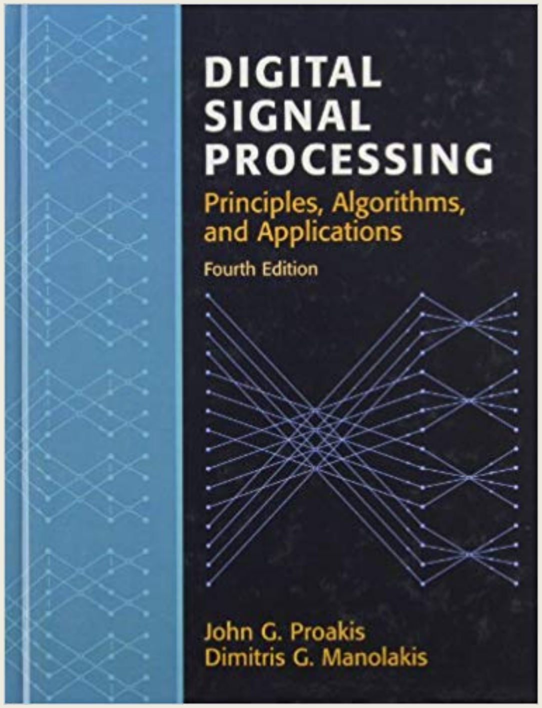
EITF75 Systems and Signals

Lecture 1
Introduction

Fredrik Rusek

EITF75 Systems and Signals



Schedule:

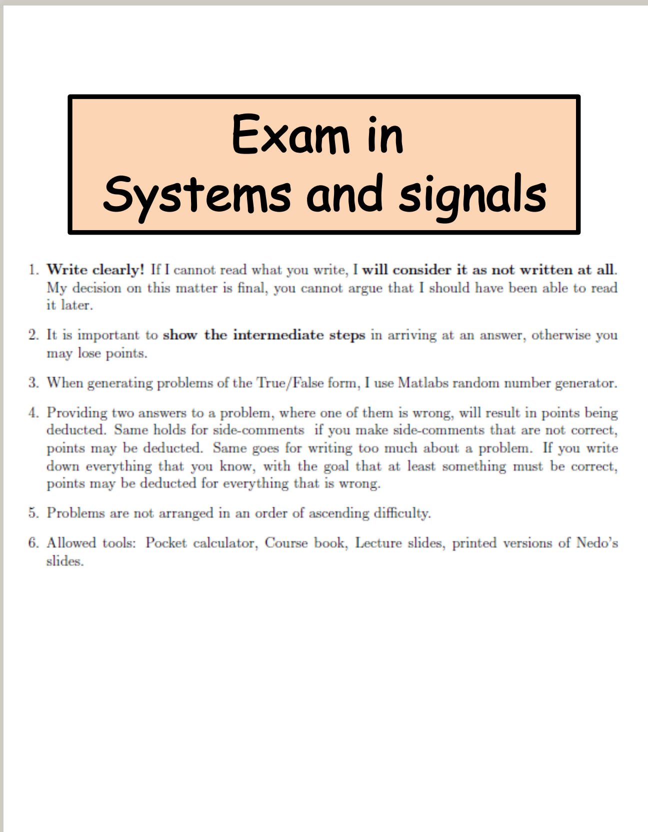
Lectures, F. Rusek, E:2377
12 lectures, 2/week
1 old exam solving
1 reserve

Exercises, G. Tian (E:2367), A. Sheikhi (E:2366)
14 exercises, 2/week

Labs, G. Tian, A. Sheikhi
2 Labs

Self-study time
96 hours

EITF75 Systems and Signals



Exam in
Systems and signals

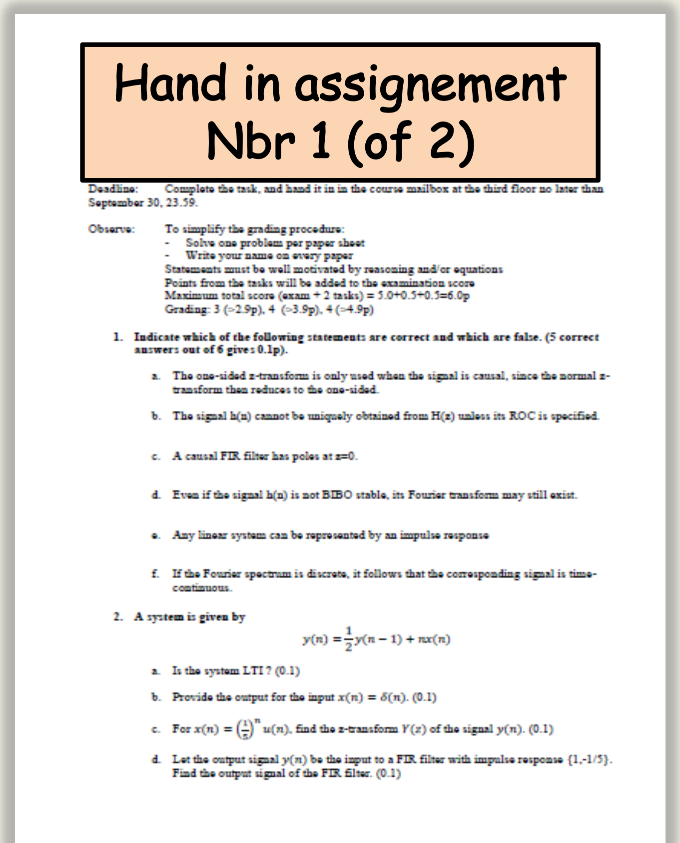
1. **Write clearly!** If I cannot read what you write, I **will consider it as not written at all**. My decision on this matter is final, you cannot argue that I should have been able to read it later.
2. It is important to **show the intermediate steps** in arriving at an answer, otherwise you may lose points.
3. When generating problems of the True/False form, I use Matlabs random number generator.
4. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments if you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that at least something must be correct, points may be deducted for everything that is wrong.
5. Problems are not arranged in an order of ascending difficulty.
6. Allowed tools: Pocket calculator, Course book, Lecture slides, printed versions of Nedo's slides.

Allowed tools:
Whatever you want to bring that does not have internet access

Two retake exams:
April, August

Exam gives maximum 5.0 points

EITF75 Systems and Signals



Hand in assignement Nbr 1 (of 2)

Deadline: Complete the task, and hand it in in the course mailbox at the third floor no later than September 30, 23.59.

Observe: To simplify the grading procedure:
- Solve one problem per paper sheet
- Write your name on every paper

Statements must be well motivated by reasoning and/or equations
Points from the tasks will be added to the examination score
Maximum total score (exam + 2 tasks) = 5.0+0.5+0.5=6.0p
Grading: 3 (>2.9p), 4 (>3.9p), 4 (>4.9p)

1. **Indicate which of the following statements are correct and which are false. (5 correct answers out of 6 gives 0.1p).**
   a. The one-sided z-transform is only used when the signal is causal, since the normal z-transform then reduces to the one-sided.
   b. The signal h(n) cannot be uniquely obtained from H(z) unless its ROC is specified.
   c. A causal FIR filter has poles at z=0.
   d. Even if the signal h(n) is not BIBO stable, its Fourier transform may still exist.
   e. Any linear system can be represented by an impulse response
   f. If the Fourier spectrum is discrete, it follows that the corresponding signal is time-continuous.
2. **A system is given by**

$$y(n) = \frac{1}{2}y(n-1) + nx(n)$$

   a. Is the system LTI ? (0.1)
   b. Provide the output for the input $x(n) = \delta(n)$. (0.1)
   c. For $x(n) = \left(\frac{1}{5}\right)^n u(n)$, find the z-transform $Y(z)$ of the signal $y(n)$. (0.1)
   d. Let the output signal $y(n)$ be the input to a FIR filter with impulse response {1,-1/5}. Find the output signal of the FIR filter. (0.1)

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So, total points: 6.0

Grades: 3.0-3.9: 3 4.0-4.9: 4 5.0-6.0: 5

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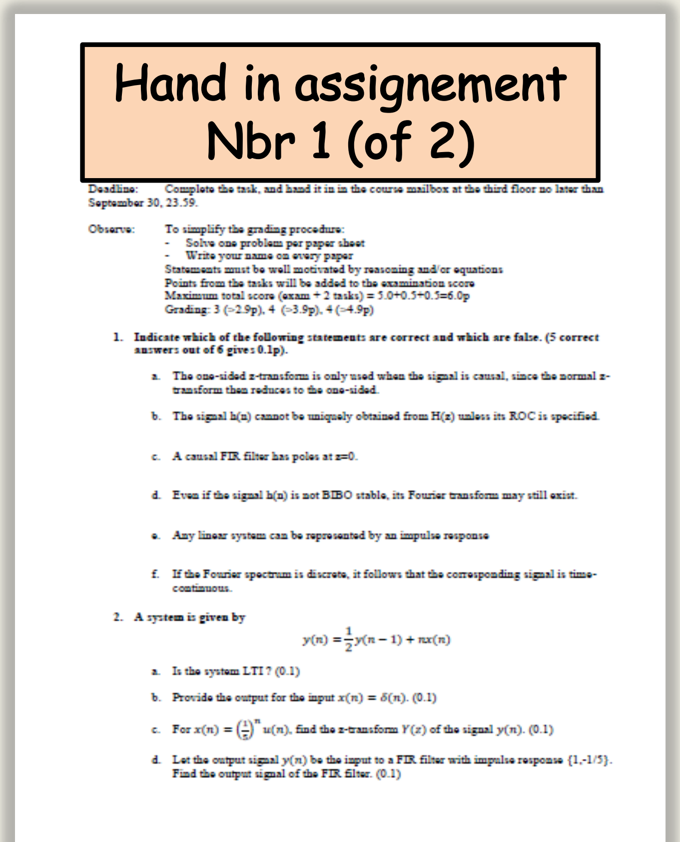

