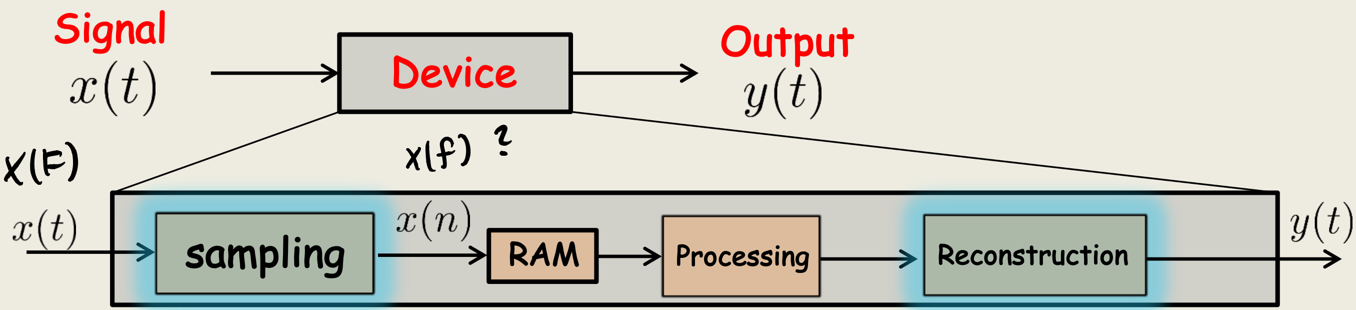
EITF75 Systems and Signals

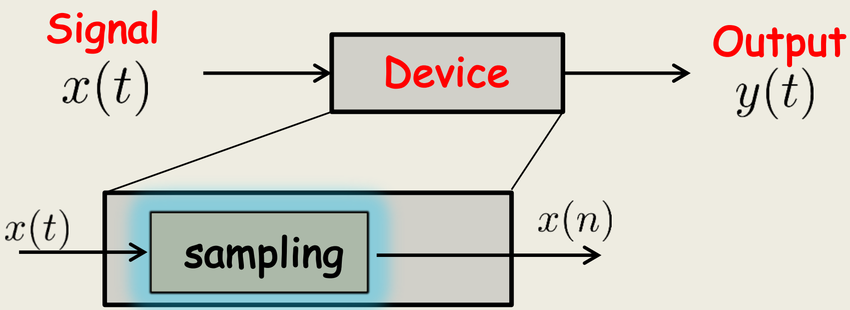


An important part of digital signal processing:
Process time-continuous signals digitally

To do so, we need

1. Study effects of sampling (A/D)
2. Study ways to implement (D/A)
3. Understand when 1&2 are optimally done

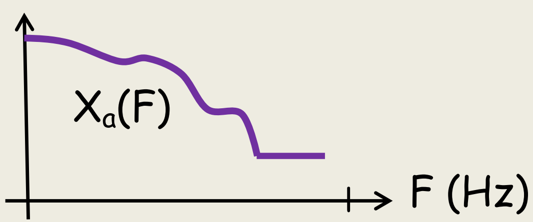
EITF75 Systems and Signals



When is it Sampling optimal? When X(f) looks the same as $X_a(F)$

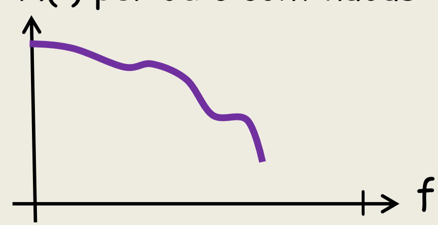
x(t) aperiodic continuous

$X_a(F)$ aperiodic continuous



x(n) aperiodic discrete

X(f) periodic continuous



Ex: F=20.000.000. (20 MHz)

f=1/2

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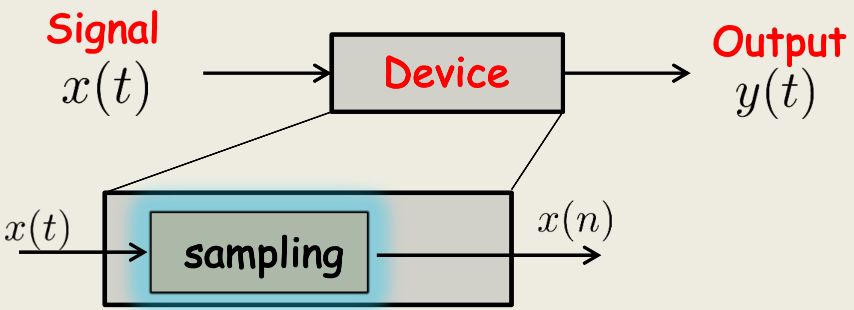


