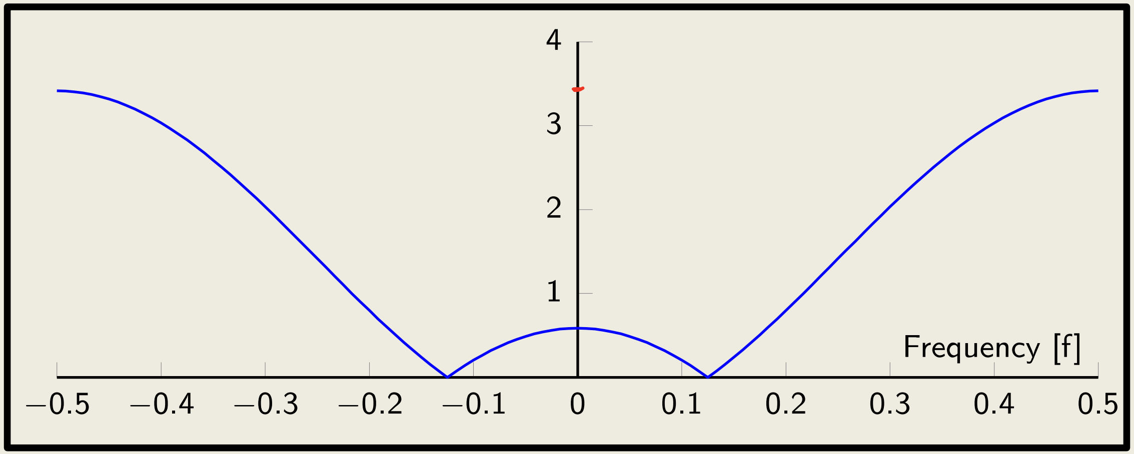
EITF75 Systems and Signals

$$|H(f)| = \left|1 - 2\cos(\omega_0)\mathrm{e}^{-i2\pi f} + \mathrm{e}^{-i4\pi f}\right|$$

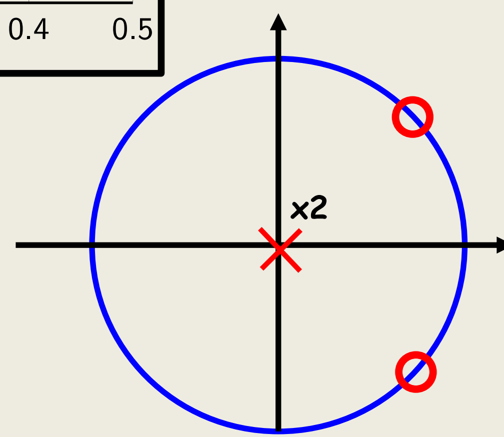


Magnitude response

FIASCO
distorts s(n)