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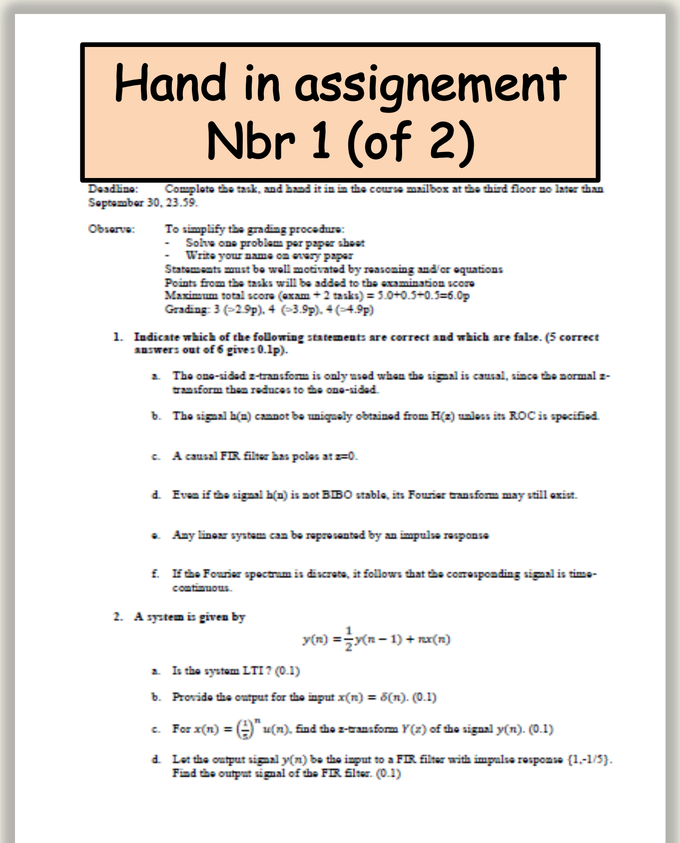
Points from hand-in assignements valid for 1 year, i.e., the October, April, and August exams

EITF75 Systems and Signals



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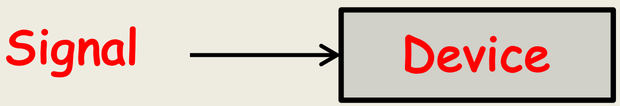
Deadline for handing in: end of September, mid-October (see web)

EITF75, Introduction

Continuous time vs. Discrete time

Consider an electronic device, observing a signal



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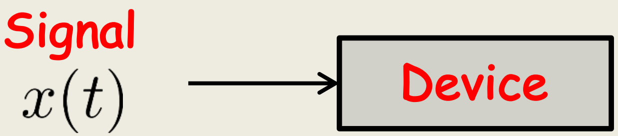

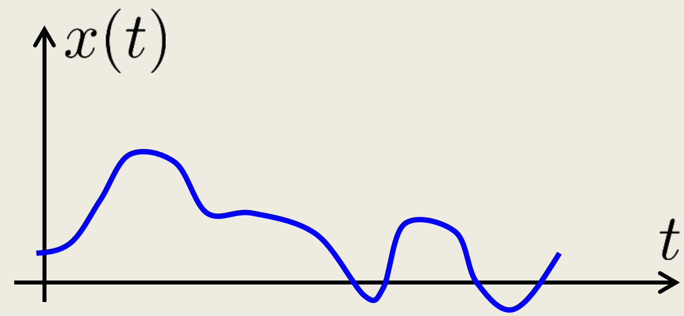

In reality, a signal is continuous.

EITF75, Introduction

Continuous time vs. Discrete time

Consider an electronic device, observing a signal

The device has a task - it should output something

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Continuous time vs. Discrete time

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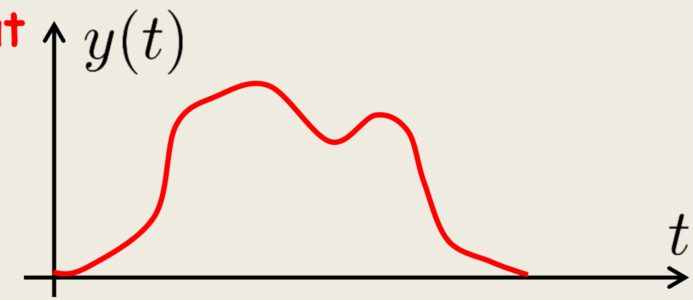

Most general case is continuous output

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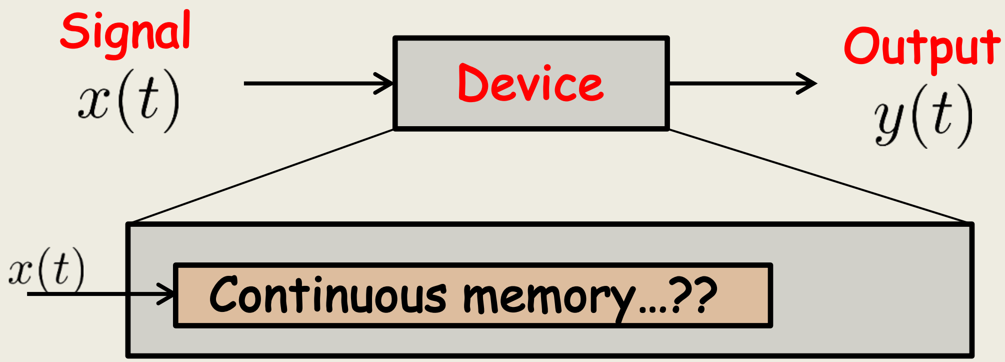

Obvious questions:

1. How should the device store/represent the signal $x(t)$?
   - Almost impossible…

EITF75, Introduction

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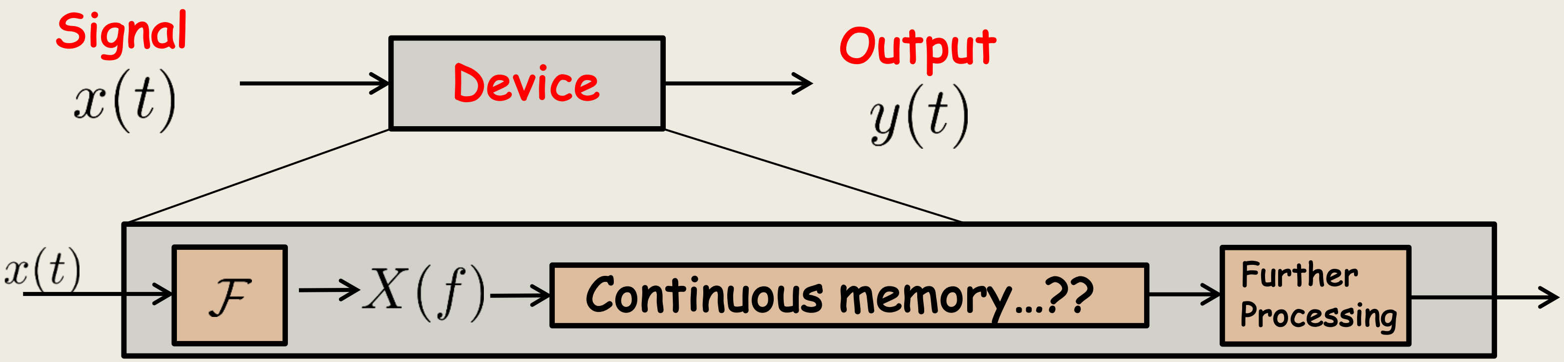

Obvious questions:

2. Suppose a Fourier transform of $x(t)$ is needed.
   How to compute $X(f)$ for every f ?

   -Almost impossible…

$$X(f) = \int_{-\infty}^{\infty} x(t) \exp(-i2\pi f t) \mathrm{d}t$$

EITF75, Introduction

Continuous time vs. Discrete time

Consider an electronic device, observing a signal

Summary

It is very hard for a computer to work with continuous signals

(Think for example of Matlab)

EITF75, Introduction

Discrete time signals

What is the difference between

- Continuous signals
- Discrete signals
- Digital signals

Digital signals:

Continuous signals:

Discrete signals:

EITF75, Introduction

Discrete time signals

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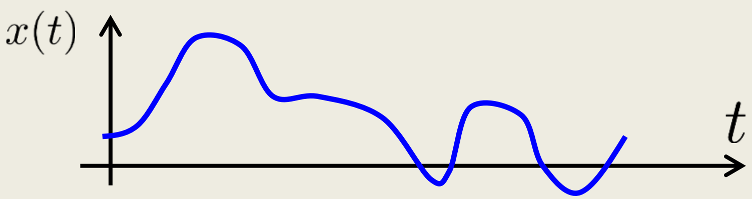
Continuous signals: Simple…



Discrete signals:

Digital signals:

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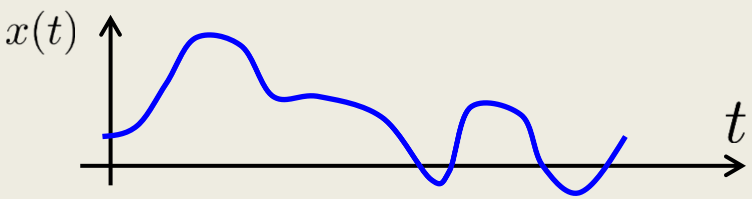
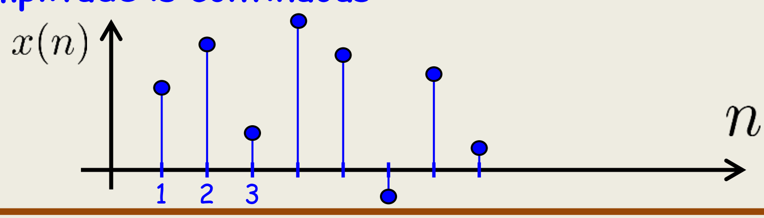


Discrete signals: time is discrete, amplitude is continuous



Digital signals:

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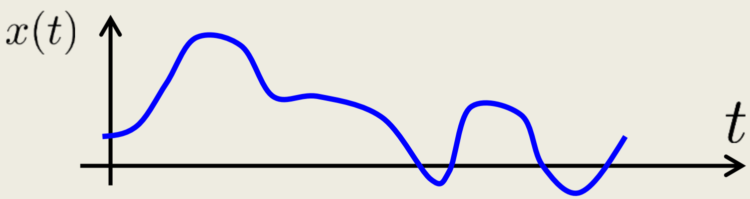

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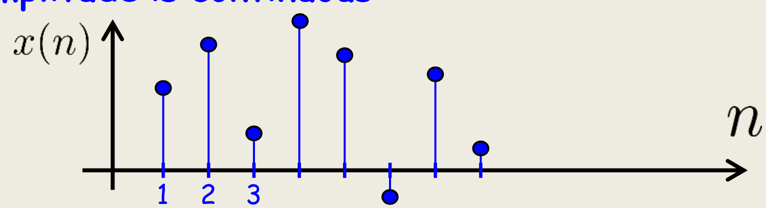

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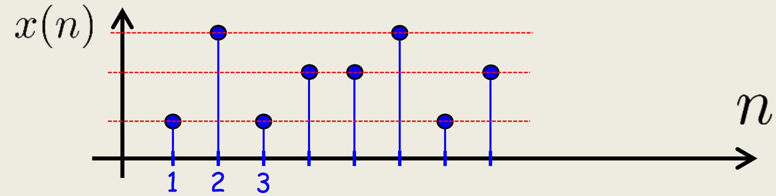

