y(n) = \begin{cases} z\left(\frac{n}{2}\right) & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \tag{26}$$

The reconstruction is ideal with the samplng rate $2F_s$.

The Discrete Fourier Transform DFT, Chapter 7

Uppgift 7.1

The first five values in an eight-point DFT of a real sequence are given by

$$\left\{ \begin{array}{ccccc} 0.25 & 0.125 - j0.3018 & 0 & 0.125 - j0.0518 & 0 \end{array} \right\} \tag{27}$$

Compute the remaining three points.

MATLAB: Check the result in MATLAB.

Uppgift 7.2

Compute the circular convolution between the following signals($N = 8$)

a) $x_1 = \left\{ \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{array} \right\}$ and $x_2 = \sin(\frac{3\pi}{8} n)$ for $0 \le n \le N-1$.

Uppgift 7.3

Given an N-point DFT $X(k)$, $0 \le k \le N-1$ of $x(n)$, $\le n \le N-1$. Define

$$X_0(k) = \begin{cases} X(k) & 0 \le k \le k_c \quad N - k_c \le k \le N-1 \\ 0 & k_c < k < N - k_c \end{cases} \tag{28}$$

Compare $x_0(n) = \text{IDFT}\{X_0(k)\}$ with $x(n)$. Vhat happens?

Uppgift 7.4

The sequences $x_1(n) = \cos \frac{2\pi}{N} n$ and $x_2(n) = \sin \frac{2\pi}{N} n$ are given for $0 \le n \le N-1$.

a) Compute the circular convolution between $x_1(n)$ and $x_2(n)$.

b) Compute the circular correlation between $x_1(n)$ och $x_2(n)$.

c) Compute the circular auto-correlation for för $x_1(n)$.

d) Compute the circular auto-correlation for för $x_2(n)$.

Uppgift 7.7

The sequence $x(n)$, $0 \le n \le N-1$, has the DFT $X(k)$. Compute the DFTs of tghe following sequences, using terms of $X(k)$

1) $x_c(n) = x(n) \cos\left(2\pi \cdot \frac{k_0}{N} \cdot n\right)$.

2) $x_s(n) = x(n) \sin\left(2\pi \cdot \frac{k_0}{N} \cdot n\right)$.

Uppgift 7.8

Compute the circular convolution between $x_1(n) = \left\{ \underline{1} \quad 2 \quad 3 \quad 1 \right\}$ and $x_2(n) = \left\{ \underline{4} \quad 3 \quad 2 \quad 2 \right\}$. Compare with the linear convolution.

Uppgift 7.9

Compute the circular convolution between $x_1(n) = \left\{ \underline{1} \quad 2 \quad 3 \quad 1 \right\}$ and $x_2(n) = \left\{ \underline{4} \quad 3 \quad 2 \quad 2 \right\}$ by using DFT and IDFT.

Uppgift 7.10

Compute the energy for the sequence $x(n) = \cos\left(2\pi \cdot \frac{k}{N} \cdot n\right)$ for $0 \le n \le N-1$.

MATLAB: Compute the energy in MATLAB by using `fft`.

Uppgift 7.11

The sequence $x(n) = \left\{ 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \right\}$ has the DFT $X(k)$. Compute the DFTs of the following sequences

a) $\left\{ 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \right\}$.

b) $\left\{ 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \right\}$.

Uppgift 7.18

The input signal to an LTI system $H(\omega)$ is $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-kN)$. We compute an N-point DFT for for the output signal for the points $0 \le n \le N-1$. Compute the connection between $Y(k)$ and $H(\omega)$.