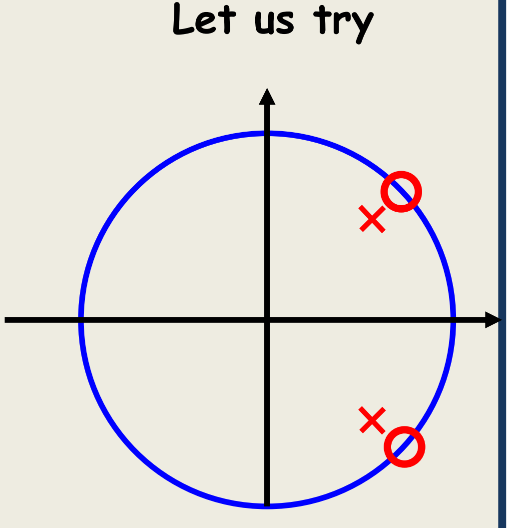
$$x(n) = s(n) + \sin(\omega_0 n)$$

$$\omega_0 = 2\pi \cdot \frac{F_0}{F_s} = 2\pi \cdot 0.125$$

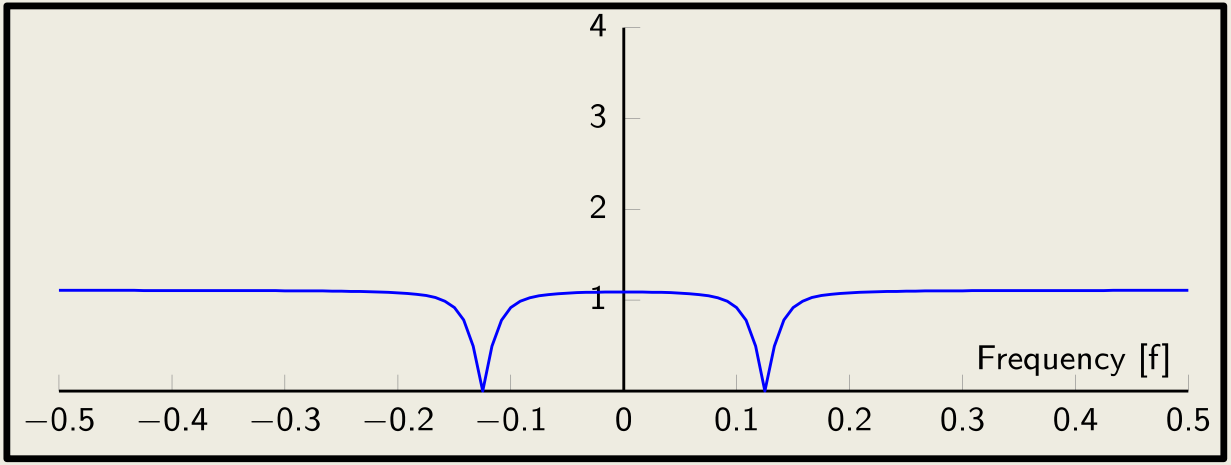


h(n) = { $\underline{1}$ -2cos(w_0) 1 }

EITF75 Systems and Signals



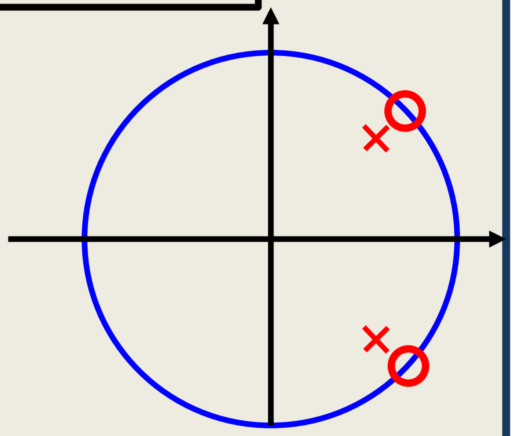
EITF75 Systems and Signals



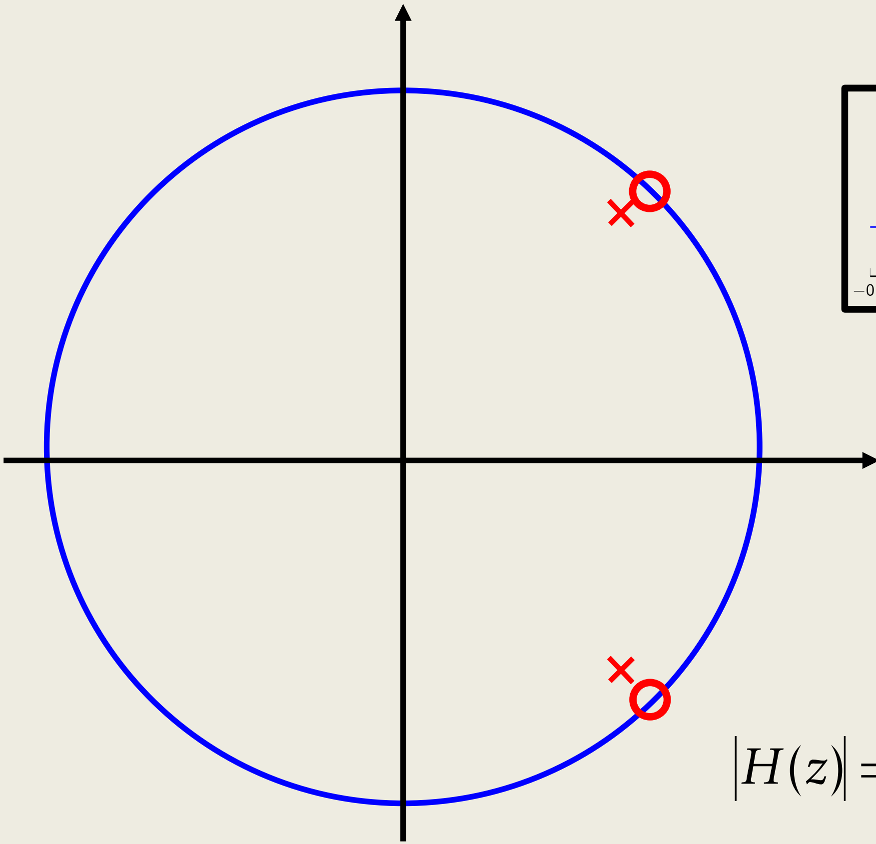
Magnitude response

Much better

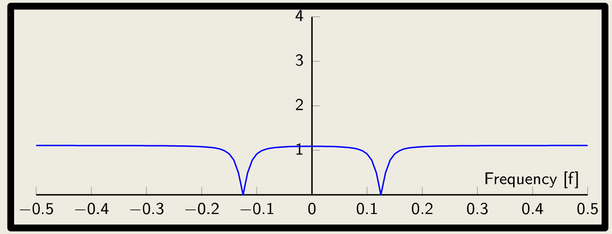
NOTCH filter



EITF75 Systems and Signals



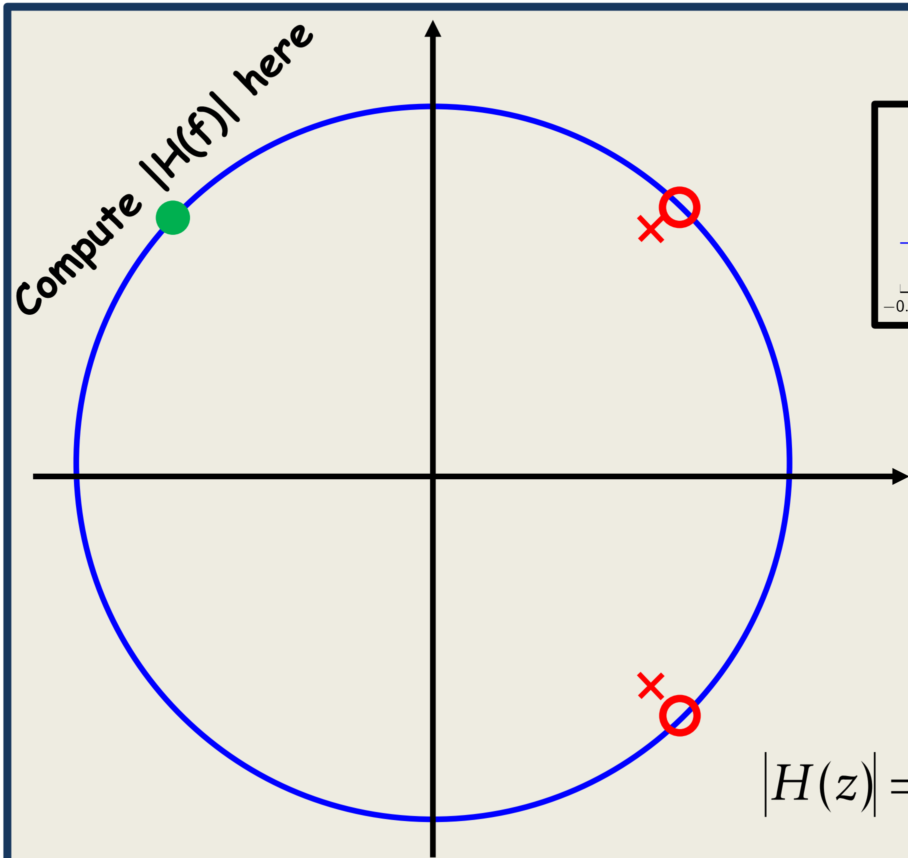
Why is it so much better?



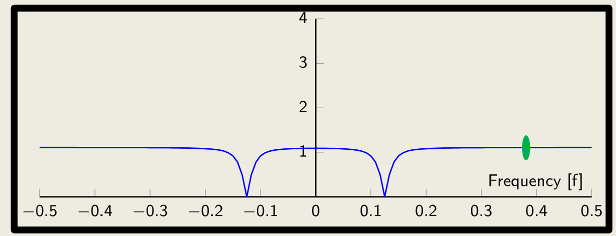
$$\left|H(z)\right| = \frac{\left|z - e^{-j\omega_0}\right| \left|z - e^{j\omega_0}\right|}{\left|z - \alpha e^{-j\omega_0}\right| \left|z - \alpha e^{j\omega_0}\right|}$$

$\alpha < 1$ $\alpha \approx 1$

EITF75 Systems and Signals



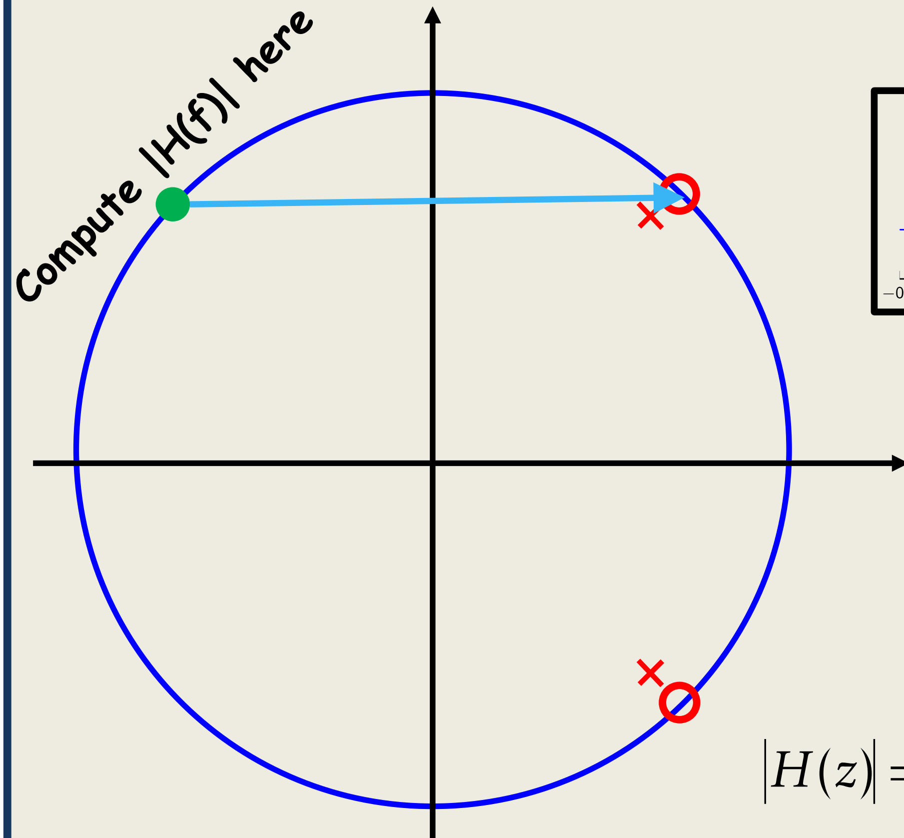
Why is it so much better?



$z = e^{j2\pi\frac{3}{8}}$

$$|H(z)| = \frac{\left|z - e^{-j\omega_0}\right| \left|z - e^{j\omega_0}\right|}{\left|z - \alpha e^{-j\omega_0}\right| \left|z - \alpha e^{j\omega_0}\right|}$$

EITF75 Systems and Signals



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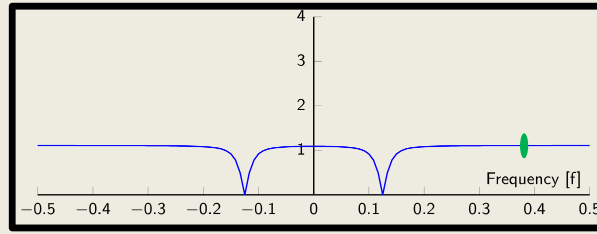
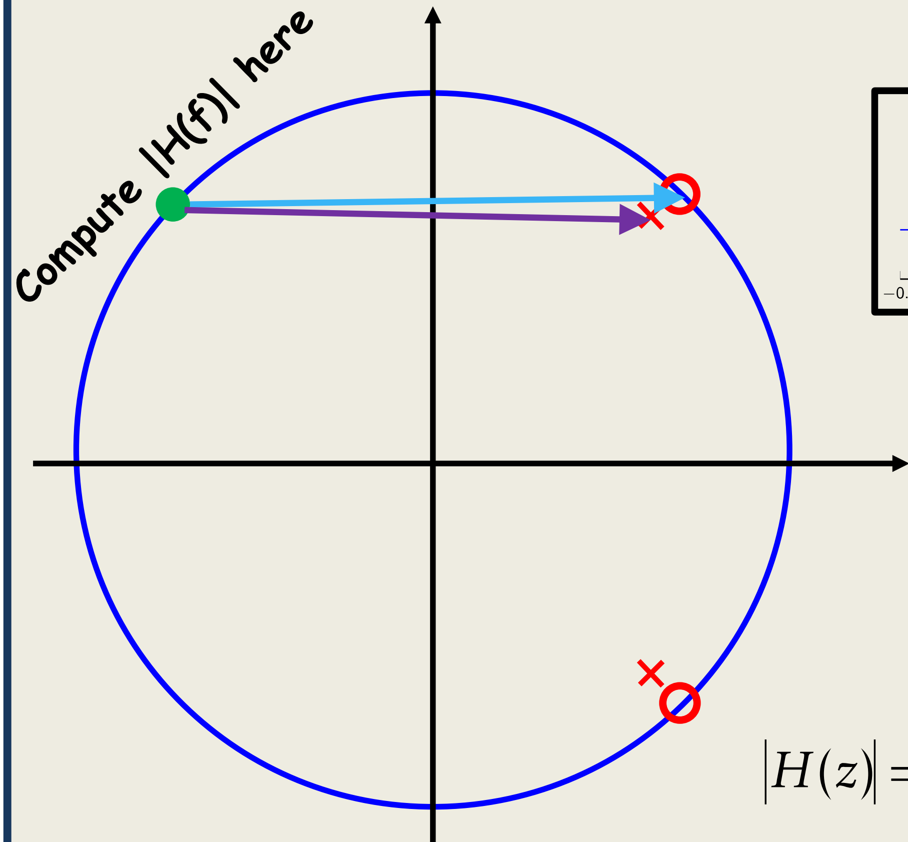