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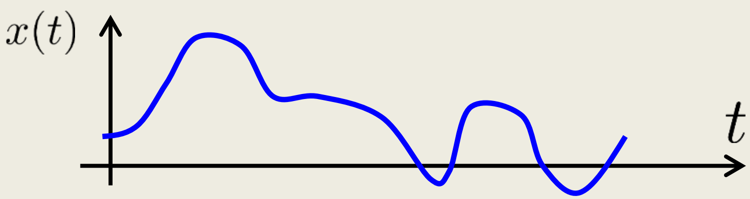

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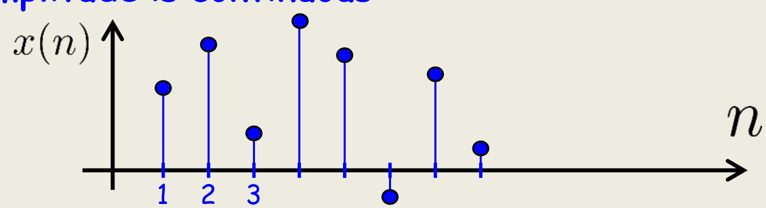

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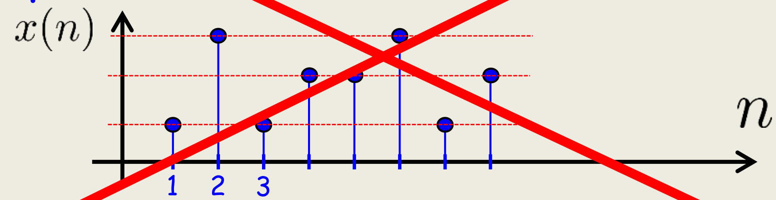

Essentially not treated in course.
However, the class of digital signals is a subset of the class of Discrete signals, so everything we study applies to digital signals as well

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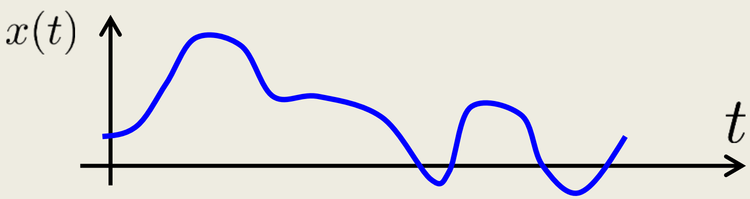
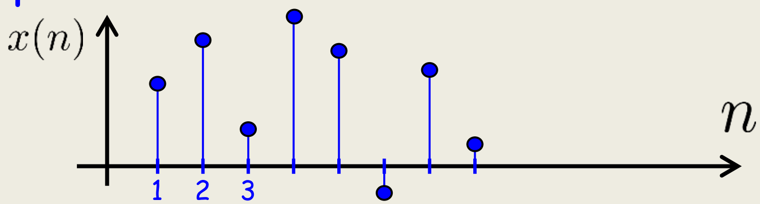
Continuous signals: Simple…



Discrete signals: time is discrete, amplitude is continuous



Where does a discrete signal appear in nature?

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Discrete time signals

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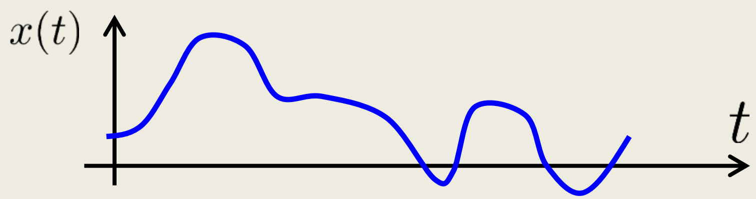
Continuous signals: Simple...



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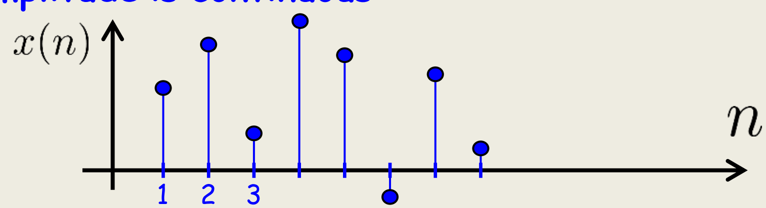

Where does a discrete signal appear in nature?

Nowehere (that I know). All natural signals are continuous/analog.

Discrete signals are man-made.

Examples:
- Read temperature at constant interval
- Read stock-price once/sec
- Read an audio signal 44100 times/sec
- Let a cell-phone read an incoming wave 10000000 times/sec

In all cases (except maybe stock price), the underlying signal is continuous, not discrete

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Discrete time signals

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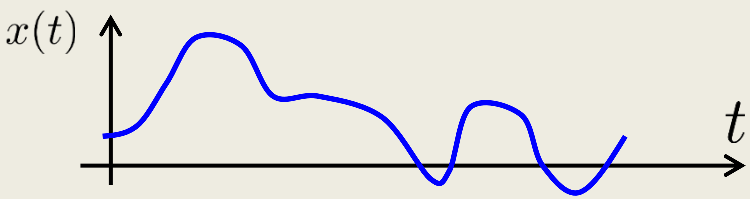
Continuous signals: Simple...



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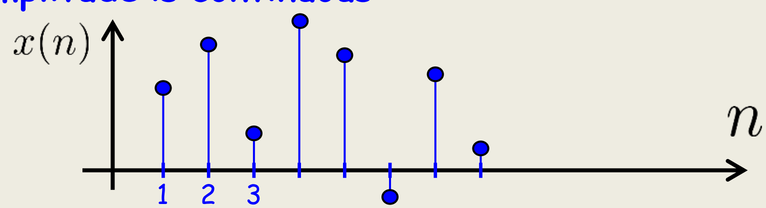
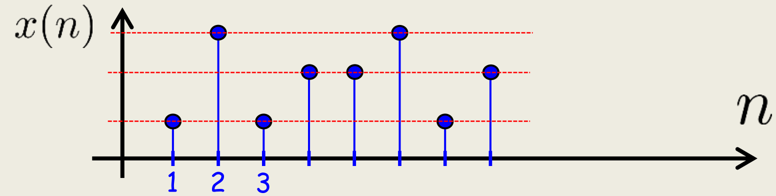


Digital signals: time is discrete, amplitude is discrete



Digital signals come about since it is hard for a computer to store all possible values. So it quantizes them.

We will briefly touch upon the losses involved in quantization

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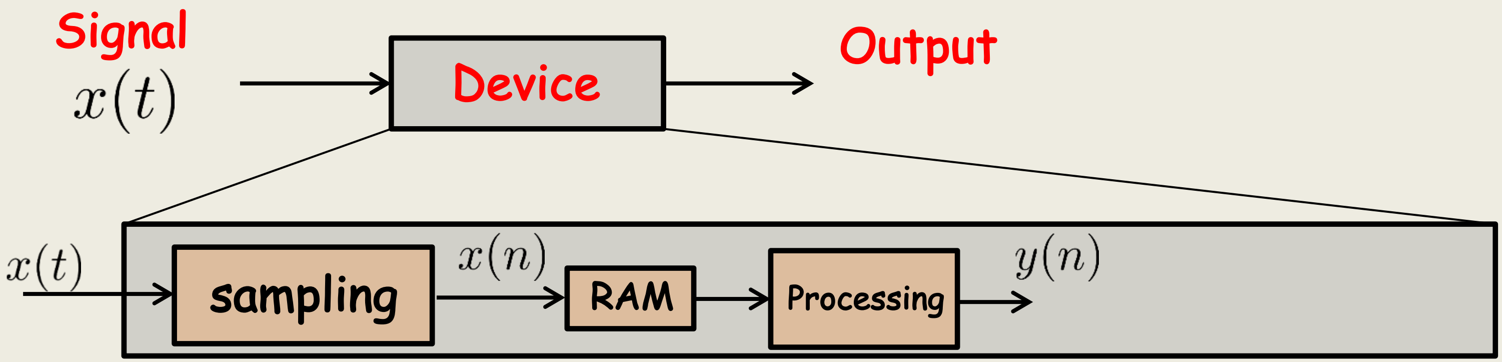
Discrete time signals

We study the following system



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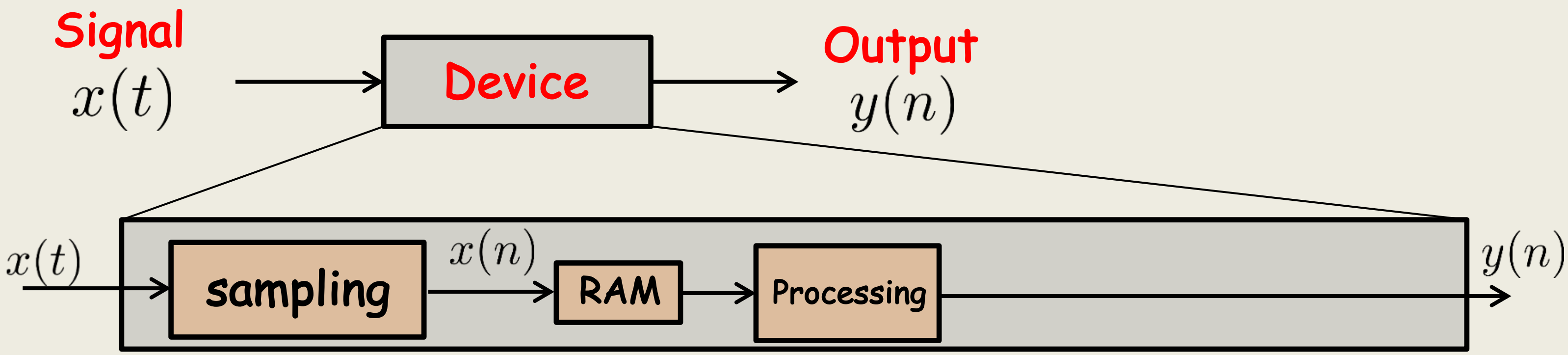
Discrete time signals