Altogether,

1. $X_a(F)$ contains all information about $x(t)$

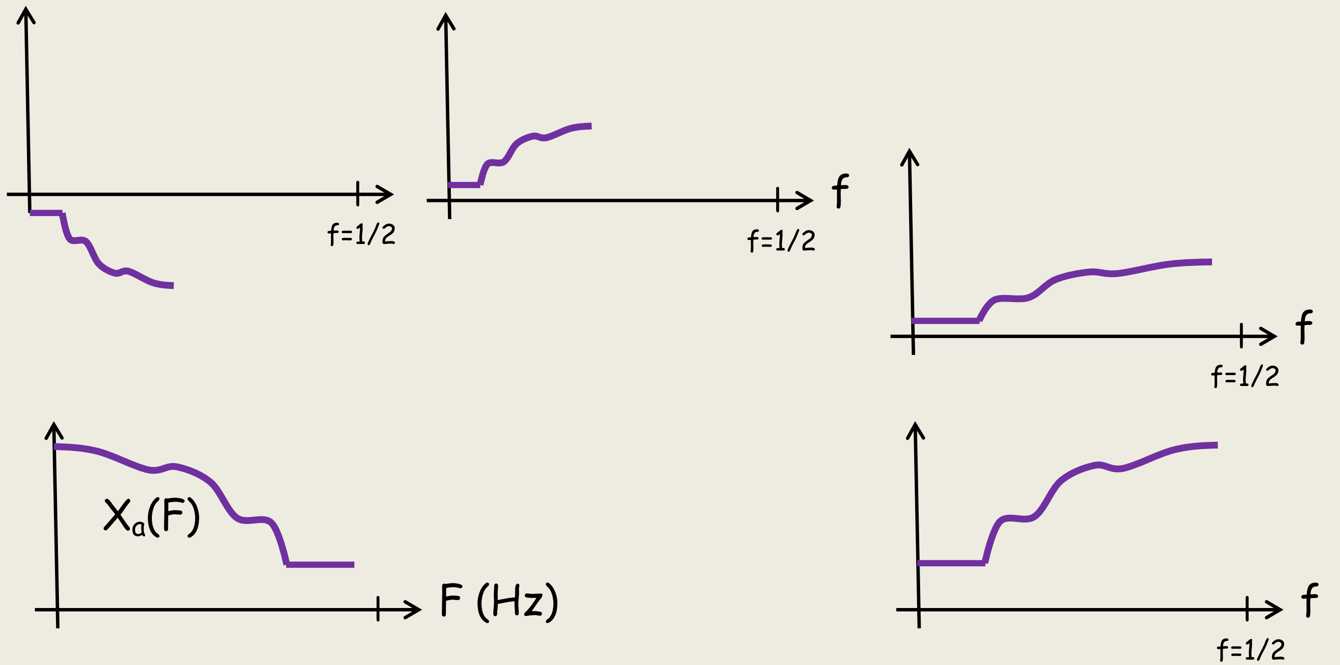
2. $X_a(F)$ is aperiodic

3. $x(n)$ sampled version of $x(t)$

4. $X(f)$ sufficient to recover $X_a(F)$ -> $x(n)$ sufficient to recover $x(t)$

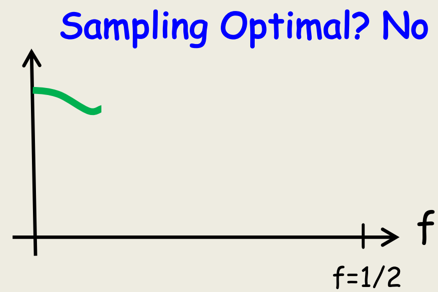
5. -> sampling optimal

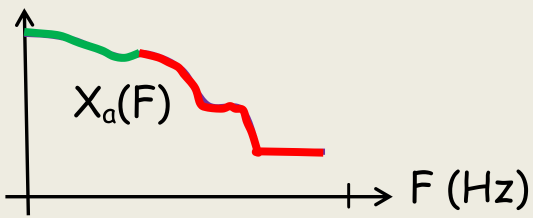
EITF75 Systems and Signals

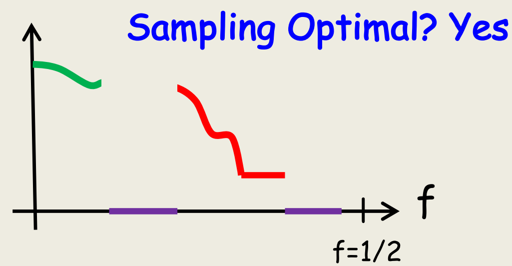


4. $X(f)$ sufficient to recover $X_a(F)$ -> $x(n)$ sufficient to recover $x(t)$
   Still true
5. -> sampling optimal

EITF75 Systems and Signals



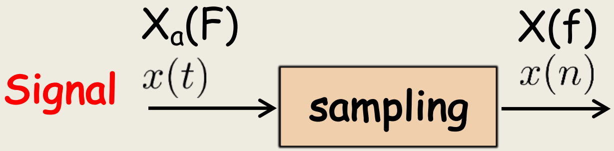




4. X(f) **sufficient to recover** $X_a(F)$ -> x(n) **sufficient to recover** x(t)

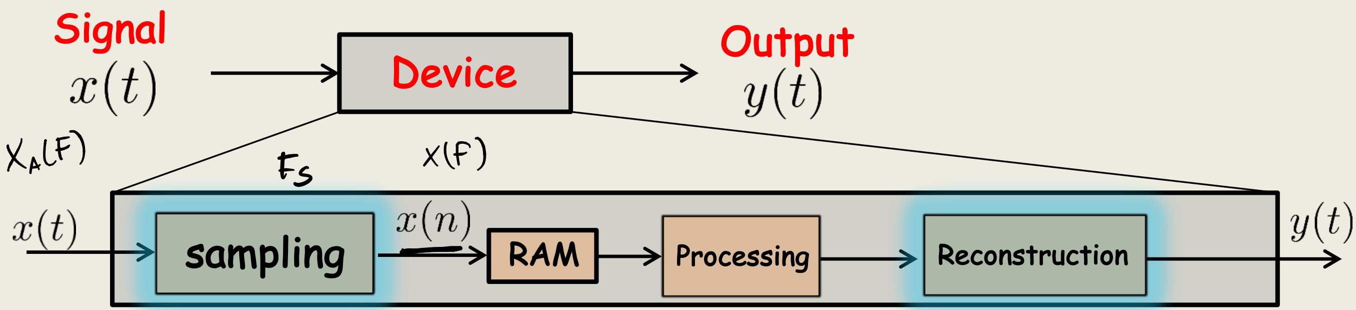
5. -> **sampling optimal**

EITF75 Systems and Signals



Status

1. We know that if $X_a(F)$ can be recovered from $X(f)$, the sampling is optimal
2. **Thus, we must study what $X(f)$ looks like**