Uppgift 7.23

Compute the N-point DFT for

a) $x(n) = \delta(n)$

b) $x(n) = \delta(n-n_0)$, $0 < n_0 < N$

c) $x(n) = a^n$, $0 \le n \le N-1$

d) $x(n) = \begin{cases} 1 & 0 \le n \le N/2-1 \quad N \text{ even} \\ 0 & N/2 \le n \le N-1 \end{cases}$

e) $x(n) = \mathrm{e}^{\mathrm{j}2\pi/N\, k_0 n}$, $0 \le n \le N-1$

f) $x(n) = \cos(2\pi/N\; k_0\; n)$, $0 \le n \le N-1$

g) $x(n) = \sin(2\pi/N\; k_0\; n)$, $0 \le n \le N-1$

h) $x(n) = \begin{cases} 1 & n \quad \text{even} \\ 0 & n \quad \text{odd} \quad 0 \le n \le N-1 \end{cases}$

Uppgift 7.24

Given $x(n) = \left\{ 1 \quad 2 \quad 3 \quad 1 \right\}$. Compute the 4-point DFT of $x(n)$ by solving a 4th-order equation system for the inverse DFT.

Uppgift 7.25

a) Compute the Fourier transform of $x(n) = \left\{ 1 \quad 2 \quad \underline{3} \quad 2 \quad 1 \quad 0 \right\}$.

b) Compute the six-point DFT of $v(n) = \left\{ 3 \quad 2 \quad 1 \quad 0 \quad 1 \quad 2 \right\}$.

c) Compare $X(k)$ and $V(k)$ i a) och b).

Uppgift E5.1

Compute and draw $|X(k)|$ where $X(k)$ is the eight-point DFT of $x(n)$ with $x(n) = \left\{ 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \right\}$

Uppgift E5.2

Let $X(k)$ be the eight-point DFT of $x(n) = \left\{ 0 \quad 1 \quad 1 \quad 3 \quad 8 \quad 7 \quad 2 \quad 2 \right\}$.

a) Compute $y(n) = \text{IDFT}\{X^*(k)\}$.

b) Compute $y(n) = \text{IDFT}\left\{(-1)^k \cdot X(k)\right\}$.

MATLAB: Check the calculations in MATLAB with `fft` and `ifft`. Let $x(n)$ be a sound signal and listen to $y(n)$ in a) and b).

Uppgift E5.3

The input signal $x(n)$ to the system $y(n) = ay(n-1) + x(n)$ where $0 < a < 1$ is periodical with the period N. Compute the impulse response for an FIR filter, which gives the same stationary solution as the system above with the input signal $x(n)$.

Uppgift E5.4

The student Hans is deeply interested in signal processing and finds a tuning fork. Sadly, there is no information about what pitch the tuning fork has. Hans decides to use a spectral analyzer. This machine is digital and performs a DFT on the input signal. Hans sets the spetctral analyzer from 0 Hz to 200 Hz (sampling rate 400 Hz) and sees a top at the frequency 138 Hz. Which values can the pitch of the tuning fork have? Give a motivation for your answer.

Uppgift E5.5

Below are four sequences $x_i(n)$ (1–4) and eight sequences $X_j(k)$ (a–h). Choose the correct pairs $x(n) \longleftrightarrow X(k)$.

1 $\left\{ 1+j \quad 0 \quad 0 \quad 0 \right\}$	a $\left\{ 1 \quad 1 \quad 0 \quad 0 \right\}$	e $\left\{ 1 \quad -j \quad j \quad 1 \right\}$
2 $\left\{ 0.5 \quad 0.5 \quad 0.5 \quad -0.5 \right\}$	b $\left\{ j \quad 0 \quad j \quad 0 \right\}$	f $\left\{ 1 \quad -j \quad 1 \quad j \right\}$
3 $\left\{ 1+j \quad j \quad 0 \quad 1 \right\}$	c $\left\{ 1+j \quad 1+j \quad 1+j \quad 1+j \right\}$	g $\left\{ 2+2j \quad 2+2j \quad 0 \quad 0 \right\}$
4 $\left\{ 0.75 \quad 0.25 \quad -0.25 \quad 0.25 \right\}$	d $\left\{ j \quad 0 \quad -j \quad 0 \right\}$	h $\left\{ 1 \quad 1 \quad 0 \quad 1 \right\}$

MATLAB: Use MATLAB to check the result.