$\underline{z} = e^{j2\pi\frac{3}{8}}$

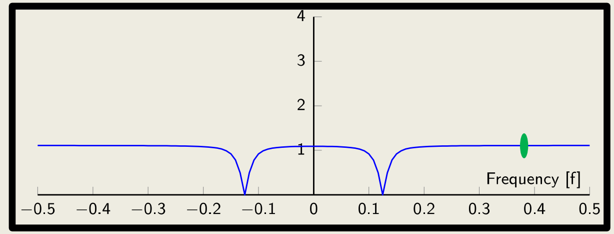
= length of the arrow

$$|H(z)| = \frac{\left|z - e^{-j\omega_0}\right| \left|z - e^{j\omega_0}\right|}{\left|z - \alpha e^{-j\omega_0}\right| \left|z - \alpha e^{j\omega_0}\right|}$$

EITF75 Systems and Signals



Why is it so much better?



$$z = e^{j2\pi\frac{3}{8}}$$

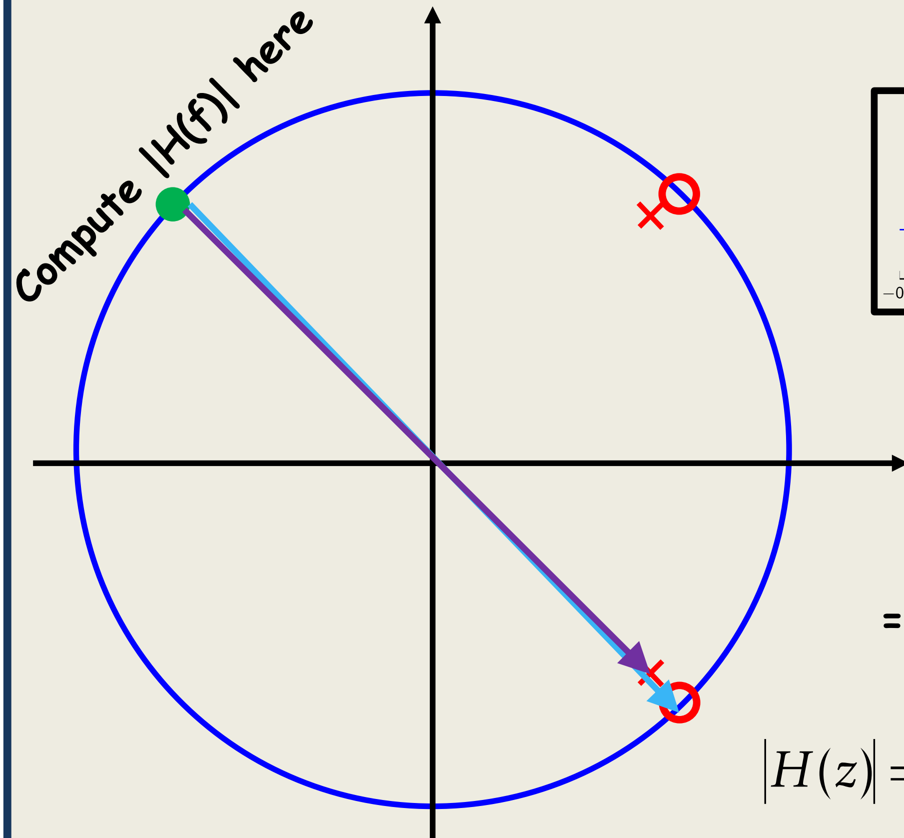
= length of the arrow

$$\left|H(z)\right| = \frac{\left|z-e^{-j\omega_0}\right|\left|z-e^{j\omega_0}\right|}{\left|z-\alpha e^{-j\omega_0}\right|\left|z-\alpha e^{j\omega_0}\right|}$$

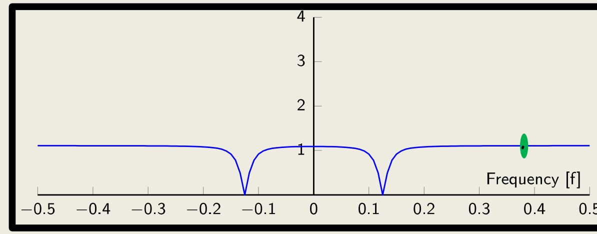
= length of the other arrow

≈ the same

EITF75 Systems and Signals



Why is it so much better?



$z = e^{j2\pi \frac{3}{8}}$

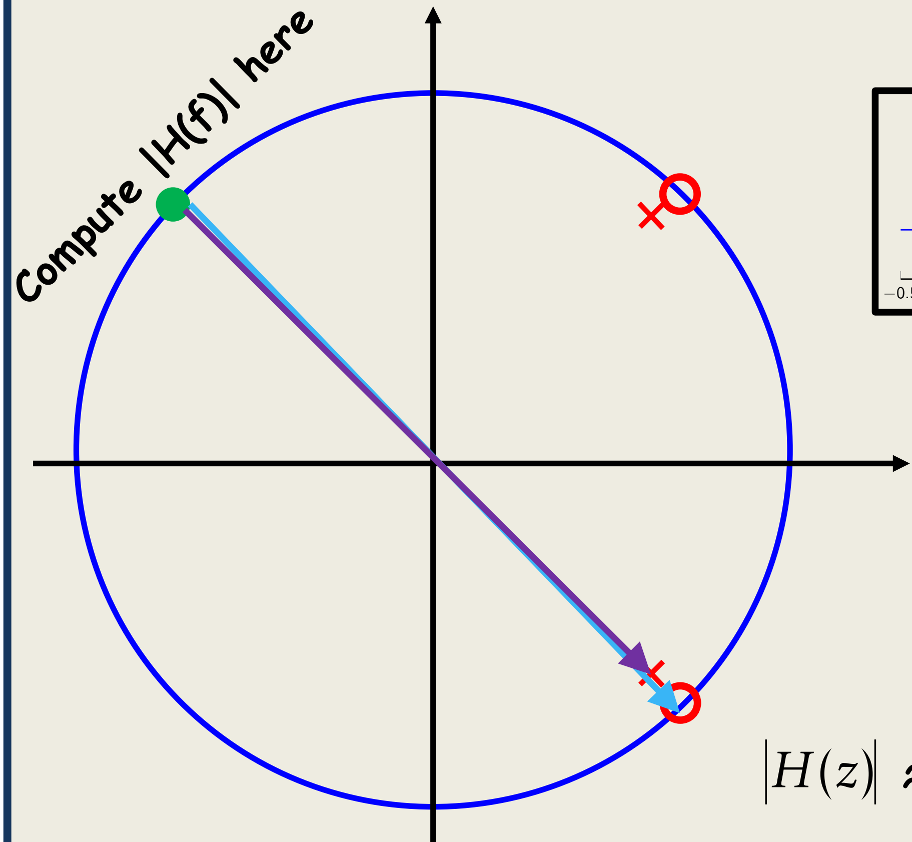
= length of the arrow

$$|H(z)| = \frac{\left|z - e^{-j\omega_0}\right|\left|z - e^{j\omega_0}\right|}{\left|z - \alpha e^{-j\omega_0}\right|\left|z - \alpha e^{j\omega_0}\right|}$$