An important part of digital signal processing:
Process time-continuous signals digitally

To do so, we need

1. Study effects of sampling (A/D). DONE

$$X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)F_s)$$

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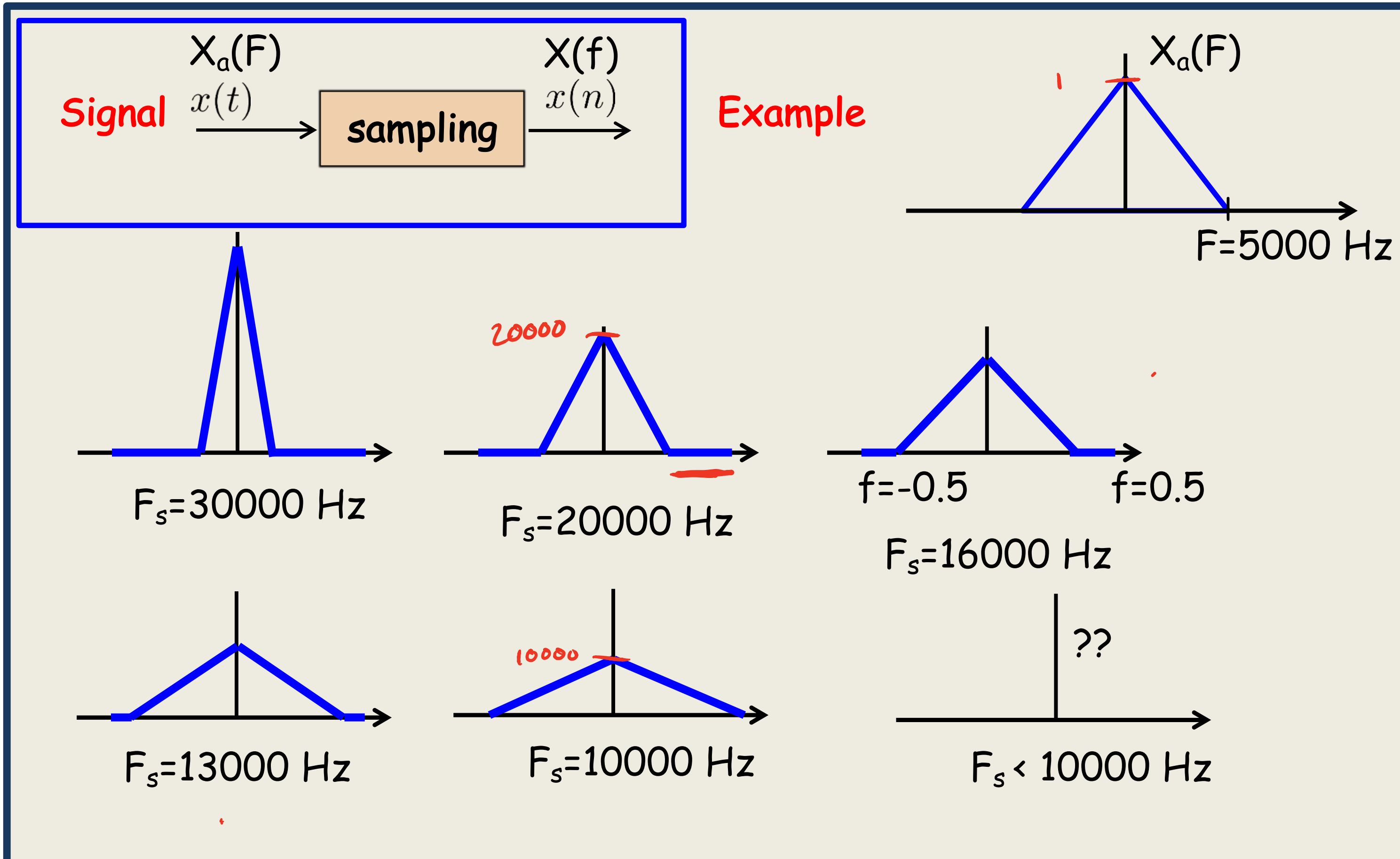