Uppgift E5.6

Given the impulse response

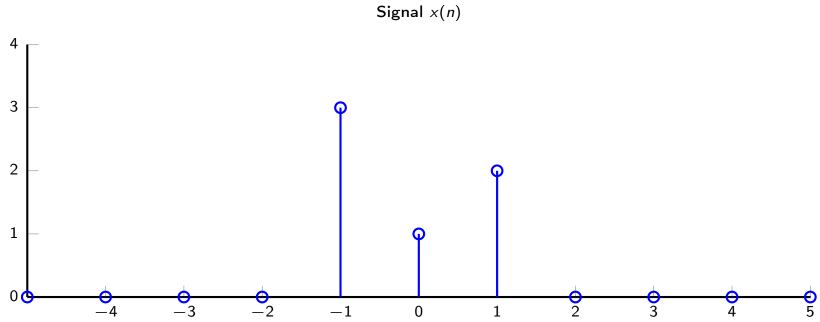
$$h(n) = \frac{1}{4}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{4}\delta(n-2) + \frac{1}{4}\delta(n-3) \tag{29}$$

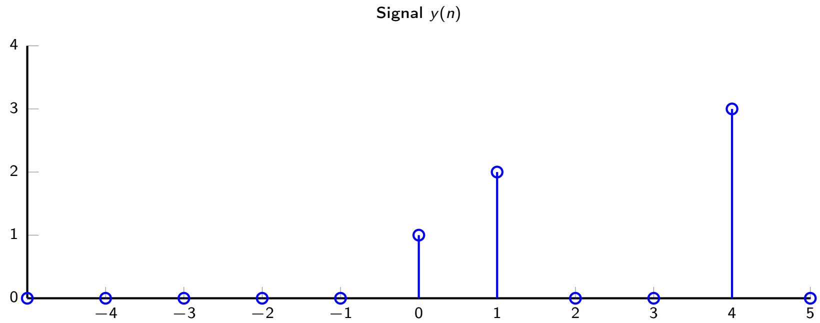
a) Compute the Fourier transform. Compute the DFT for $N \geq 4$. Connection?

b) What is $H(k)$ for $k = 0 \ldots 3$ when $N = 4$? What is $h_p(n) = \text{IDFT}\{H(k)\}$?

Uppgift E5.7

The following two signals are given





The Fourier transforms of the signals are sampled in the points $f = k/N$ för $k = 0 \dots N-1$ where $N = 5$. How is $Y(k/N)$ related to $X(k/N)$?

Uppgift E5.8

The impulse response to an LTI system is

$$h(n) = \delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2) \tag{30}$$

and the input signal is

$$x(n) = \delta(n) + \delta(n-1) \tag{31}$$

a) Compute the output signal with convolution.

b) Compute the output signal with the Fourier transform.

c) Let $f = k/N$ för $k = 0 \dots N-1$ in the expression for $Y(f)$. If N is less than a certain number M there is aliasing in the time domain. Conclusion? How large is M? How large is M in the genearl case, when the impulse response has length P and the input signal has length Q ?

Uppgift E5.9

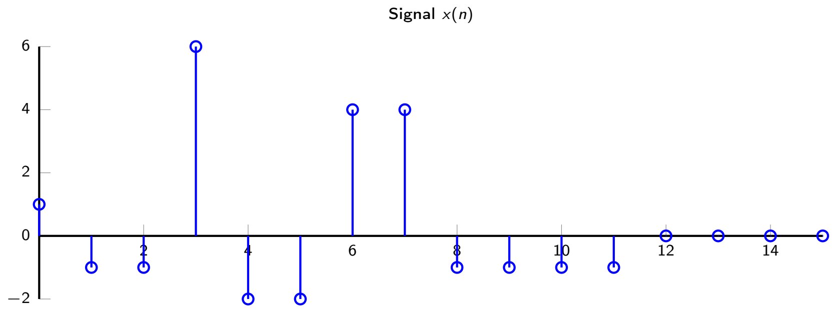
The sequence $h(n)$ is the impulse response to a transversal filter of order L with the coefficients b_k, $k = 0 \dots L$. The signal $x(n)$ is a truncated step function with length Q, that is,