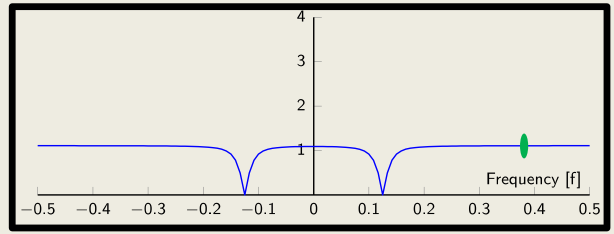
= length of the other arrow

≈ the same

EITF75 Systems and Signals



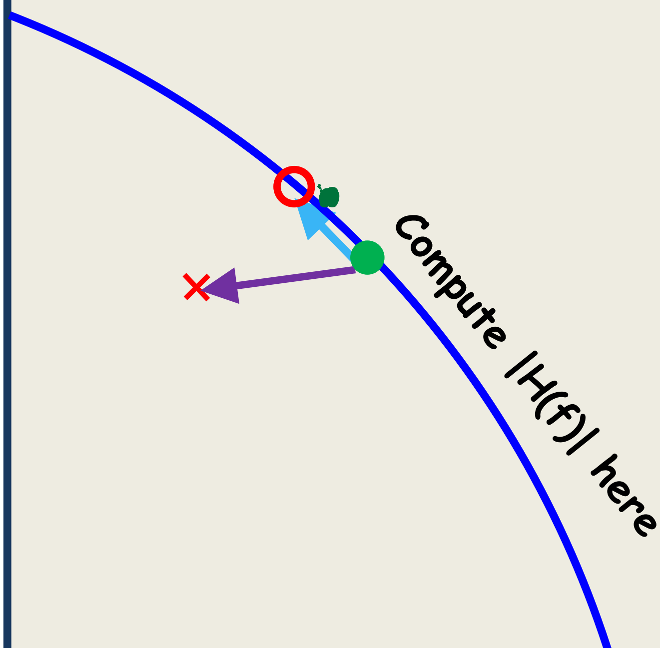
Why is it so much better?



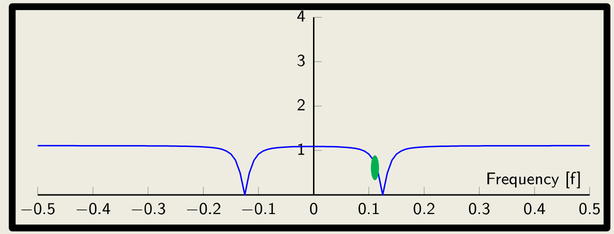
$$z = e^{i2\pi \frac{3}{8}}$$

$$\left|H(z)\right| \approx 1$$

EITF75 Systems and Signals



Why is it so much better?



$z \approx e^{i2\pi \cdot 0.1}$

Arrow lengths are different

$$|H(z)| = \frac{\left|z - e^{-j\omega_0}\right|\left|z - e^{j\omega_0}\right|}{\left|z - \alpha e^{-j\omega_0}\right|\left|z - \alpha e^{j\omega_0}\right|}$$

EITF75 Systems and Signals

Summary:

A pole close to a zero "stabilizes" the magnitude response

A causal FIR filter has poles at the origin

If no poles at all, not a causal filter

Indeed possible to remove interference digitally

Comb filters

Assume a FIR filter $$H(z) = \sum_{k=0}^{K} h(k) z^{-k}$$

Comb filters

Assume a FIR filter $$H(z) = \sum_{k=0}^{K} h(k) z^{-k}$$

Construct another filter as $$H_L(z) = \sum_{k=0}^{K} h(k) z^{-kL}$$

Comb filters

Assume a FIR filter $H(z) = \sum_{k=0}^{K} h(k) z^{-k}$

Construct another filter as $H_L(z) = \sum_{k=0}^{K} h(k) z^{-kL}$

Fourier transform $H_L(f) = \sum_{k=0}^{K} h(k) \mathrm{e}^{-i2\pi kLf} = H(fL)$

Discrete-time Fourier transform

From Wikipedia, the free encyclopedia

Properties of discrete-time Fourier transforms [edit]

Time Expansion	$\begin{cases} x[n/M] & n = \text{multiple of M} \\ 0 & \text{otherwise} \end{cases}$	$X_{2\pi}(M\omega)$

Fourier transform $H_L(f) = \sum_{k=0}^{K} h(k) e^{-i2\pi kLf} = H(fL)$

EITF75 Systems and Signals

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Fourier transform $$H_L(f) = \sum_{k=0}^{K} h(k) e^{-i2\pi kLf} = H(fL)$$

$$h_L(nL) = h(n)$$

Comb filters

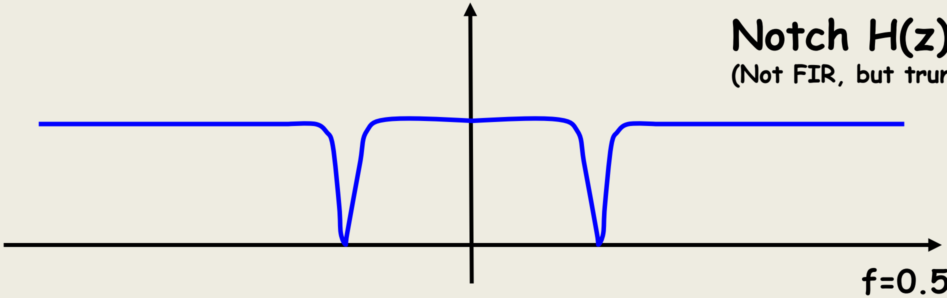
Assume a FIR filter $$H(z)=\sum_{k=0}^{K}h(k)z^{-k}$$

Construct another filter as $$H_L(z)=\sum_{k=0}^{K}h(k)z^{-kL}$$

Fourier transform $$H_L(f)=\sum_{k=0}^{K}h(k)e^{-i2\pi kLf}=H(fL)$$

Notch H(z)

(Not FIR, but truncate to large length)

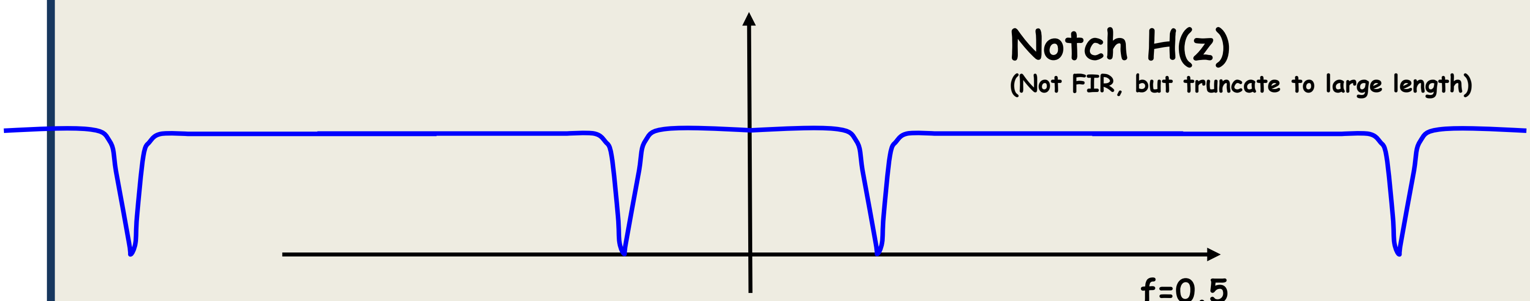


Comb filters

Assume a FIR filter $H(z) = \sum_{k=0}^{K} h(k) z^{-k}$

Construct another filter as $H_L(z) = \sum_{k=0}^{K} h(k) z^{-kL}$

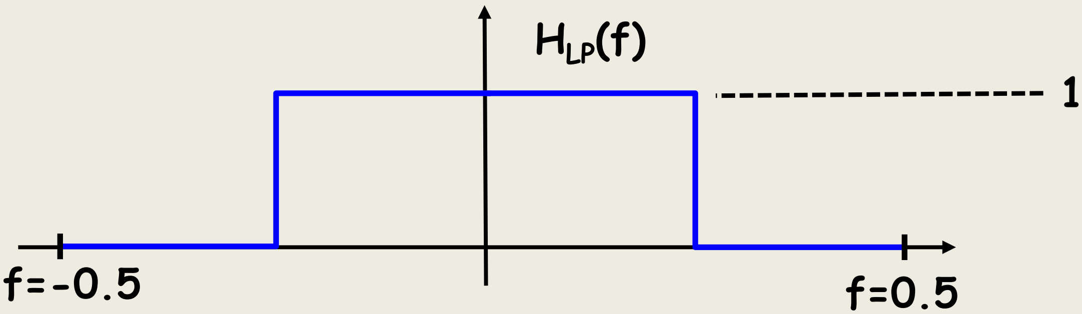
Fourier transform $H_L(f) = \sum_{k=0}^{K} h(k) e^{-i2\pi kLf} = H(fL)$