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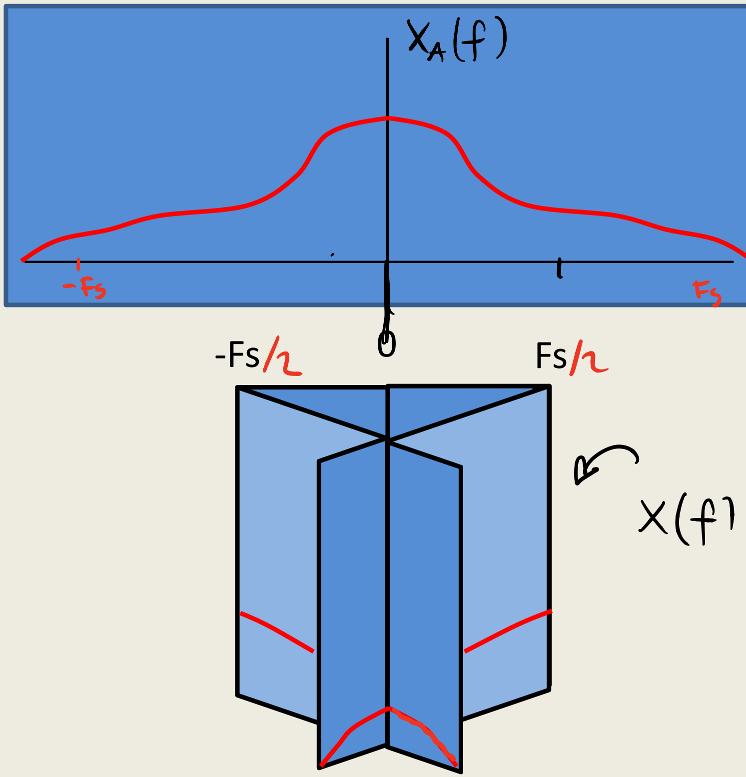
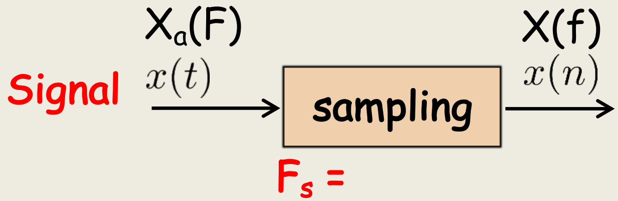
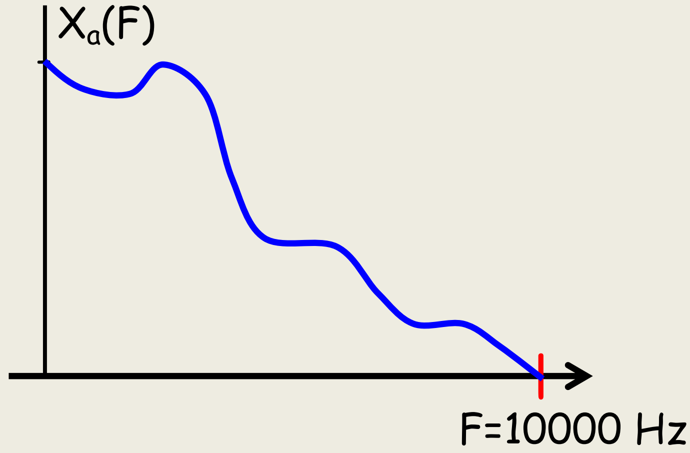
$$X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)F_s)$$ = Folding



EITF75 Systems and Signals



When is sampling lossless?



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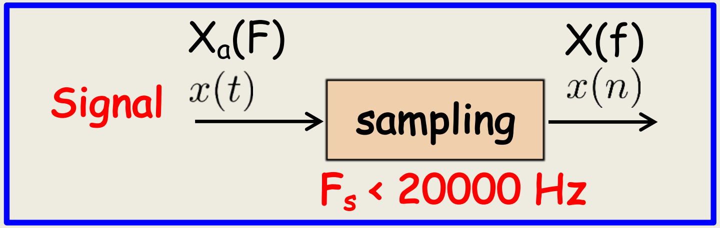

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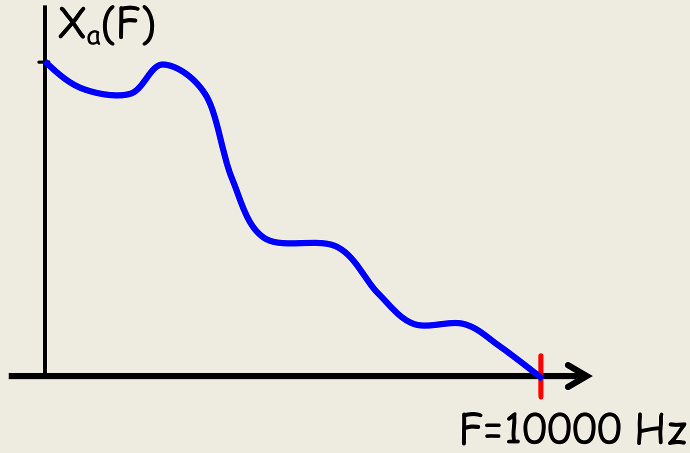

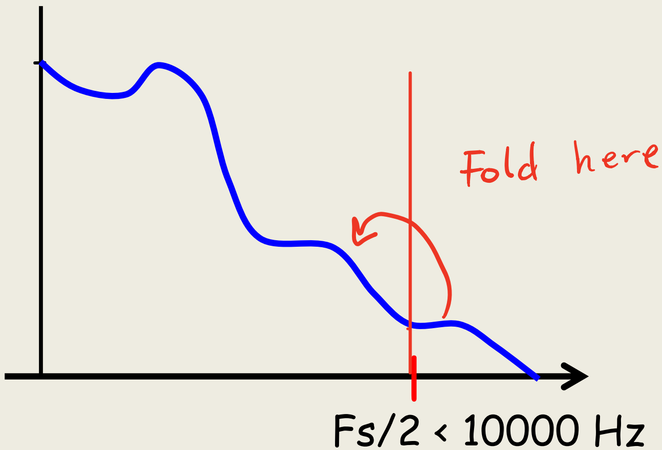

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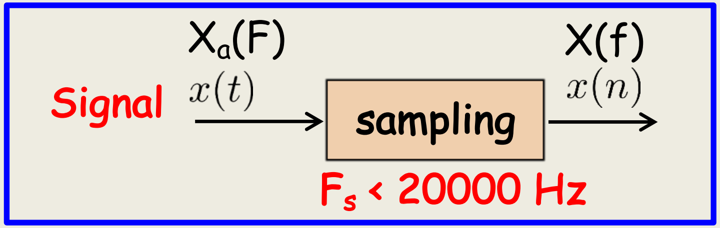