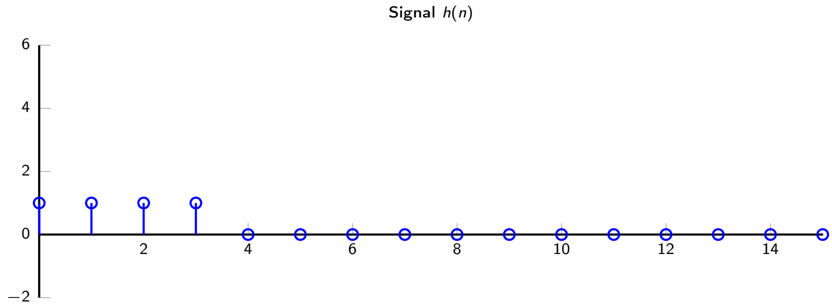
$$x(n) = \begin{cases} 1 & n = 0 \dots Q-1 \quad Q > L+1 \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

We want to compute the output signal from the filter, but mistakenly uses the DFT length $N = Q$ instead of $N \geq Q + P - 1 = Q + L$, where P is the length of $h(n)$). Given this choice of N compute the erroneous output signal $y_p(n)$, $n = 0 \dots N-1$.

Uppgift E5.10

The signals $x(n)$ and $h(n)$ in the figures are the input signalks and the impulse response to an LTI sytem. We want to compoute the output signal $y(n) = h(n) * x(n)$ with an eight-point DFT. This can be done by splitting the input signals in segments of the length four. Do this and compute each output signal segmnt with a convolution.





Uppgift E5.11

We are gibe an infinite sequence $x(n) = (\frac{1}{2})^n u(n)$ with the Fourier transform $X(f)$. From this, we form a finite sequence $y(n)$ so that $y(n) = 0$ where $k < 0$ and $k > 9$ by computing

$$y(n) = \text{IDFT}\{Y(k)\} \quad n = 0 \ldots 9 \tag{33}$$

where

$$Y(k) = X(f)|_{f=\frac{k}{10}} \quad k = 0 \ldots 9 \tag{34}$$

Compute $y(n)$.

Uppgift E5.12

It is possible to computefiltering wiht the overlap-and-save-method. The input signal $x(n)$ is split in segments with the length N

$$x_i(n) = x(n + i(N - M + 1)) \quad 0 \leq n \leq N - 1 \tag{35}$$

where M id the lengthof filter's impulse response. For each segment we calculate

$$X_i(k) = \text{DFT}\{x_i(n)\} \tag{36}$$

and

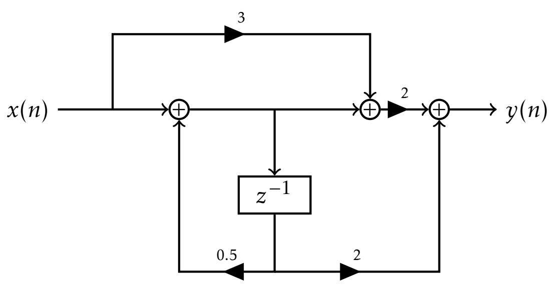
$$y_i(n) = \text{IDFT}\{X_i(k)\, H(k)\} \tag{37}$$

where $H(k)$ is the DFT$\{h(n)\}$. Decide how $y_i(n)$, $i = 0, 1, 2 \ldots$ shoudlk be combined to give $y(n) = x(n) * h(n)$. Show this in a figure.