We study the following system

Output can be either **discrete** or continuous



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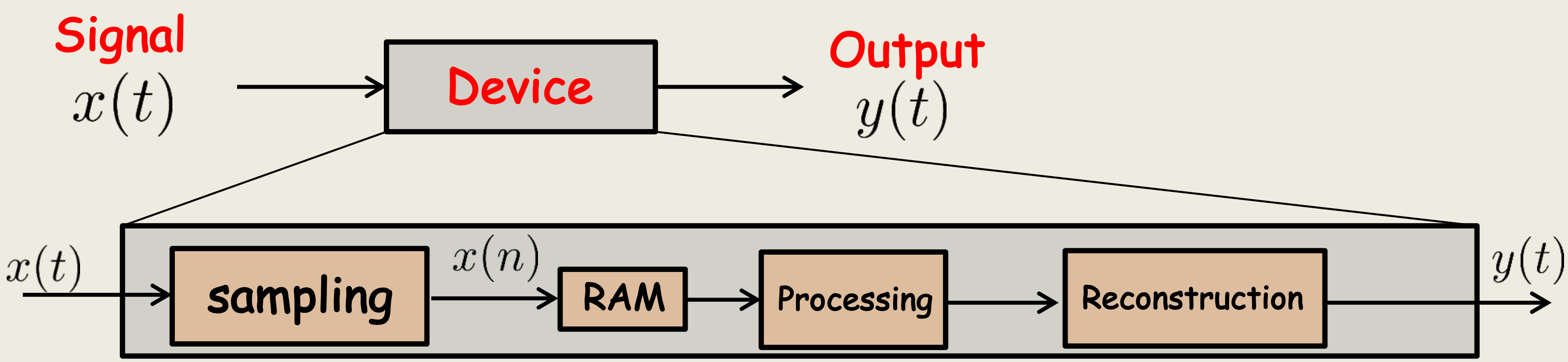

EITF75, Introduction

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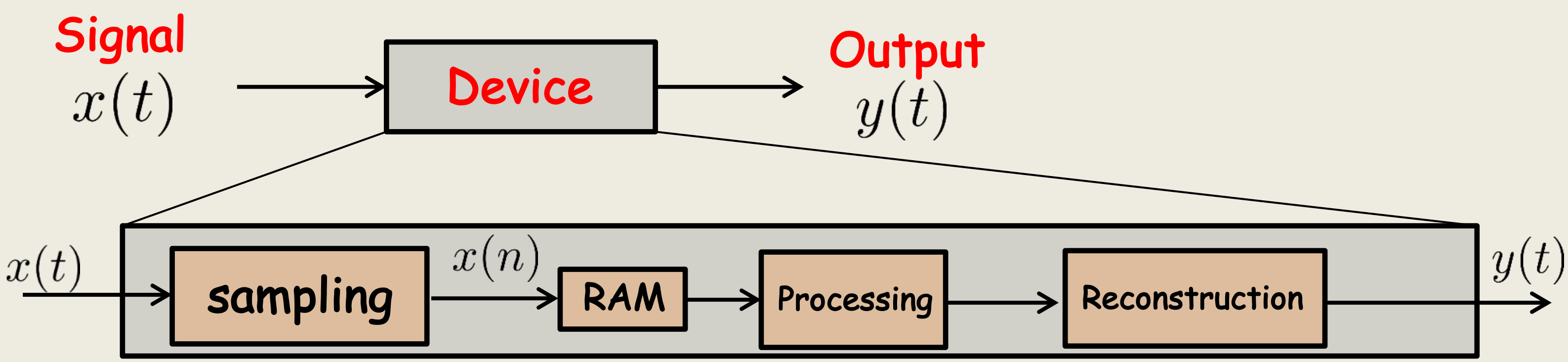
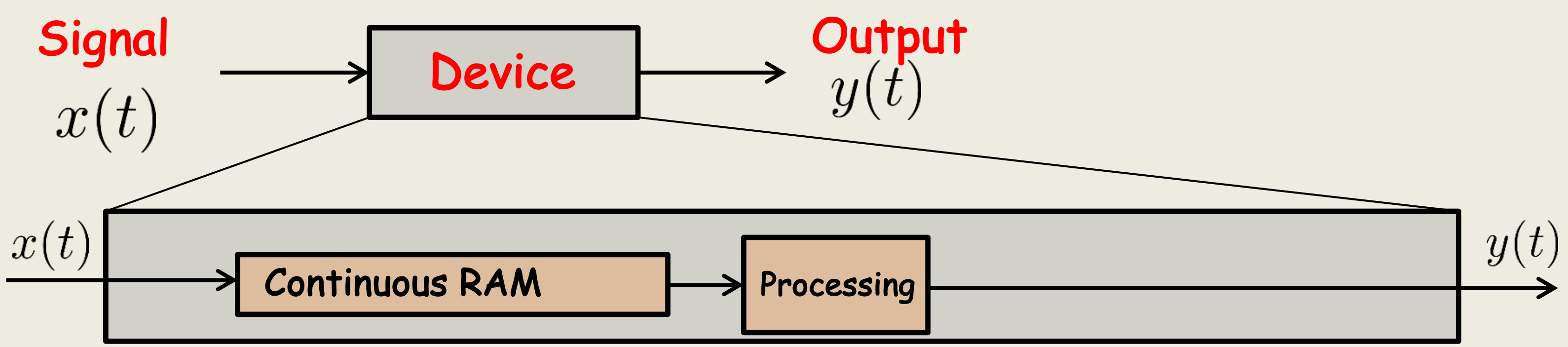


The target is to carry out the same task as an anolog device would



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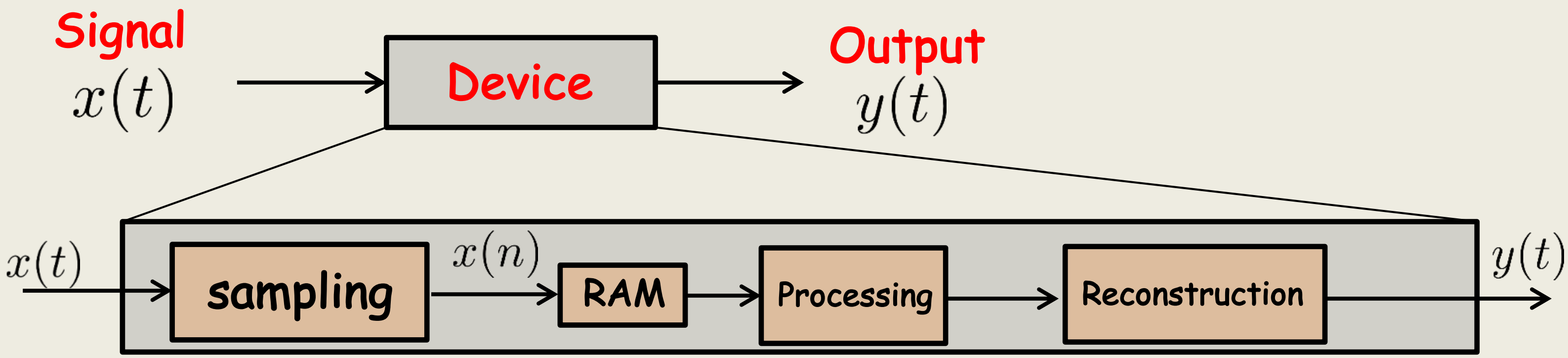

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Example: "The output should be the mean of the input during the last 8 second plus the mean outputs during the last 5 seconds"

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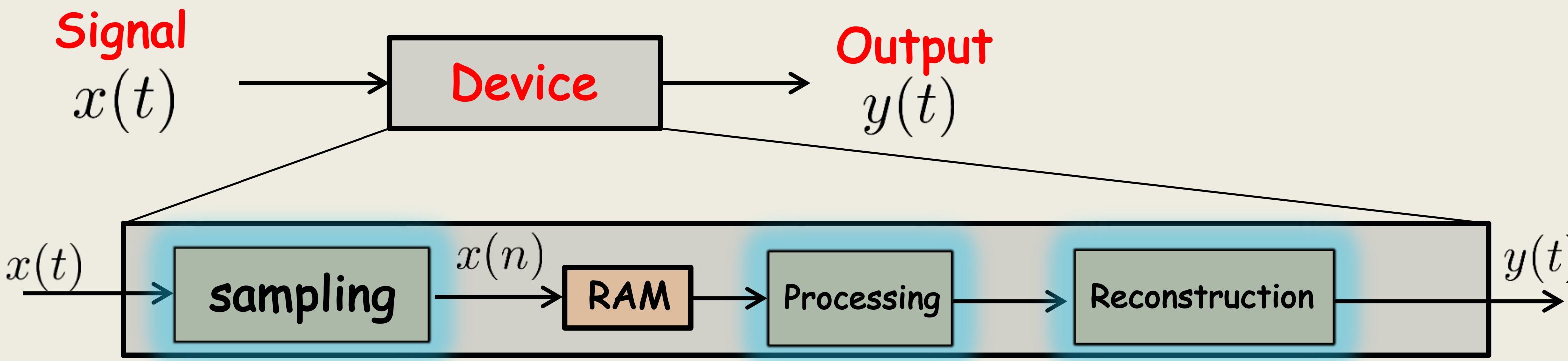

Thus, we need to understand how

- Sampling should be done so that nothing is lost
- How to process a discrete signal
- How to convert a discrete signal into a continuous one so that it behaves well

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EITF75, Introduction

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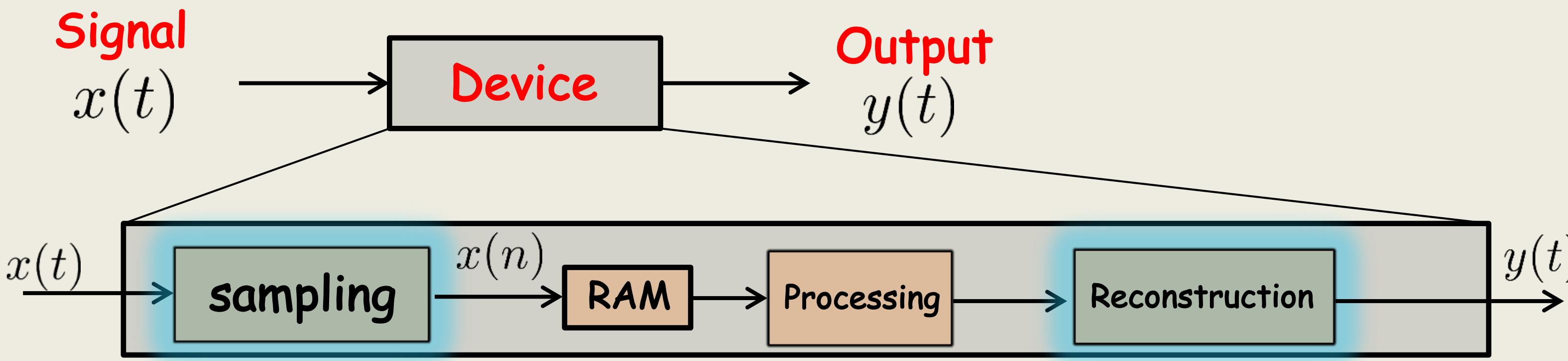