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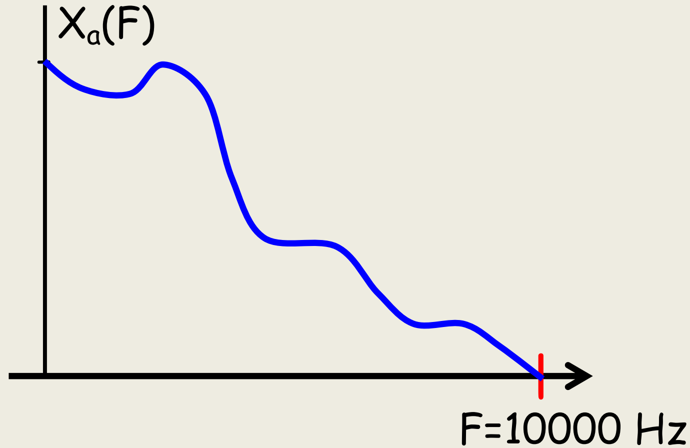
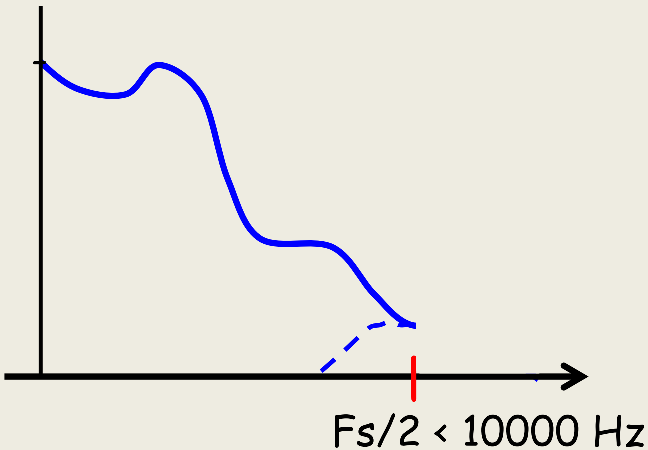
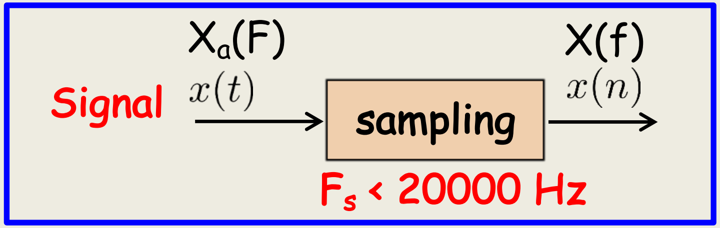




EITF75 Systems and Signals



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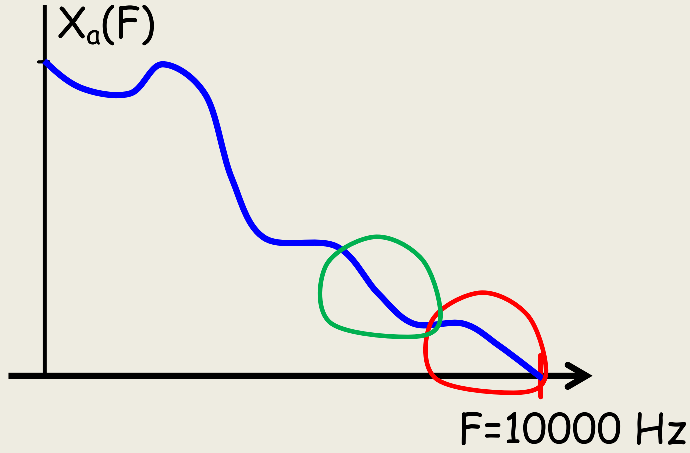
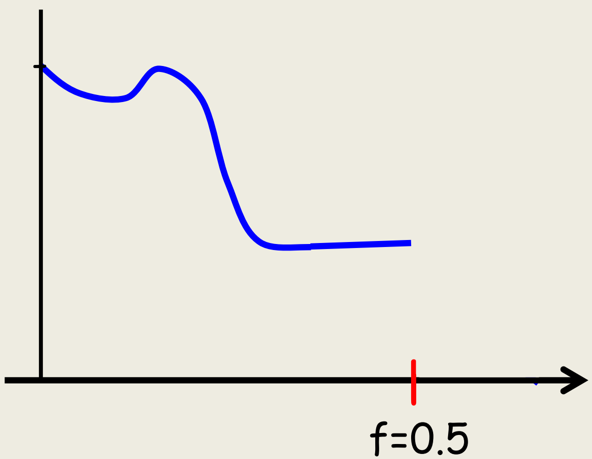




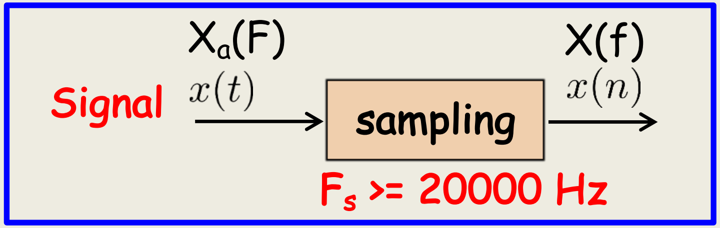
We cannot recover x(t) from x(n)