EITF75 Systems and Signals

Low-pass to High-pass conversion

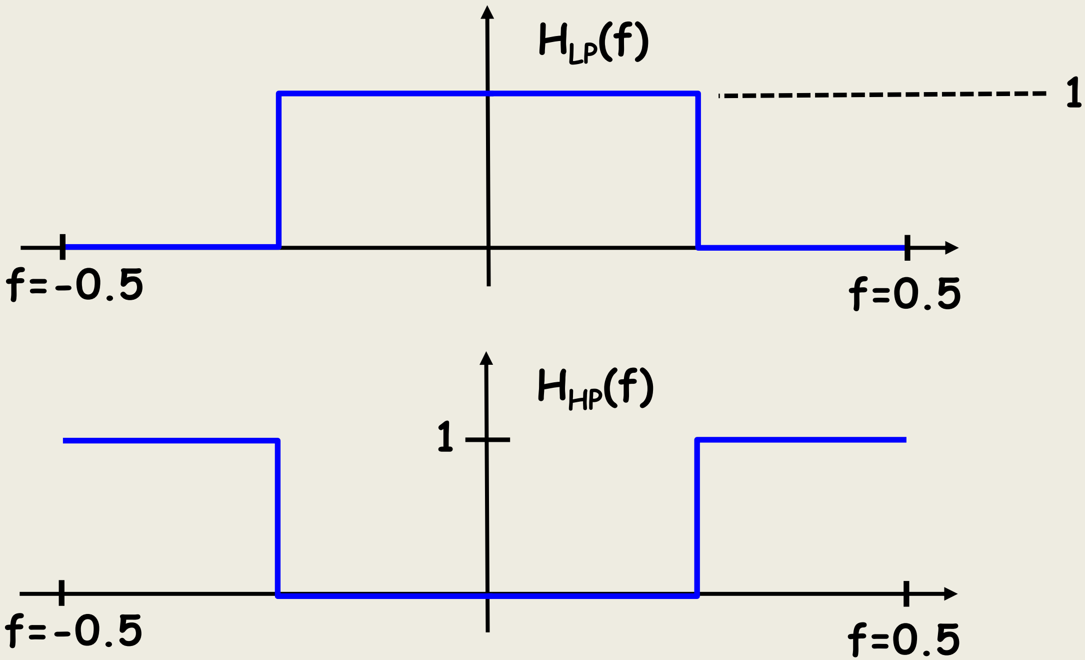
Method A



EITF75 Systems and Signals

Low-pass to High-pass conversion

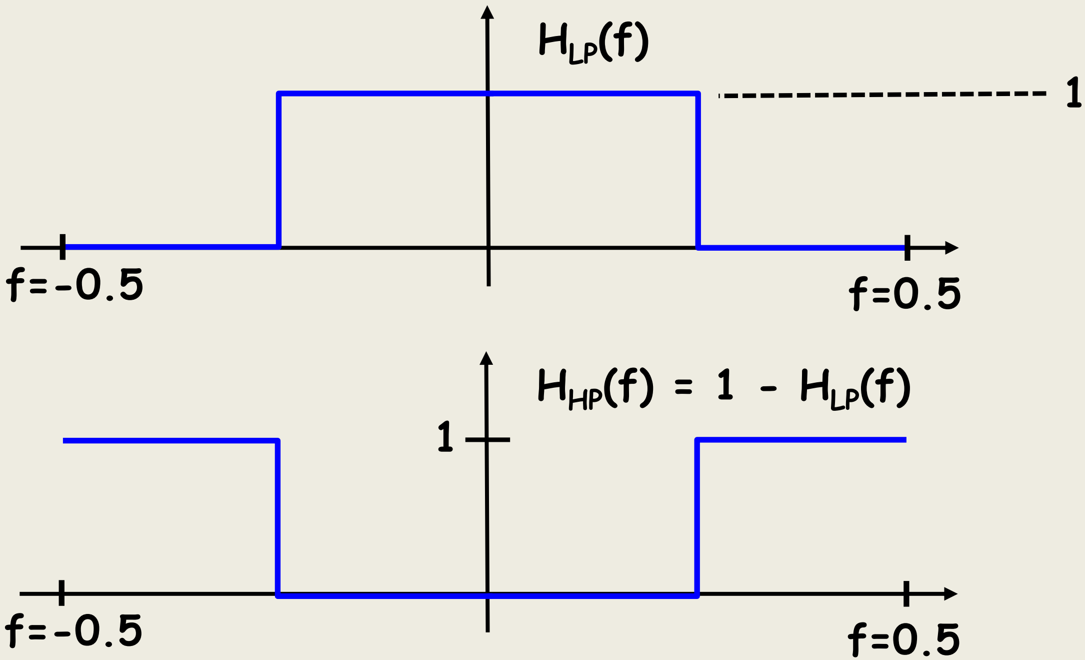
Method A



EITF75 Systems and Signals

Low-pass to High-pass conversion

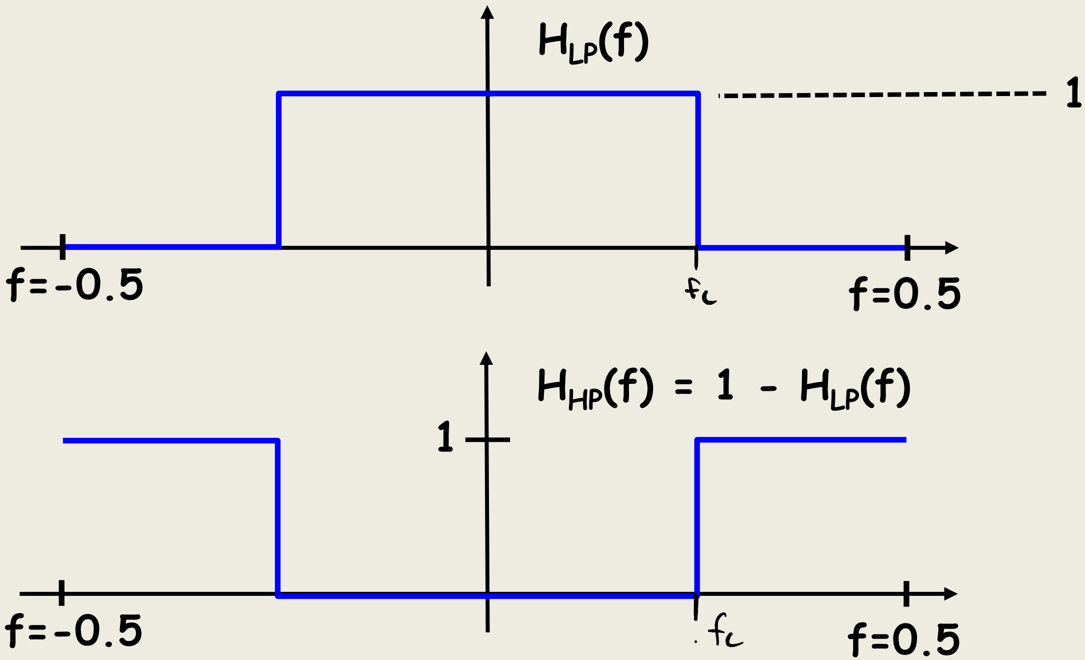
Method A



EITF75 Systems and Signals

Low-pass to High-pass conversion

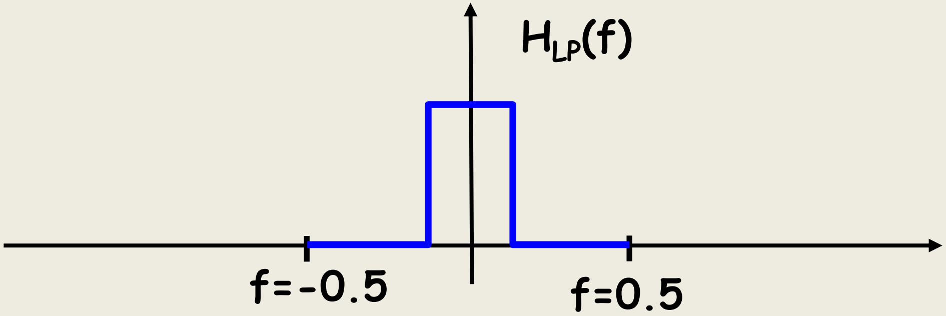
Method A $$h_{\mathrm{HP}}(n) = \delta(n) - h_{\mathrm{LP}}(n)$$