Realizations, Chapter 9

Uppgift 9.3

Compute the system function and impulse response for the following circuit.



Uppgift 9.9

Compute the realization in Direkt Form I, Direkt Form II, cascade form, and parallel form for the systems below.

a) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$.

f) $y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$.

Which of the systems are stable?

Uppgift 9.15

Compute the parameters K_m for a lattice FIR filter with the equation

$$H(z) = A_2(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2} \tag{38}$$

MATLAB: Also compute the parameters with MATLAB.

Uppgift 9.19

a) Compute and draw the zeros for the lattice FIR filtet with the parameters$K_1 = \frac{1}{2}$, $K_2 = -\frac{1}{3}$, and $K_3 = 1$.

b) Same as in a) with $K_3 = -1$.

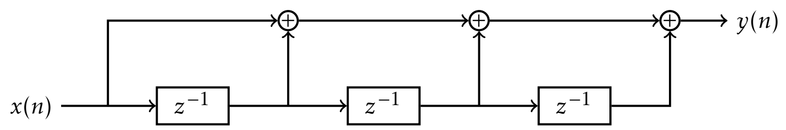
c) In a) and b) all zeros are on the unit circle. Can this result be generalized? How?

d) Compute the phase function for the filters in a) and b). Conclusion?

Examples of Filter Design

Uppgift E8.1

Given the FIR filter below.



a) Compute poles and zeros.

b) Compute and draw the amplitude function $|H(f)|$.

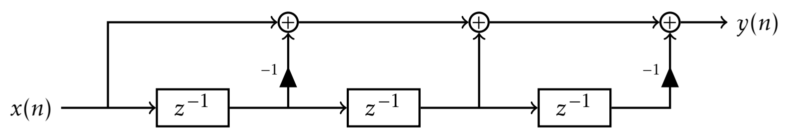
c) Compute and draw the phase function.

d) At what frequencies are $|H(f)| = 0$?

e) What type of filter (LP, BP, HP) is it?

Uppgift E8.2

Given the FIR filter below.



a) Compute poles and zeros.

b) Compute and draw the amplitude function $|H(f)|$.

c) Compute and draw the phase function.

d) At what frequencies are $|H(f)| = 0$?

e) What type of filter (LP, BP, HP) is it?

Uppgift E8.3

We want to filter a signal by creating an output signal as the moving average of the five last values, like this

$$y(n) = \frac{1}{5}\sum x(k) = 0.2\left[x(n-4)+x(n-3)+x(n-2)+x(n-1)+x(n)\right] \quad (39)$$

a) Compute the impulse response $h(n)$.

b) Compute the amplitude function $|H(f)|$.

c) Draw $|H(f)|$ for $0 \le f \le 1$.

Uppgift E8.4

Using the signal$x(n) = \sin(\pi/2n)$ you want to create the signal $y(n) = \sin(\pi/2n + \pi/3)$. Design a circuit that accomplishes this. Try with a FIR filter.

Uppgift E8.5

Compute the impulse response $h(n)$ for a circuit with the following demands:

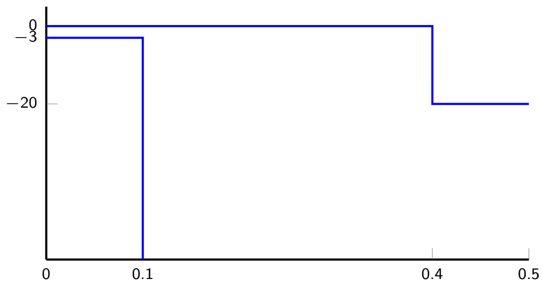
- $|H(0)| = |H(-1/5)| = |H(1/5)| = 1$
- $H(2/5) = H(-2/5) = 0$
- $h(n)$ is real
- $h(n)$ is causal
- the circuit has linear phase

MATLAB: Plot the spectrum in MATLAB and see if the demadns are fulfilled.