Red frequencies have been mixed with green

This <u>effect</u> is called <u>Aliasing</u>

<u>Method</u> to find X(f) is called <u>folding</u>

EITF75 Systems and Signals



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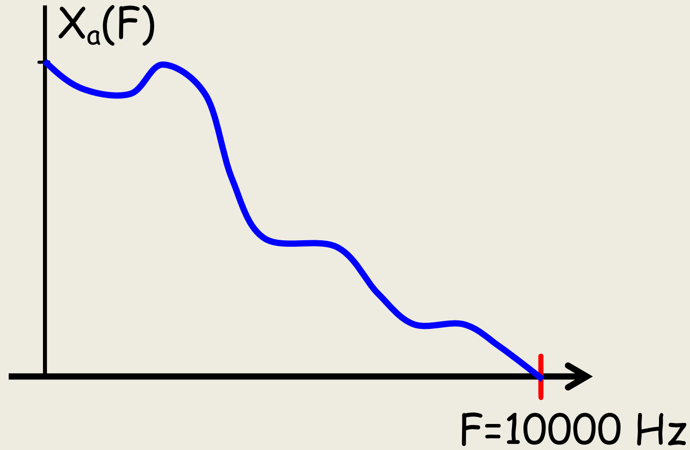
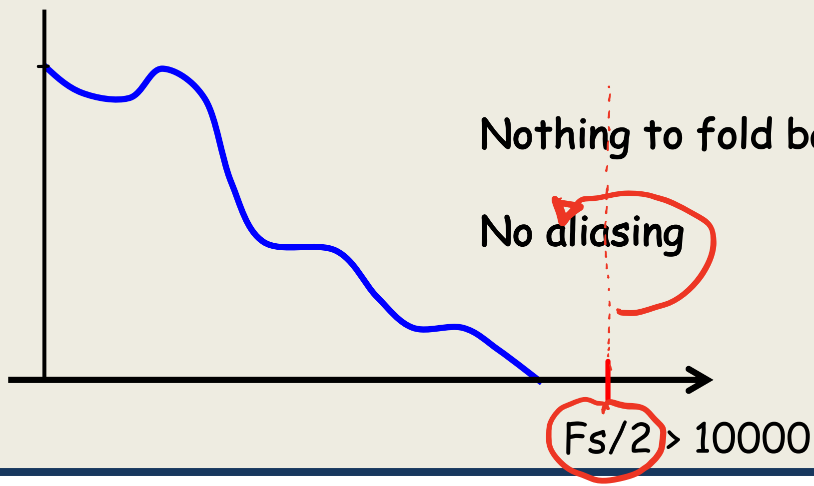
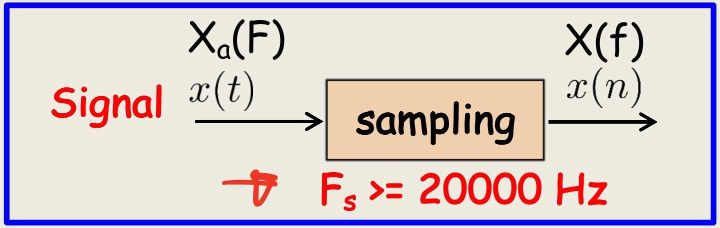
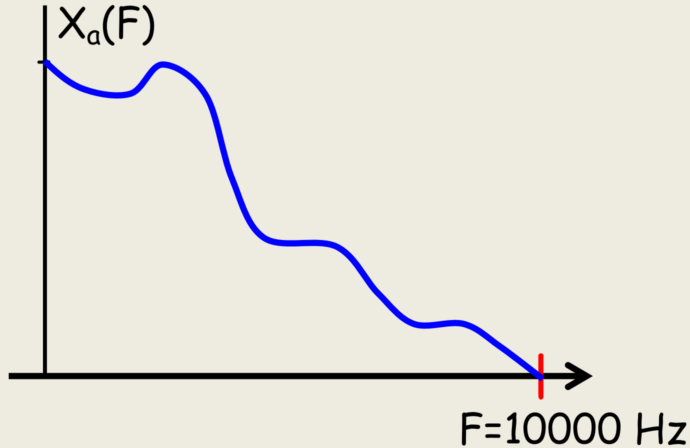


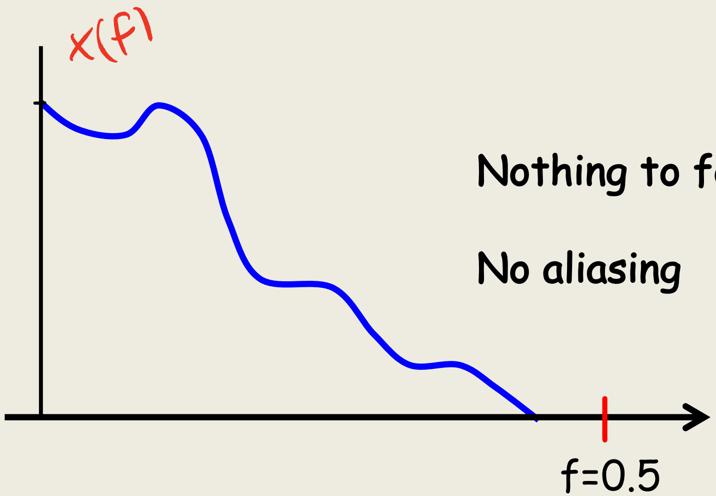


EITF75 Systems and Signals



When is sampling lossless?



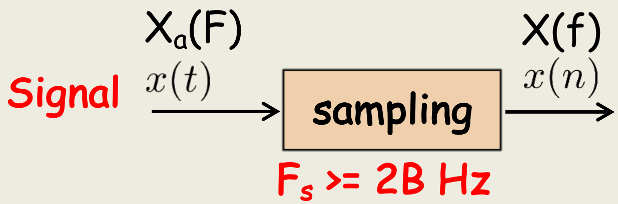


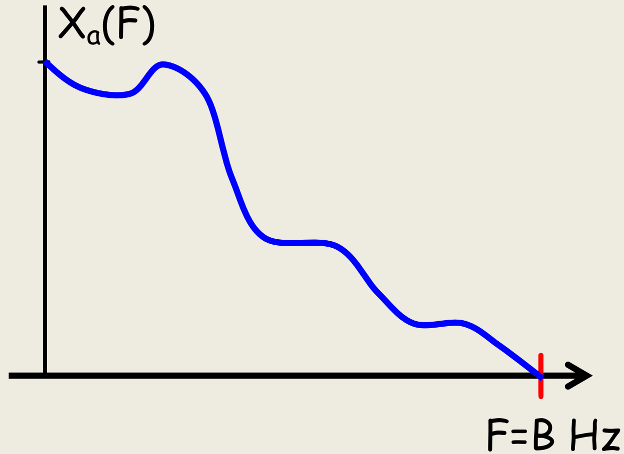
Nothing to fold back

No aliasing

We can recover x(t) from x(n)