EITF75 Systems and Signals

Low-pass to High-pass conversion

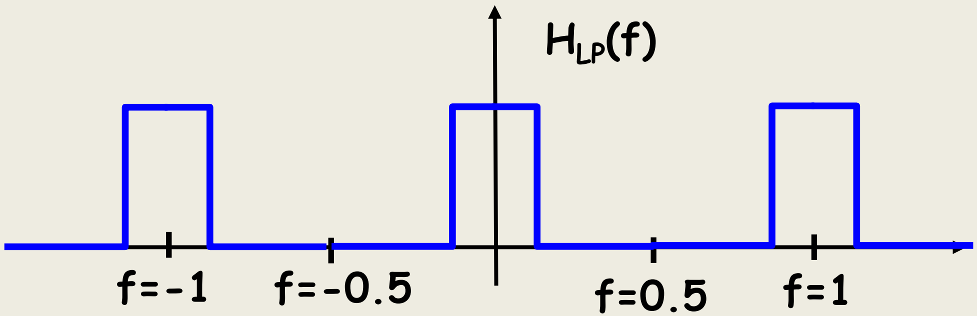
Method B



EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B

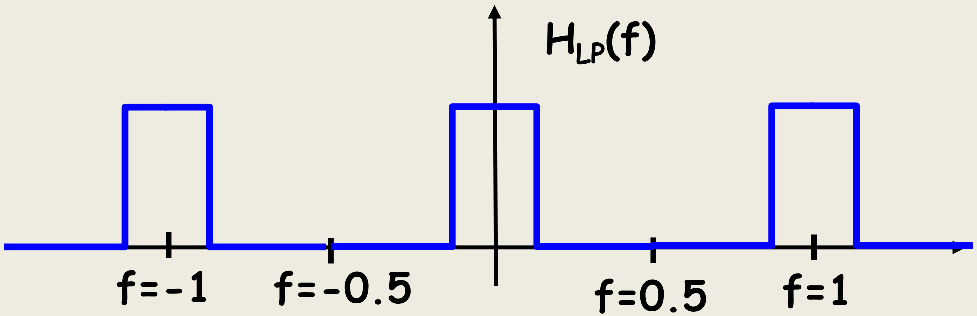


$H_{LP}(f)$ is periodic with period 1

EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B



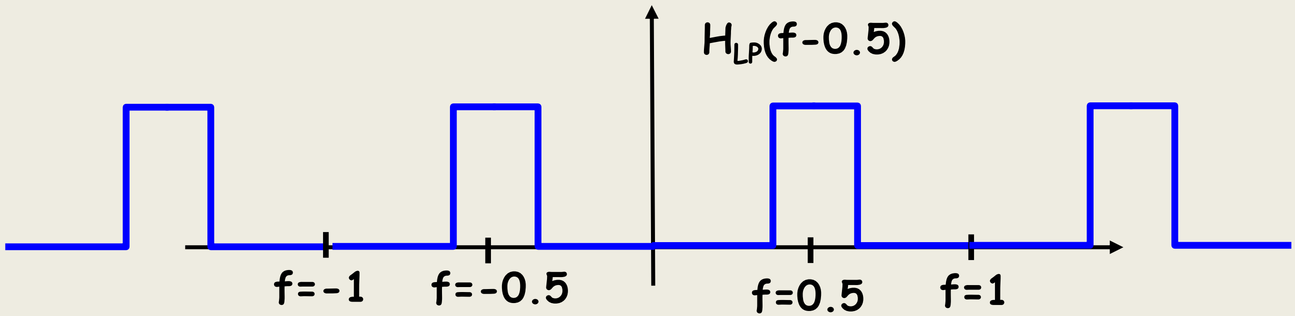
$H_{LP}(f)$ is periodic with period 1

Next slide: Study $H_{LP}(f-0.5)$

EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B



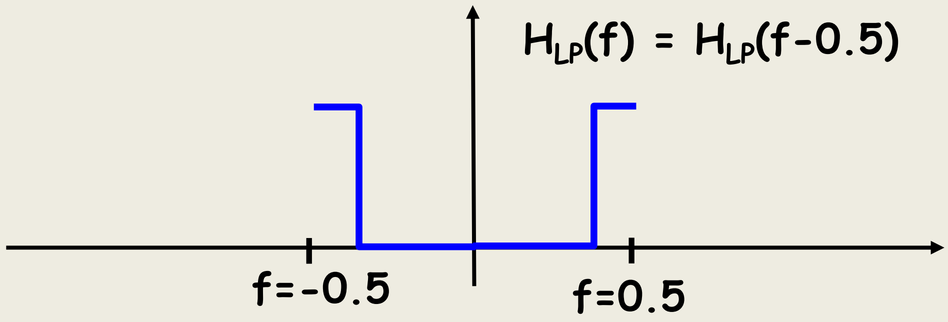
$h(n) \longleftrightarrow H_{LP}(f)$

$? \longleftrightarrow H_{LP}(f-0.5)$

EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B



This is a high pass filter

EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B

$H_{LP}(f) = H_{LP}(f-0.5)$

Discrete-time Fourier transform

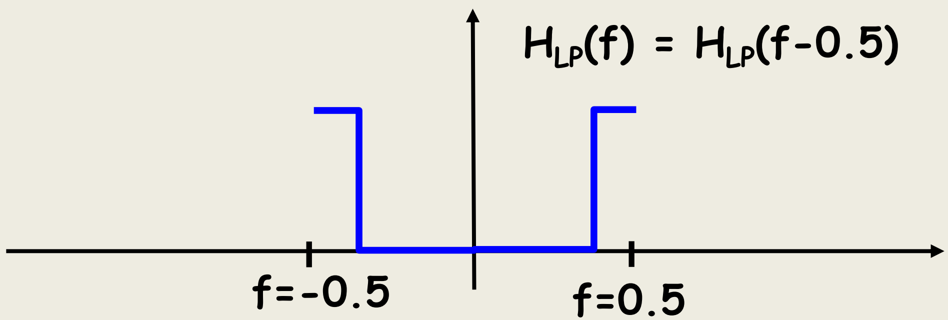
From Wikipedia, the free encyclopedia

Properties of discrete-time Fourier transforms [edit]

Shift in time / Modulation in frequency	$x[n-k]$	$X_{2\pi}(\omega) \cdot e^{-i\omega k}$
Shift in frequency / Modulation in time	$x[n] \cdot e^{2\pi a n i}$ $e^{\pi n i}$	$X_{2\pi}(f-a)$ $a = \frac{1}{2}$

Low-pass to High-pass conversion

Method B



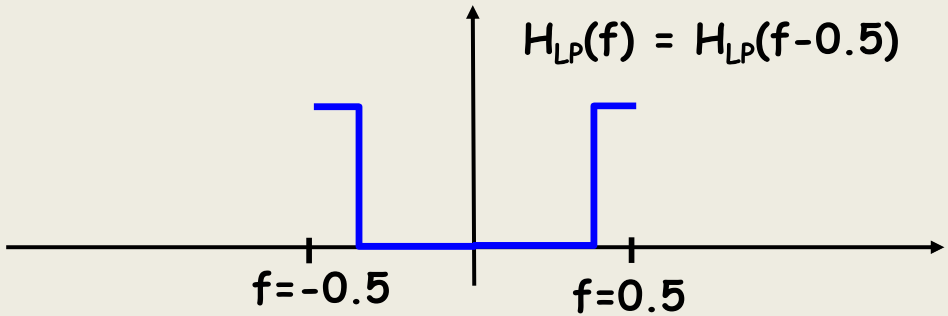
This is a high pass filter

$$h_{\mathrm{HP}}(n) = \mathrm{e}^{i\pi n} h_{\mathrm{LP}}(n)$$

EITF75 Systems and Signals

Low-pass to High-pass conversion

Method B $h_{\mathrm{HP}}(n) = (-1)^n h_{\mathrm{LP}}(n)$



This is a high pass filter

$$h_{\mathrm{HP}}(n) = \mathrm{e}^{i\pi n} h_{\mathrm{LP}}(n)$$

EITF75 Systems and Signals

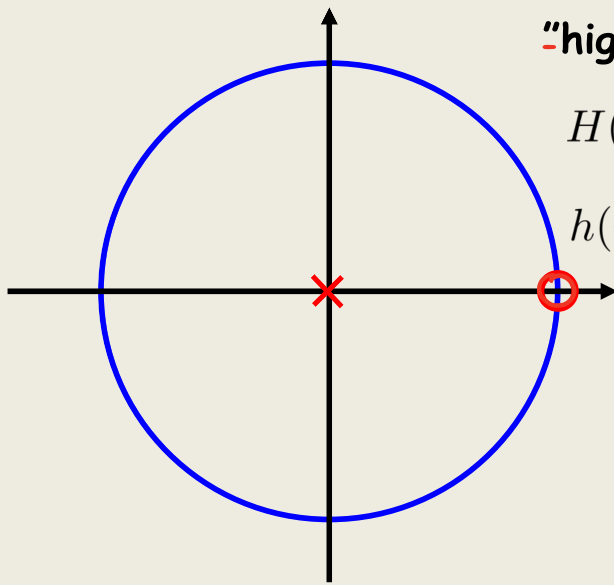
Viewed in another way

If $h_{\mathrm{HP}}(n) = (-1)^n h_{\mathrm{LP}}(n)$ **then** $h_{\mathrm{LP}}(n) = (-1)^n h_{\mathrm{HP}}(n)$

EITF75 Systems and Signals

Viewed in another way

If $h_{\mathrm{HP}}(n) = (-1)^n h_{\mathrm{LP}}(n)$ **then** $h_{\mathrm{LP}}(n) = (-1)^n h_{\mathrm{HP}}(n)$