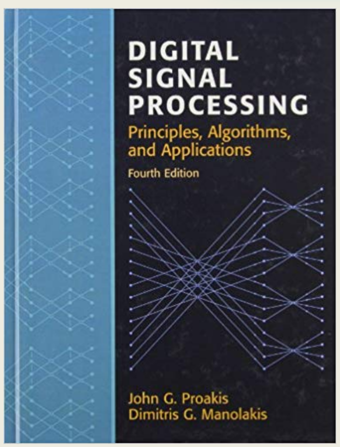

Chapters

1 Introduction
2 Discrete-Time Signals And Systems
3 The Z-Transform And Its Application To The Analysis Of LTI Systems
4 Frequency Analysis Of Signals And Systems
5 Frequency Domain Analysis Of Lti Systems
6 Sampling And Reconstruction Of Signals
7 The Discrete Fourier Transform: Its Properties And Applications
~~8 Efficient Computation Of The DFT: Fast Fourier Transform Algorithms~~
9 Implementation Of Discrete-Time Systems

EITF75, Introduction

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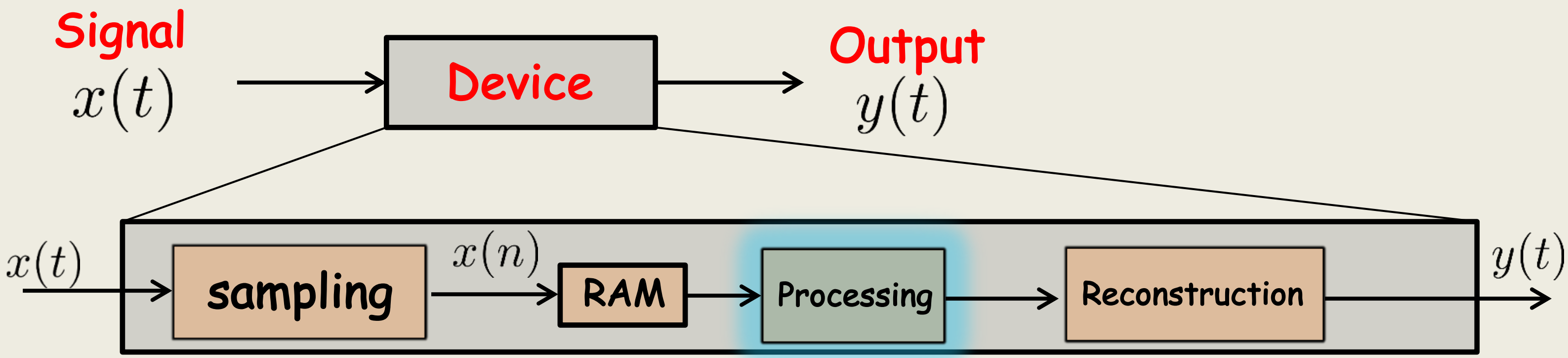

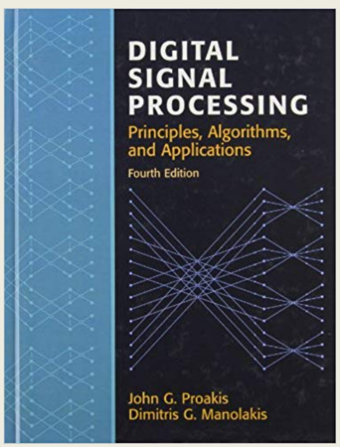

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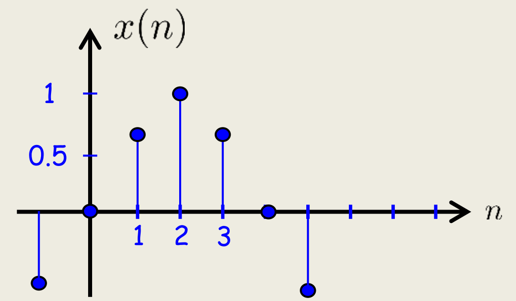
EITF75, Introduction

Some recap, notation and other basics

How we express a discrete signal

$$x(n) = \sin\left(2\pi\frac{1}{8}n\right) \approx \{\ldots \quad -1 \quad -0.7 \;\; \underline{0} \;\; 0.7 \;\; 1 \;\; 0.7 \quad \ldots\}$$

The bar below "0" marks that this is at time n=0



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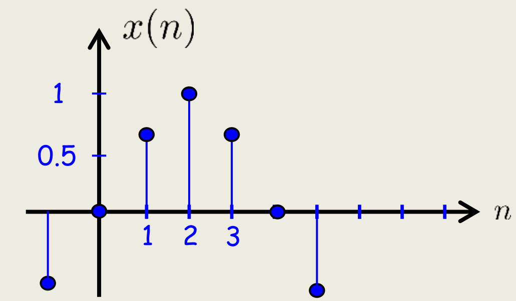

Examples of systems

$$x(n) \longrightarrow \boxed{\text{system}} \longrightarrow y(n)$$

$$y(n) = \frac{1}{5}x(n) + \frac{1}{5}x(n-1) + \frac{1}{5}x(n-2) + \frac{1}{5}x(n-3) + \frac{1}{5}x(n-4)$$

Easily understood as average of 5 last samples

EITF75, Introduction

Some recap, notation and other basics

How we express a discrete signal

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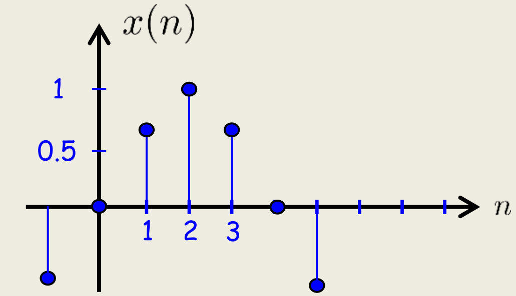

Examples of systems

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$$y(n) = \frac{1}{5}x(n) - \frac{1}{5}x(n-1) + \frac{1}{5}x(n-2) - \frac{1}{5}x(n-3) + \frac{1}{5}x(n-4)$$

But what does this system do? We'll understand how to find out…

EITF75, Introduction

Some recap, notation and other basics

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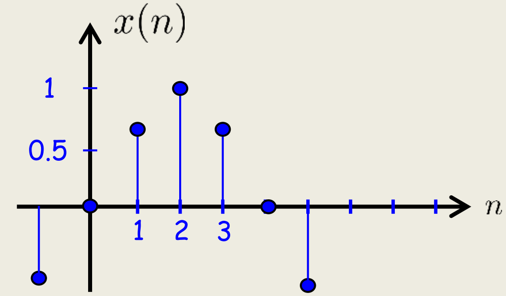

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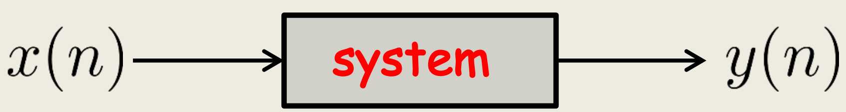

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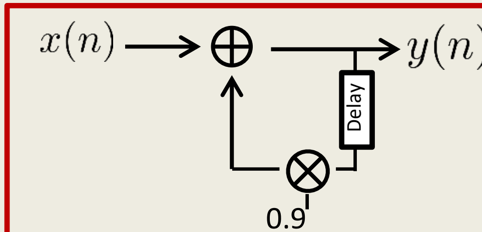
Important class:
Feedback systems

$$y(n) = 0.9y(n-1) + x(n)$$



What will happen ?

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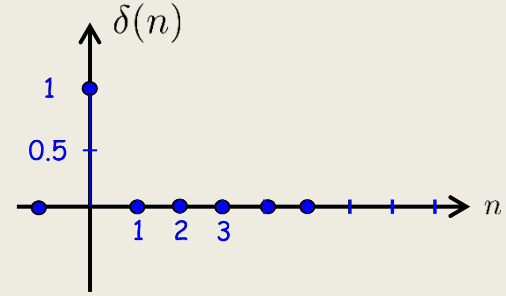
Some recap, notation and other basics

Some important discrete signals and concepts

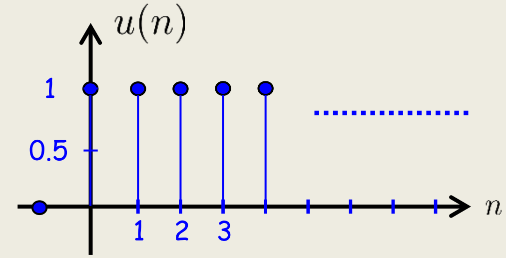
Impulse

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} = \{\ldots\ 0\ \underline{1}\ 0\ 0\ \ldots\} = \{\underline{1}\}$$



Step

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} = \{\underline{1}\ 1\ 1\ \ldots\}$$



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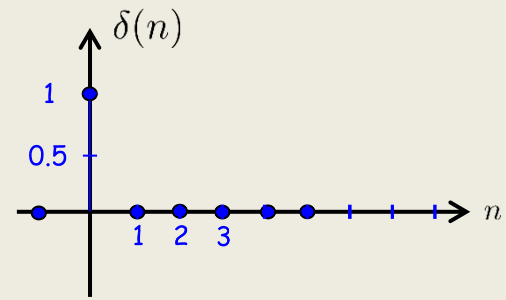

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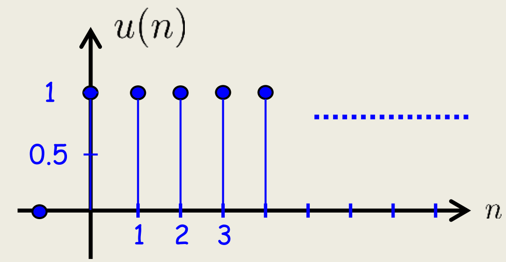

Signal is causal if it is zero for all negative indices

$$x(n) = 0, n < 0 \iff x(n) \text{ causal}$$

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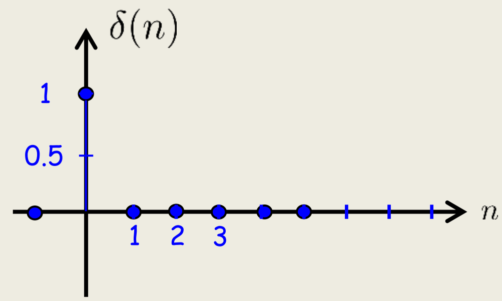

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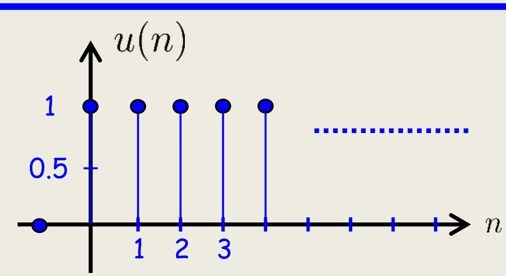