EITF75 Systems and Signals



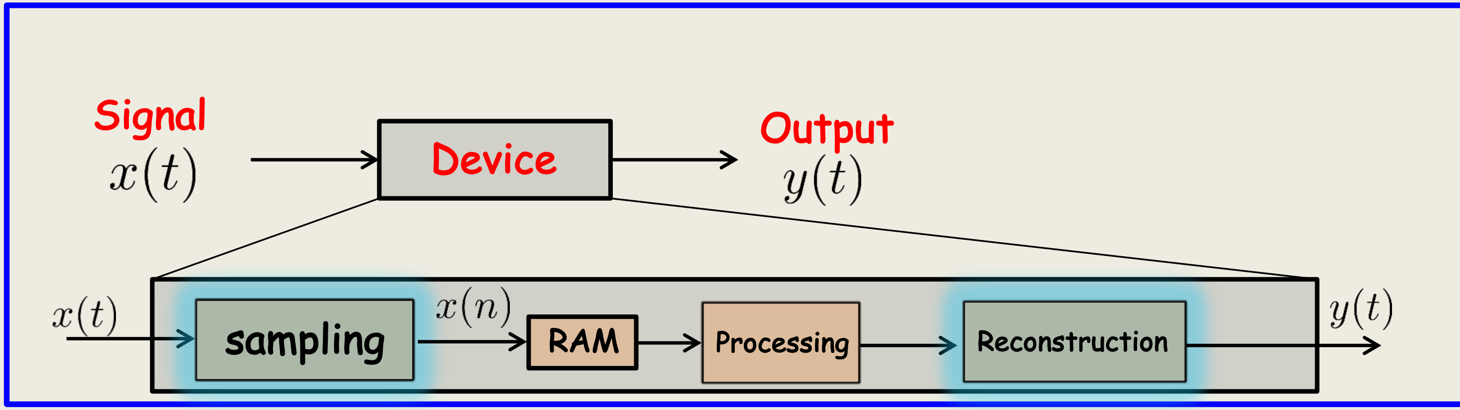


When is sampling lossless?

When F_s > 2B Hz

Sampling Theorem (Shannon 1948)

If F_s>2B, where B is the highest frequency of the analog signal, then the analog signal can be recovered from its sampled version

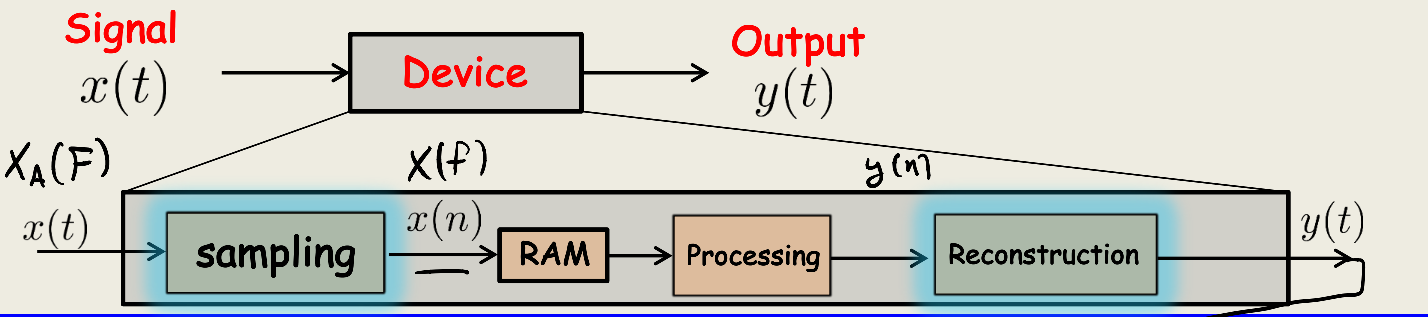


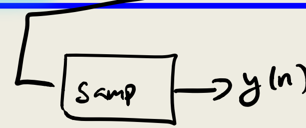
An important part of digital signal processing:
Process time-continuous signals digitally

Folding

To do so, we need

1. Study effects of sampling (A/D). DONE $\quad X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)F_s)$
2. Study ways to implement (D/A). Still unclear how to do this
3. Understand when 1 is optimally done. DONE $\quad F_s >= 2B$ Hz



An important part of digital signal processing: 

Process time-continuous signals digitally

To do so, we need

1. Study effects of sampling (A/D). DONE $$X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)F_s)$$

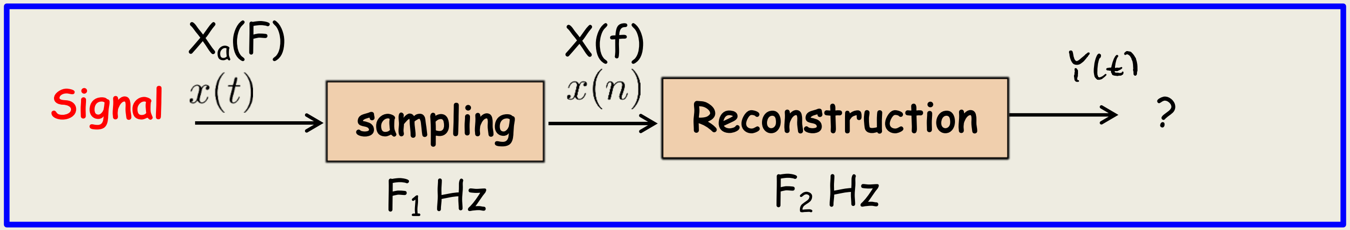
2. Study ways to implement (D/A). DONE $$x(t) = \sum_{n=-\infty}^{\infty} x(n)\mathrm{sinc}(F_s(t-n/F_s))$$

