Exam in Systems and signals, EITF75

Tuesday, August 18, 2020

1. **Note:** Maximum grade is 3. If a higher degree is desired, then you need to complement the written exam with an oral exam. Indicate that you want the oral exam when you send in your solutions.
2. Send your solutions to fredrik.rusek@eit.lth.se no later than 09.00 Wednesday August 19.
3. **Write clearly!** If I cannot read what you write, I **will consider it as not written at all**. My decision on this matter is final, you cannot argue that I should have been able to read it later.
4. It is important to **show the intermediate steps** in arriving at an answer, otherwise you may lose points.
5. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments, if you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that "at least something must be correct", points may be deducted for everything that is wrong.
6. Problems are not arranged in an order of ascending difficulty.
7. Maximum score is 3.0p. Passing grade: 1.5p. Bonus points from hand-ins will be scaled by a factor 0.5 and added.

1. Consider the difference equation $y(n) = a(1)y(n-1) + \sum_{k=0}^{L} b(k)x(n-k)$.

   a). For $a(1) = 0.5$, $b(0) = 1$, $b(k) = 0,\ k \neq 0$, and $y(-1) = 0$, find $y(n)$ for the input signal $x(n) = u(n)$ (a step). (0.4p)

   b). For $a(1) = 0.5$, $b(0) = 1$, $b(k) = 0,\ k \neq 0$, and $y(-1) = 1$, find an input signal $x(n)$ so that $y(n) = 0,\ n \geq 0$. (0.3p)

   c). Find conditions on $a(1)$, L, $b(k)$ and $y(-1)$ so that there exists a finite energy signal $x(n)$ which results in $y(n) = 0,\ n \geq 0$. (0.3p)

2. For each of the below cases, state whether or not the signal processing description has all poles inside the unit circle.

   a). Deposit 1000 SEK on the bank and keep it there indefinitely at 2% interest rate. (0.1p)

   b). Receive salary each month on the 25th, but having spent half of your total by the 24th (i.e., on August 25, your total is your savings plus your salary, but half of that total has been spent by September 24). (0.1p)

   c). The number of students taking reexam n is 70% of those taking reexam $n-1$ plus 35% of those taking reexam $n-2$. (0.1p)

3. A continuous signal $s(t)$ has frequency support in the frequency range 1000-3000 Hz. A device observes the signal $r(t) = s(t) + \iota(t)$ where $\iota(t)$ is a disturbance in the frequency range 5000-6000 Hz. Design a digital system that eliminates the disturbance $\iota(t)$. Your system should comprise 3 parts: (1) analog-to digital conversion (sampling), (2) a discrete time filter, and (3) digital-to-analog conversion. As usual, the filter needs not to be perfect, but your solution should clearly show that you understand the principles of digital filter design. (1p)

4. A signal $x(n) = (1 + \cos(2\pi 0.3n))u(n)$ is applied as input to a system with transfer function $H(z)$ where all poles are inside the unit circle.

   a). What is the general behavior of the output at small n, i.e., describe the structure of $y(n)$ for $0 \leq n \approx< 100$. What parts does it contain? (0.1p)

   b). What is the general behavior of the output at large n, i.e., describe the structure of $y(n)$ for $n \gg 100$. What parts does it contain? (0.1p)

   c). What is the condition on $H(z)$ for having $y(n) = 0$ as $n \to \infty$. (0.3p)

   d). Implement a FIR filter $H(z)$ so that $y(n) = 0$ as $n \to \infty$. (0.2p)

Good Luck