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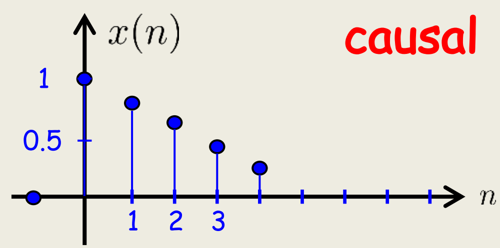

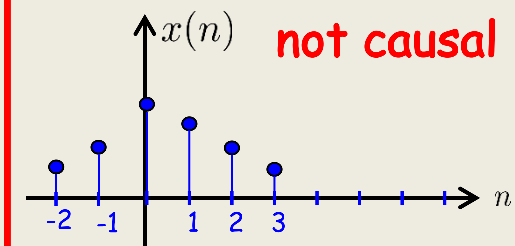

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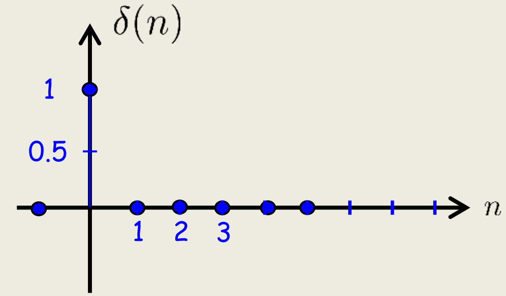

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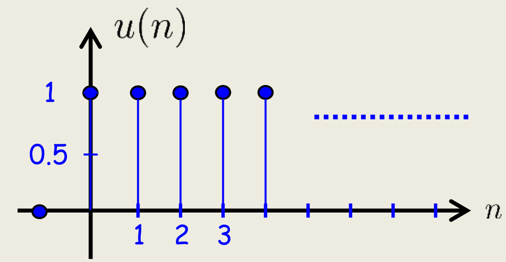

Impulses and steps are key later, but for now we just mention that they can be used to mathematically represent a signal.

Examples:

$$x(n) = \{\underline{1} \ 4 \ 1\} = 1 \cdot \delta(n) + 4 \cdot \delta(n-1) + 1 \cdot \delta(n-2)$$

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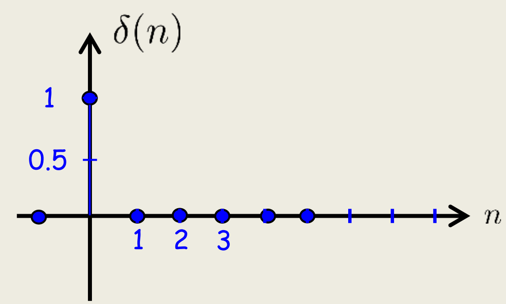

Step

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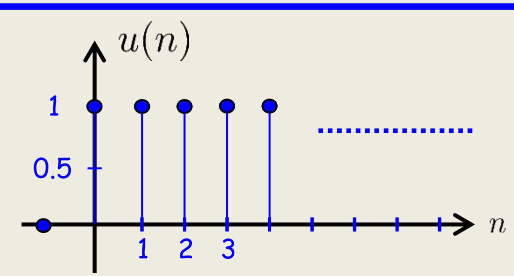

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Examples:

$$x(n) = \{\underline{1} \;\; 4 \;\; 1\} = 1 \cdot \delta(n) + 4 \cdot \delta(n-1) + 1 \cdot \delta(n-2) = \sum_k x(k)\delta(n-k)$$

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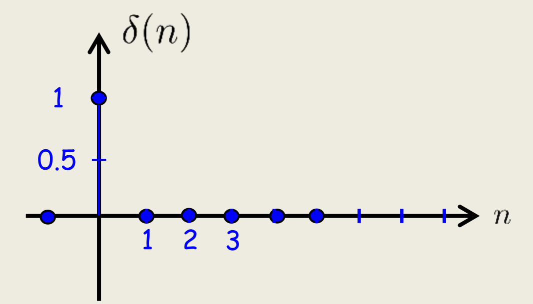

Step

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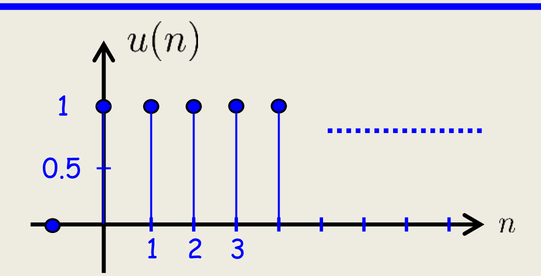

An expression of such form is important and is named convolution

$$x(n) = \{\underline{1} \;\; 4 \;\; 1\} = 1 \cdot \delta(n) + 4 \cdot \delta(n-1) + 1 \cdot \delta(n-2) = \boxed{\sum_k x(k)\delta(n-k)}$$

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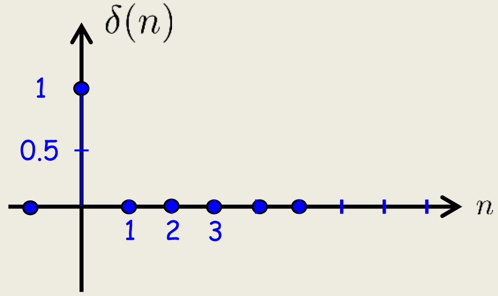
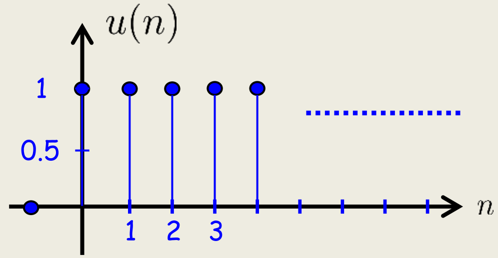
Impulse

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} = \{\ldots \; 0 \; \underline{1} \; 0 \; 0 \; \ldots\} = \{\underline{1}\}$$



Step

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} = \{\underline{1} \; 1 \; 1 \; \ldots\}$$



Note that the below equality is obvious and only means
"a convolution with an impulse changes nothing"

$$\boxed{x(n) = \sum_k x(k)\delta(n-k)}$$

However, this form will turn out to be useful later

Some recap, notation and other basics

Some important discrete signals and concepts

Impulse

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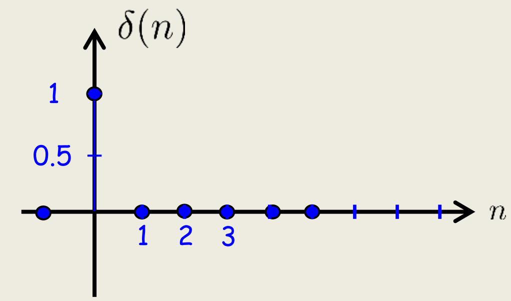

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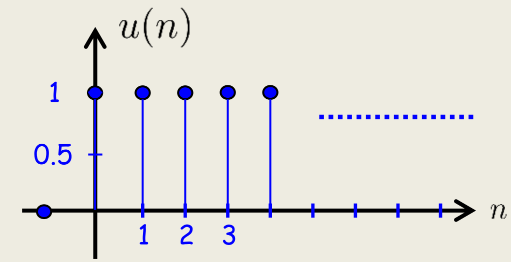
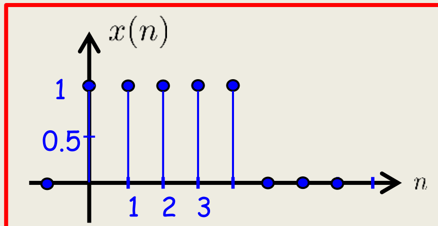


Another example:



$x(n) =$ *Can be written with step functions as...*

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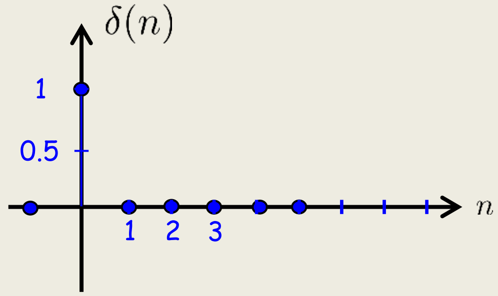

Step

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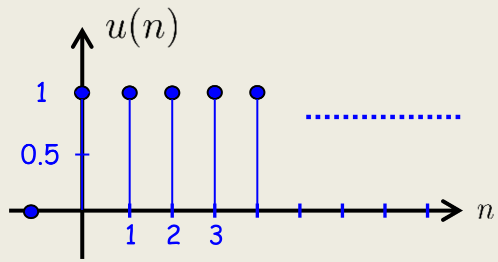

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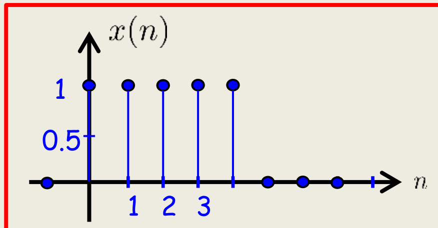

$$x(n) = u(n) - u(n-5)$$

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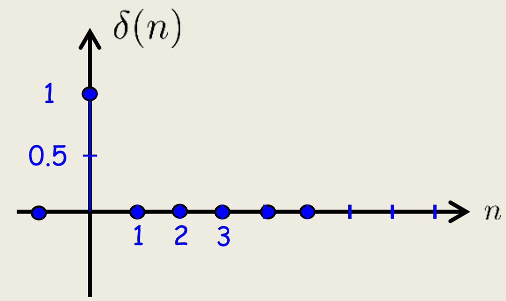
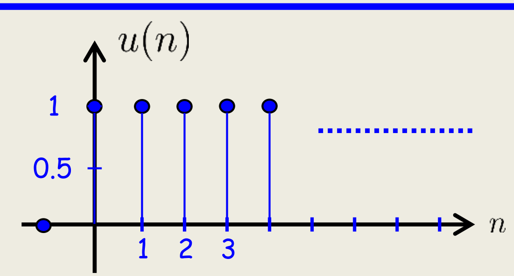
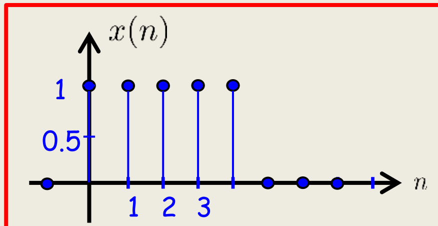