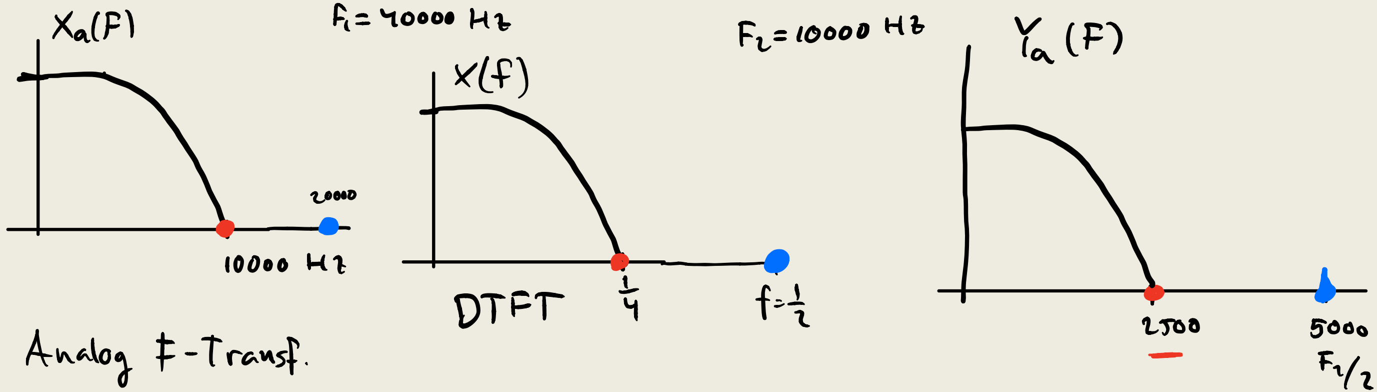
3. Understand when 1&2 are optimally done. DONE $F_s >= 2B$ Hz

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

“ $X_a(F) = X_A(F)$ ”

EITF75 Systems and Signals

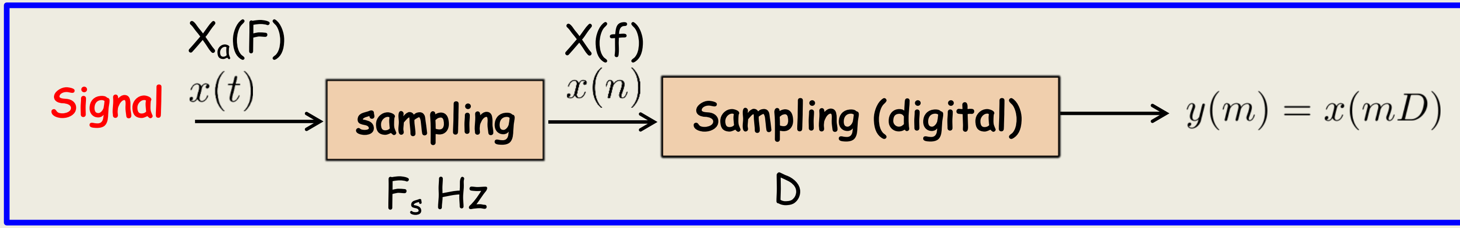




$$F_1 \geq 2 \cdot B = 2 \cdot 10000 = 20000$$

$$B \triangleq \max F \quad : \quad X_A(F) \neq 0$$

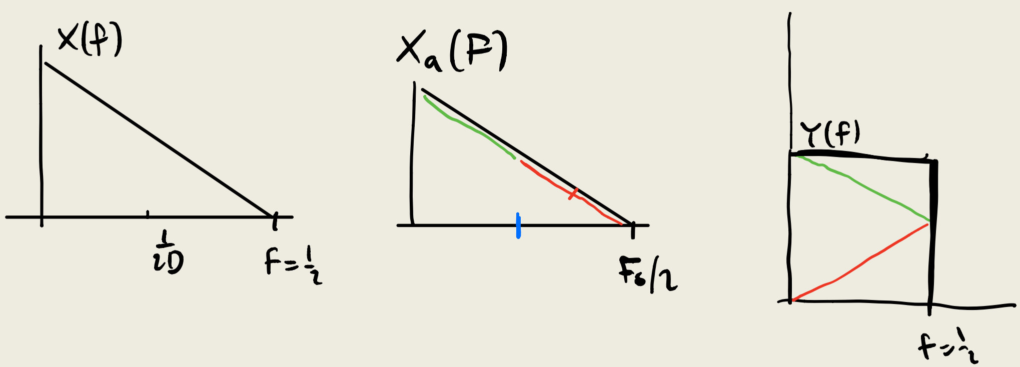
EITF75 Systems and Signals



Decimation (Used for inter connecting systems with different sampling rates)

$x(n)$: sample $x(t)$ @ F_s

$y(n)$: sample $x(t)$ @ F_s/D $\Rightarrow$ Folding at $\frac{F_s}{2D}$



$\underline{\underline{D=2}}$

Folding @ $\frac{F_s}{4}$

EITF75 Systems and Signals

Signal $\xrightarrow[x(t)]{X_a(F)}$ **sampling** (F_s Hz) $\xrightarrow[x(n)]{X(f)}$ **Up-sampling** ($\underline{I=2}$) $\longrightarrow$

$y(n) = \{x(0) \;\; 0 \;\; x(1) \; 0 \; x(2) \;\; 0 \; \cdots\}$

$\underline{Y(f)} = ?$

Interpolation (Used for inter connecting systems with different sampling rates)

$$\underline{Y(z)} = \sum_{m=-\infty}^{\infty} y(m) z^{-m}$$

$$= \sum_{m=-\infty}^{\infty} x(m) z^{-Im} = \sum x(m) (z^{I})^{-m}$$

$$= X(z^{I}) \implies \underline{Y(f) = X(If)}$$

$$Y(f) = Y(z)|_{z=e^{i2\pi f}})$$