Uppgift E8.6

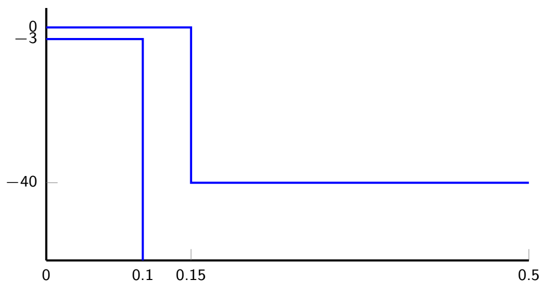
A filter $H(z)$ is given by N poles in origo and N zeros in $z = -1$. The amplification for a stationary signal is 1 (0 dB). Compute the values of N for which the damping demands in the figure are fulfilled and compute the impulse response for the minimal value of N.

MATLAB: Plot the spectrum in MATLAB and see if the demands are fulfilled.



An Example with a FIR Filter and the Windowing Method

Design a FIR filter with the damping demand below using the windoing method and the equi-ripple method.



Firts select a window for the FIR filter. Hamming has th elargest side lobe, −55 dB. Thus, we schoose a Hamming window. An approximate value for M is found in Table 5.2:

$$\frac{1.7}{M} \approx 0.15 - 0.1 \quad \Rightarrow \quad M = 34 \tag{40}$$

Note that Table 5.2 defines the lower point at −6 dB, which means that the correct value is somewhat larger. A better value is seen in Figure 36. At the lower limit $f = 0.1$, $|H(f)| = -3$ dB, we get

$$x = -0.4 = (f - f_c) \cdot M = (0.1 - f_c) \cdot M \tag{41}$$

and at $f = 0.15$, $|H(f)| = -40$ dB we get

$$x = 1.5 = (f - f_c) \cdot M = (0.15 - f_c) \cdot M \tag{42}$$

If we solve for f_c and M we get

$$f_c = 0.110 \tag{43}$$

$$M = 38 \tag{44}$$

Now let us use the equi-ripple method. The pass band ripple is

$$20 \log \frac{1+\delta_p}{1-\delta_p} = 3 \quad \Rightarrow \quad \frac{1+\delta_p}{1-\delta_p} = 10^{0.15} = 1.41 \quad \Rightarrow \quad \delta_p = \frac{0.41}{2.41} = 0.17 \tag{45}$$

and

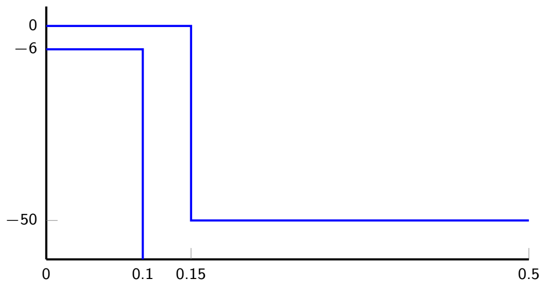
$$20 \log \delta_s = -40 \quad \Rightarrow \quad \delta_s = 0.01 \tag{46}$$

$$\begin{cases} D_\infty = \dfrac{-20 log \sqrt{0.17 \cdot 0.01} - 13}{14.6} = 1.0 \\ \Delta f = 0.15 - 0.1 = 0.05 \end{cases} \quad \Rightarrow \quad N = \frac{1.0}{0.05} + 1 = 21 \tag{47}$$

The equi-ripple filter gives the lower degree with the band-block damping 40 dB in the entire blocked band, while the FIR filter with a Hamming window has a larger side lobe, with a damping of around 55 dB.