Step

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} = \{\underline{1} \ 1 \ 1 \ \dots\}$$



$$x(n) = u(n) - u(n-5)$$



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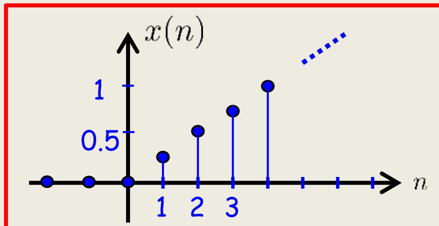

$$x(n) =$$

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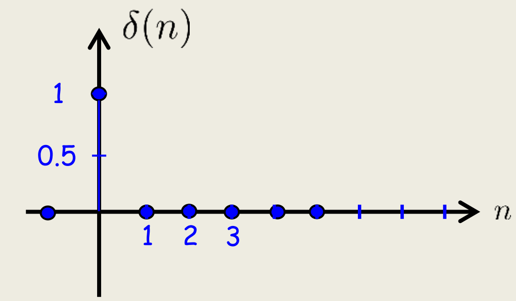

Step

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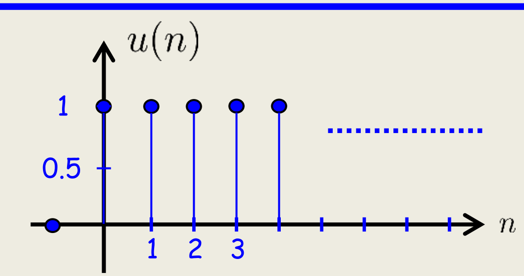

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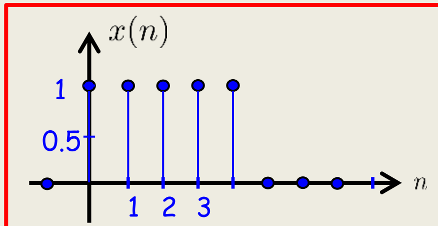

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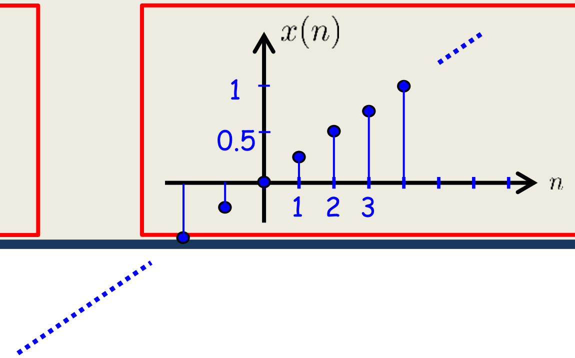

This is not correct,
since such signal is not causal

$$x(n) \neq 0.5 \cdot n$$

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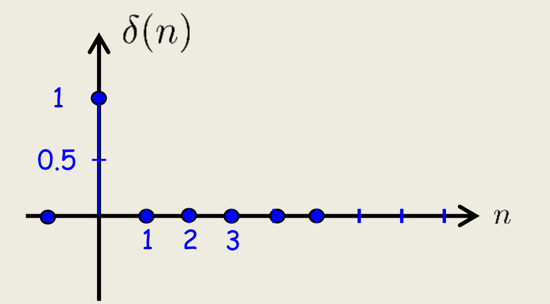

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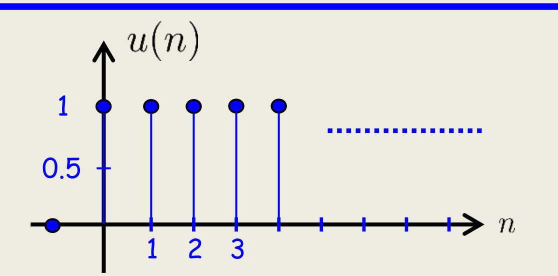

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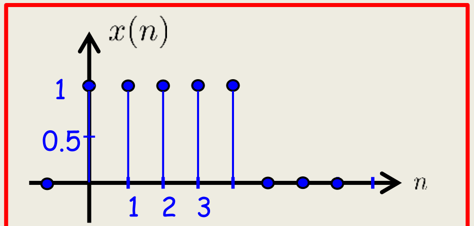

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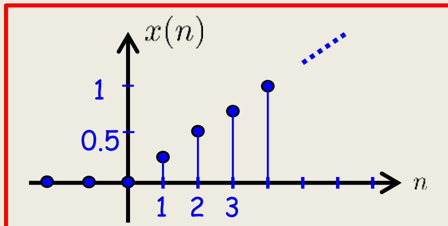

Like this

$$x(n) = 0.5 \cdot nu(n)$$

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Systems: delay

Expression

$$y(n) = x(n-1)$$

Circuit

$$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow y(n)$$

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Systems: delay

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$$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow y(n)$$

What is the expression?

Expression

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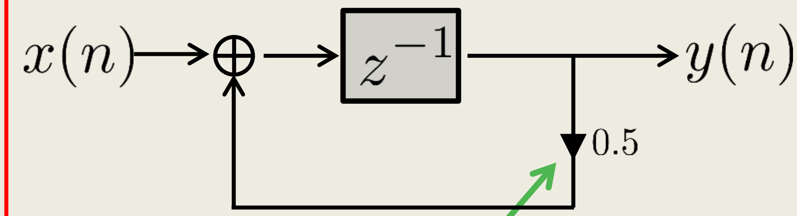

Multiplication from now and on

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Systems: delay

Expression

$$y(n) = x(n-1)$$

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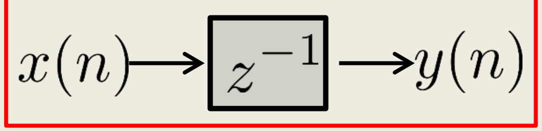

What is the expression?

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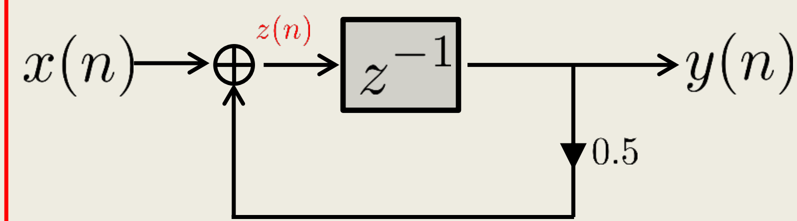

$$z(n) = x(n) + 0.5 \cdot y(n)$$

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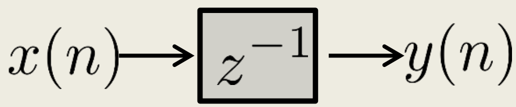

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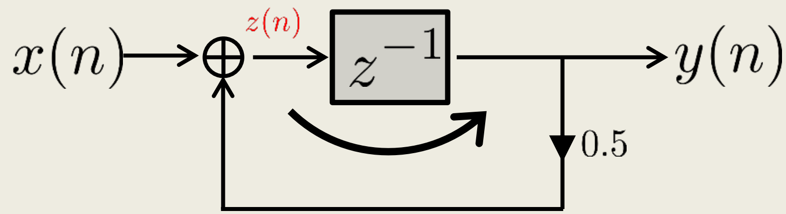

$$z(n) = x(n) + 0.5 \cdot y(n) \qquad y(n) = z(n-1)$$

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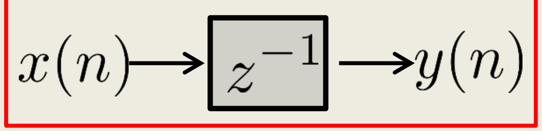
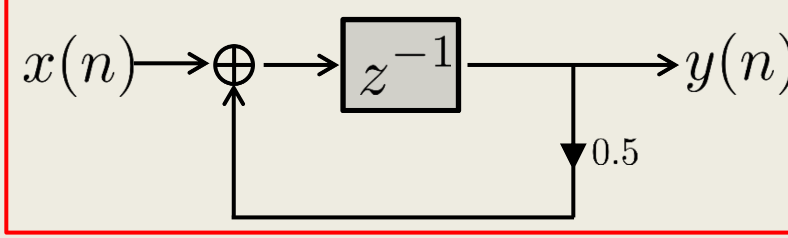


What is the expression?

Expression

$$y(n) = 0.5 \cdot y(n-1) + x(n-1)$$

Circuit



$$z(n) = x(n) + 0.5 \cdot y(n) \qquad y(n) = z(n-1)$$

EITF75, Introduction

Some recap, notation and other basics

In general, a signal $y(n)$ generated

from $x(n)$ via $\oplus$ $\boxed{z^{-1}}$ $\downarrow$

can be mathematically described by

$$\sum_{k} a(k)y(n-k) = \sum_{\ell} b(\ell)x(n-\ell)$$

We will study this type of systems in detail

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_{n} |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_n |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Ohm's law:

$$U = R \cdot I$$

Power of signal:

$$P = U \cdot I$$

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_{n} |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Ohm's law:

$$U = R \cdot I$$

Power of signal:

$$P = U \cdot I$$

So:

$$P = U^2/R$$

One measures the voltage using some equipment. Said equipment has a resistance, R, which does not change when the voltage changes.

Therefore, the power of two signals can be fairly compared using the square law.

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_n |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Even symmetry

$$x(n) = x(-n)$$

Odd symmetry

$$x(n) = -x(-n)$$

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_{n} |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Even symmetry

$$x(n) = x(-n)$$

Odd symmetry

$$x(n) = -x(-n)$$

Finite Memory

$y(n)$ depends on $x(n),\ x(n-1),\ \ldots,\ x(n-L)$

but not on $x(n-L-1),\ x(n-L-2),\ \ldots$

$$x(n) \rightarrow \boxed{\text{system}} \rightarrow y(n)$$

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_{n} |x(n)|^2$$

Average power of signal

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In words:

The output at time 10^6 does not depend on the input at time 0

Finite Memory

$y(n)$ depends on $x(n),\ x(n-1),\ \ldots,\ x(n-L)$

but not on $x(n-L-1),\ x(n-L-2),\ \ldots$

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EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_n |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Even symmetry

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In words:

The output at time 10^6 depends on the input at time 0

Finite Memory

$y(n)$ depends on $x(n),\ x(n-1),\ \ldots,\ x(n-L)$

but not on $x(n-L-1),\ x(n-L-2),\ \ldots$

Infinite Memory

$y(n)$ depends on

$x(n), \ldots, x(-\infty)$

EITF75, Introduction

Some recap, notation and other basics

Energy of signal

$$E = \sum_n |x(n)|^2$$

Average power of signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Even symmetry

$$x(n) = x(-n)$$

Odd symmetry

$$x(n) = -x(-n)$$

Finite Memory $y(n) = x(n) + x(n-1)$

$y(n)$ depends on $x(n),\ x(n-1),\ \ldots,\ x(n-L)$

but not on $x(n-L-1),\ x(n-L-2),\ \ldots$

Infinite Memory

$y(n)$ depends on

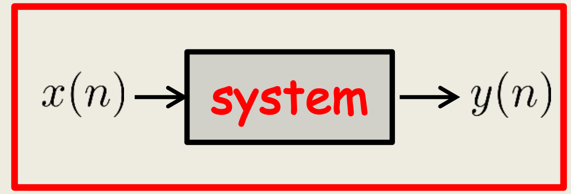
$x(n),\ \ldots,\ x(-\infty)$

$$y(n) = 0.5 \cdot y(n-1) + x(n)$$

We will study why later....