"high pass" $z_0 = 1$

$$H(z) = (z-1) = \frac{1-z^{-1}}{z^{-1}}$$

$$h(n) = \{1 \quad -1\}$$

EITF75 Systems and Signals

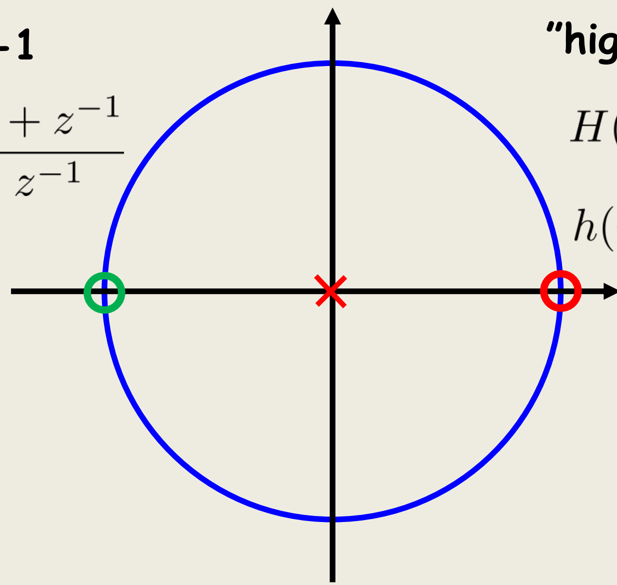
Viewed in another way

If $h_{\mathrm{HP}}(n) = (-1)^n h_{\mathrm{LP}}(n)$ then $h_{\mathrm{LP}}(n) = (-1)^n h_{\mathrm{HP}}(n)$

"low pass" $z_0 = -1$

$$H(z) = (z+1) = \frac{1+z^{-1}}{z^{-1}}$$

$$h(n) = \{1 \ \ 1\}$$



"high pass" $z_0 = 1$

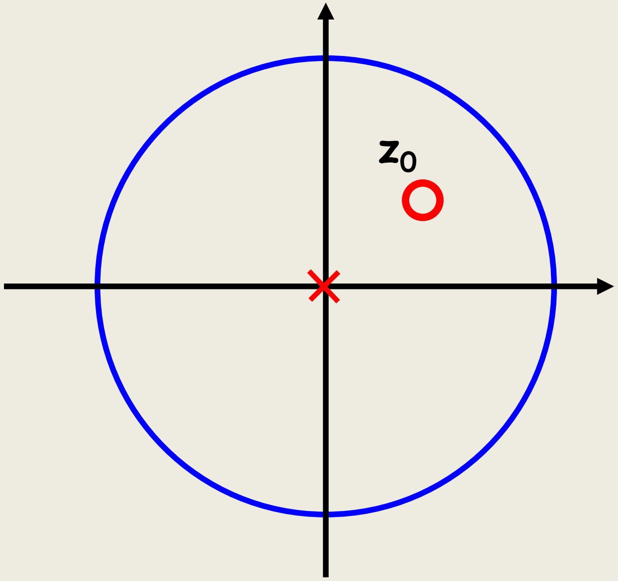
$$H(z) = (z-1) = \frac{1-z^{-1}}{z^{-1}}$$

$$h(n) = \{1 \ \ -1\}$$

EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

Linear phase <-> If z_0 is a zero, so is z_0^{-1}



EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

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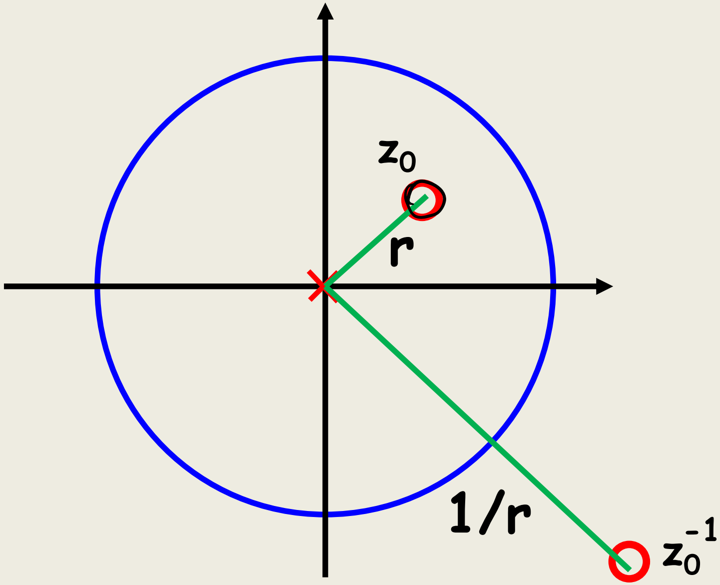

EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

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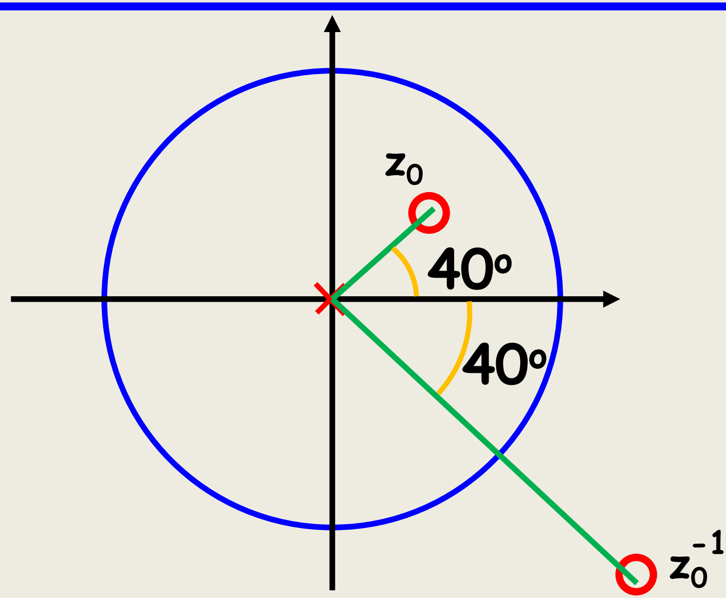

EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

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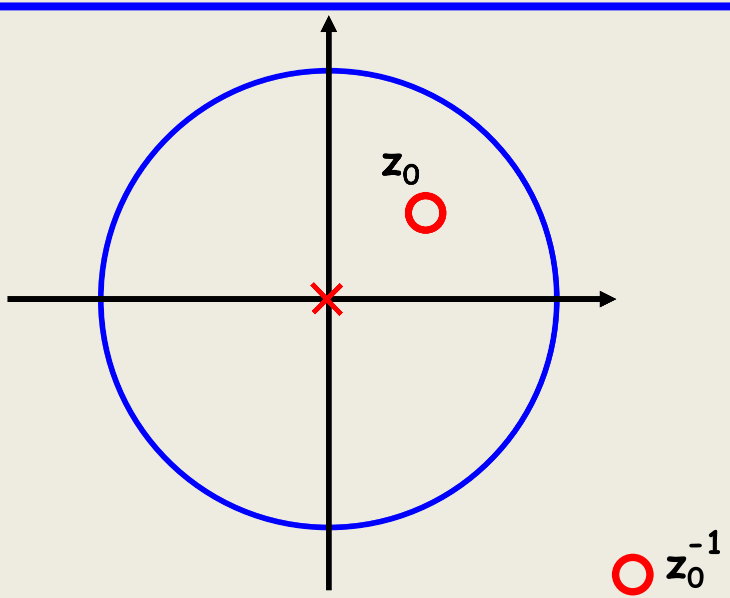
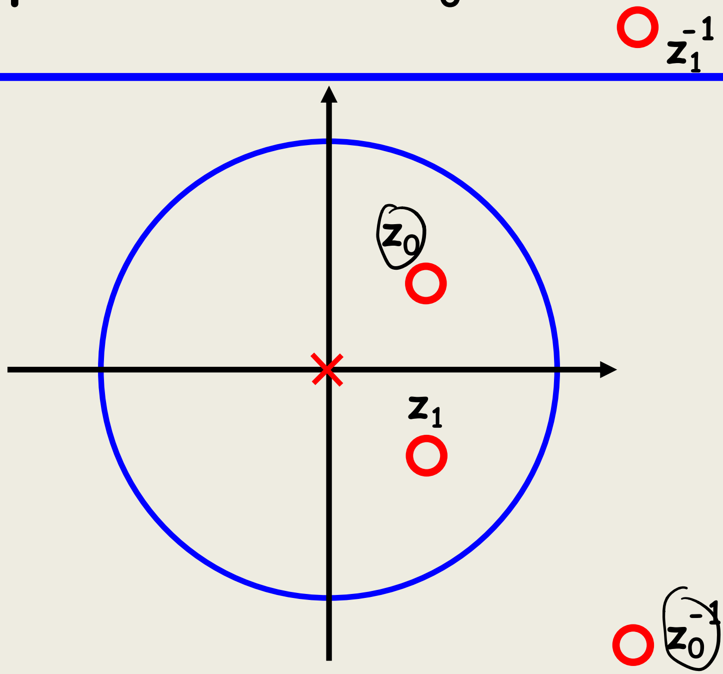