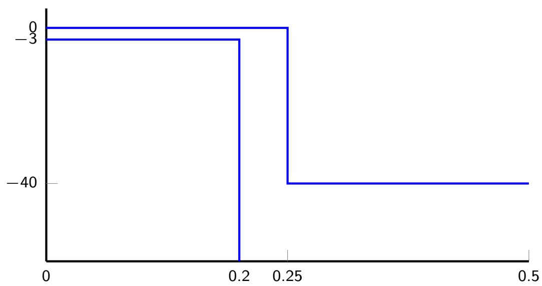
Uppgift E8.7

Compute the impulse response for a realizable filter fulfilling the demands below. The filter should have as low order as possible and exact linear phase.



Uppgift E8.8

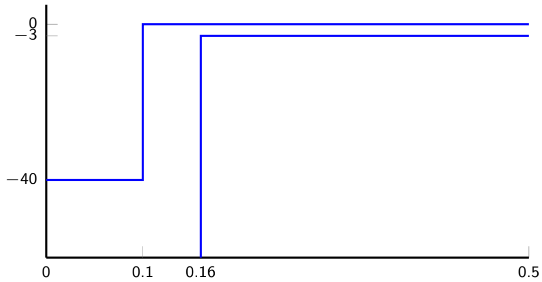
Design a FIR low pass filter fulfilling the deamdns given in the figure $H_d(f)$ with the cutoff frequency f_c and use a Hamming window, which shoudl give a band damping of at least 40 dB. Compute f_c to fulfill the demadns with a minimal M (odd). Design a realization.



MATLAB: Plot the spectrum in MATLAB and see if the demadns are fulfilled.

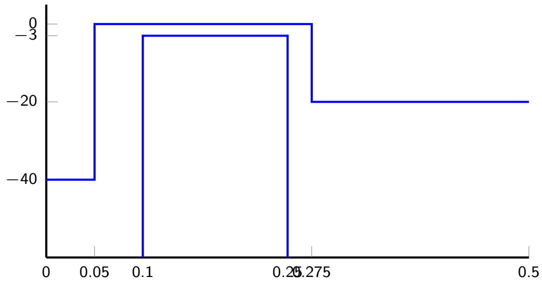
Uppgift E8.9

Design a high pass filter according to the figure below. At the frequency 0.16 the damping **must** be 3dB. Design the impulse response for a FIR filter that fulfills the demands.



Uppgift E8.10

Design a band pass filter according to the figure below. At the frequencies 0.1 and 0.25 the damping **must** be 3 dB. Design the impulse response for a FIR filter that fulfills the demands.



MATLAB: Plot the spectrum in MATLAB and see if the demadns are fulfilled.

Uppgift E8.11

The bandwidth for a band pass filter is defined as the difference in frequency between the two points where the amplitude is $|H(\omega)| = 1/\sqrt{2}$. Compute the bandwidth for an FIR filter with the impulse response

$$h(n) = \left(0.4933 \cdot \frac{\sin(0.2466\pi(n-20))}{0.2466\pi(n-20)} \cdot \cos(\pi 0.4192(n-20))\right) \tag{48}$$

$$\cdot \left(0.54 + 0.46 \cdot \cos\left(\frac{2\pi(n-20)}{40}\right)\right) \tag{49}$$

för $0 \leq n \leq 40$.

Uppgift E8.12

Two FIR filters are coupled in a cascade $H(z) = H_1(z) \cdot H_2(z)$ where

$$H_1(z) = 1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2} \tag{50}$$

Compute $H_2(z)$ so that the entire system has linear pahse and stationary amplification$|H(0)| = (1 - 2r\cos(\theta) + r^2)^2$. Give $\arg H(f)$ for $0 \leq f \leq 1/2$.

Uppgift E8.13

We wish to study a band pass signal with components in 2.5 MHz–3 MHz. The signal is sampled the the frequency $F_s = 10$ MHz and AD converted. However, the signal has generated overtones in 5 MHz–6 MHz and it has added disturbances in 9 MHz–10 MHz. We want to suppress the disturbances with an FIR filter with a dampoinmg of at least 40 dB. The filter should not affect the signal with more than 3 dB.