This is not a real-valued h(n)

EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

Linear phase <-> If z_0 is a zero, so is z_0^{-1}

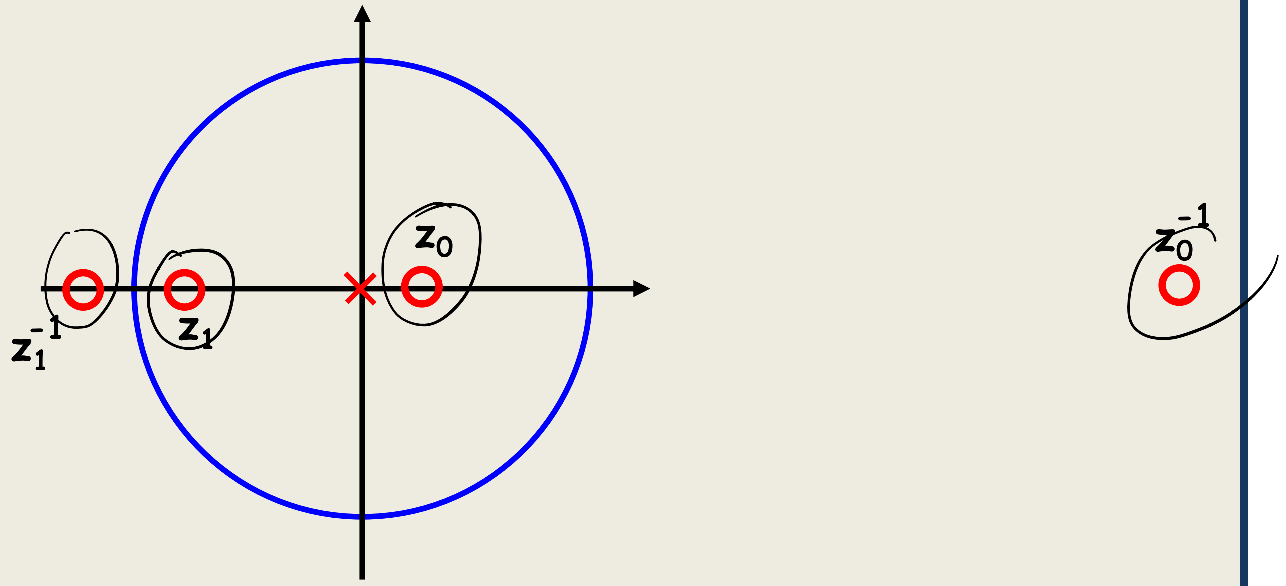


This is

EITF75 Systems and Signals

Pole-Zero diagram for linear phase FIR filters

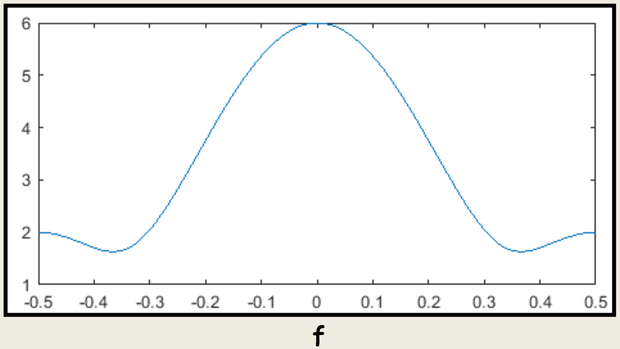
Linear phase <-> If z_0 is a zero, so is z_0^{-1}

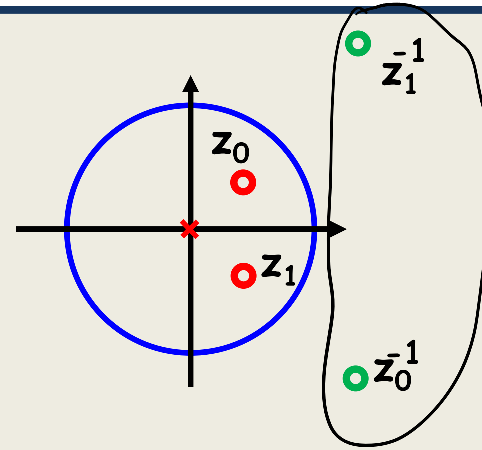


EITF75 Systems and Signals

Minimum phase filters

$$|H(f)| = |H^*(f)|$$

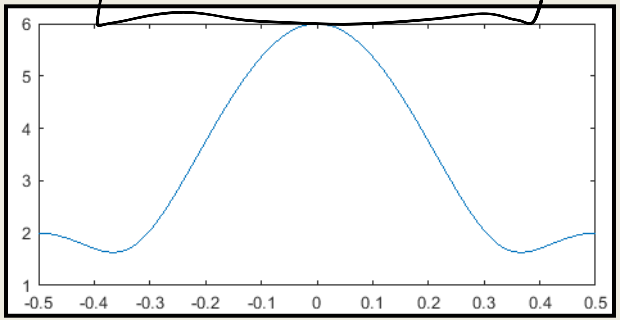




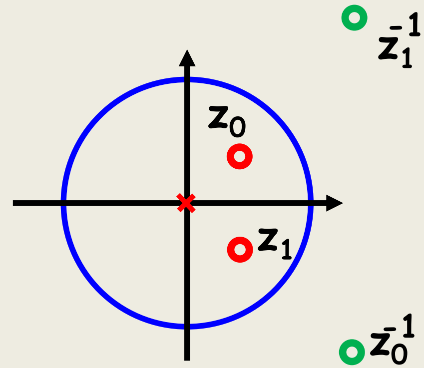
EITF75 Systems and Signals

Minimum phase filters

$$\boxed{|H(f)| = |H^*(f)|}$$

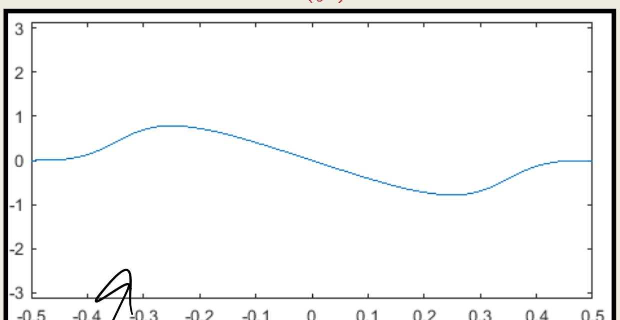


f

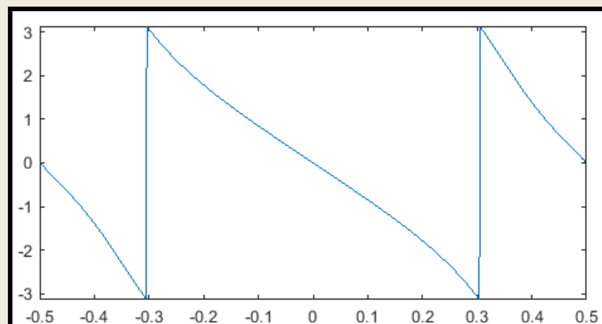


$$\frac{\phi(f)}{f}$$

$\Theta(f)$



$\Theta(f)$

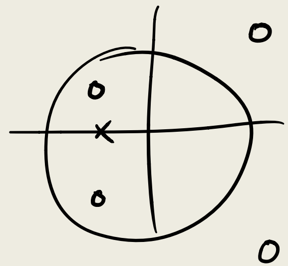


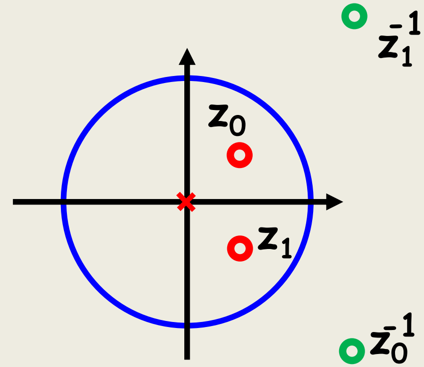
EITF75 Systems and Signals

Minimum phase filters

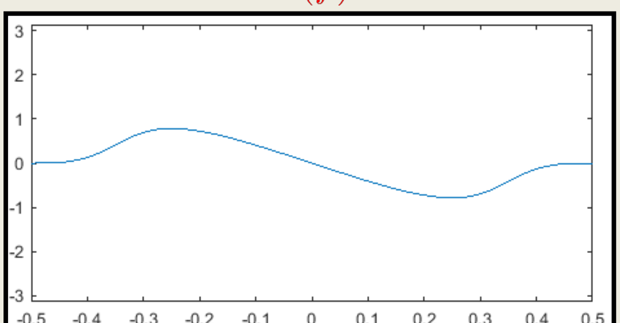
This is a general rule:
A filter with all zeros inside the unit circle has smaller phase.

Minimum phase filter
Maximum phase filter





$\Theta(f)$



$\Theta(f)$