EITF75, Introduction

Some recap, notation and other basics



Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
$$\Longleftrightarrow$$
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

EITF75, Introduction

Some recap, notation and other basics

Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$

$$\Longleftrightarrow$$

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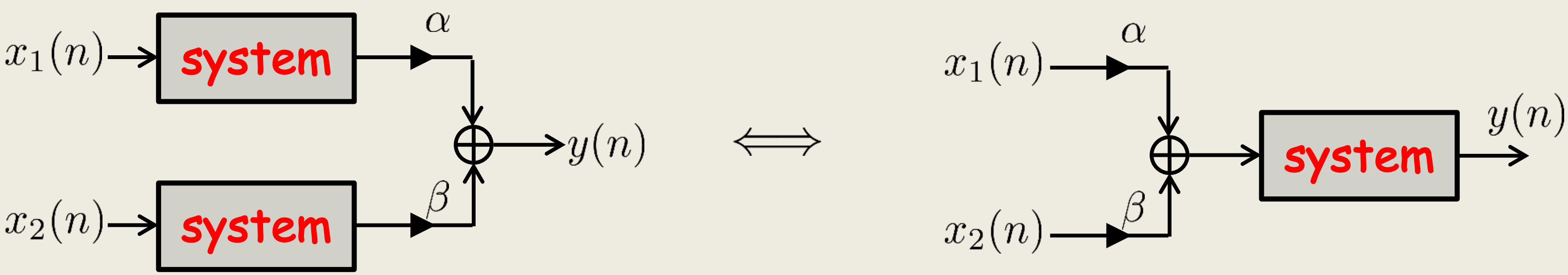
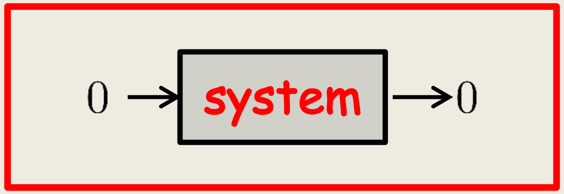


EITF75, Introduction

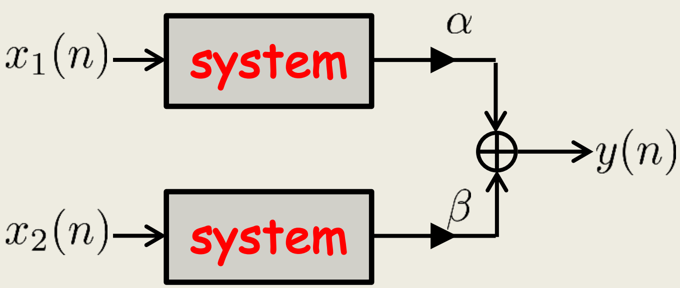
Some recap, notation and other basics



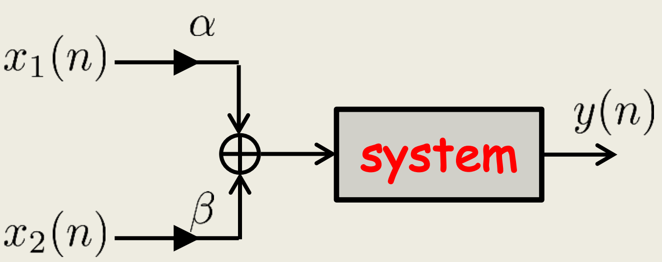
Holds for all linear systems

Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
$$\Longleftrightarrow$$
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

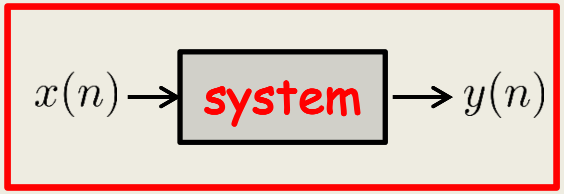


$$\Longleftrightarrow$$



EITF75, Introduction

Some recap, notation and other basics



Linear system

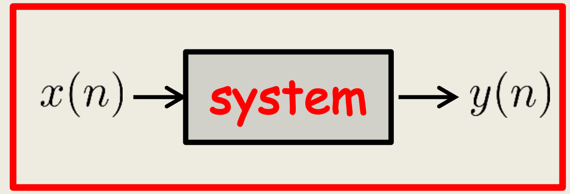
$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
$$\Longleftrightarrow$$
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

Time invariant system

$$x(n) \text{ replaced by } x(n-D)$$
$$\Longleftrightarrow$$
$$y(n) \text{ replaced by } y(n-D)$$

EITF75, Introduction

Some recap, notation and other basics



Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
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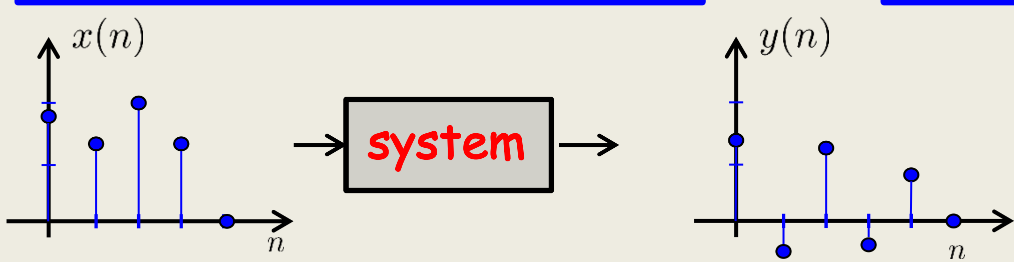
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

Time invariant system

$$x(n) \text{ replaced by } x(n-D)$$
$$\Longleftrightarrow$$
$$y(n) \text{ replaced by } y(n-D)$$



EITF75, Introduction

Some recap, notation and other basics

$$x(n) \rightarrow \boxed{\text{system}} \rightarrow y(n)$$

Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
$$\Longleftrightarrow$$
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

Time invariant system

$$x(n) \text{ replaced by } x(n-D)$$
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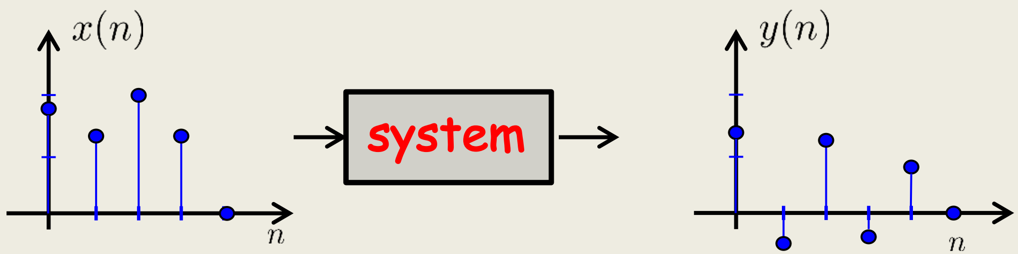
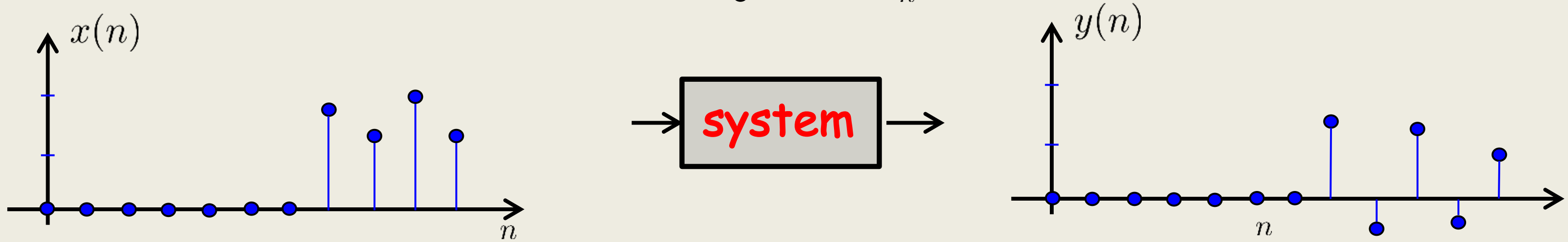
$$y(n) \text{ replaced by } y(n-D)$$





EITF75, Introduction

Some recap, notation and other basics

$$x(n) \longrightarrow \boxed{\text{system}} \longrightarrow y(n)$$

Linear system

$$x(n) = \alpha x_1(n) + \beta x_2(n)$$
$$\Longleftrightarrow$$
$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

Time invariant system

$$x(n) \text{ replaced by } x(n-D)$$
$$\Longleftrightarrow$$
$$y(n) \text{ replaced by } y(n-D)$$

BIBO-stability

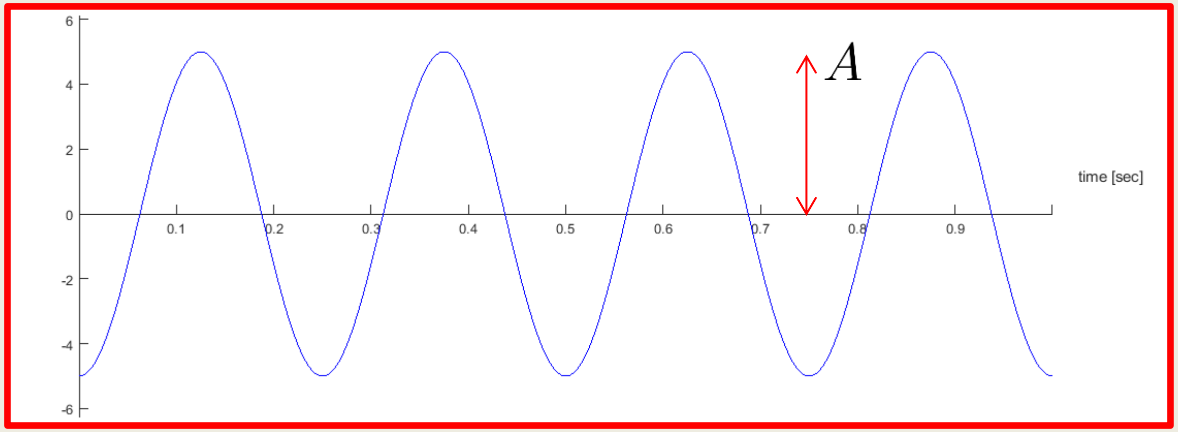
$$|x(n)| < M_x \implies |y(n)| < M_y < \infty$$

BIBO = Bounded input,
bounded output

EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi)$$



A Amplitude

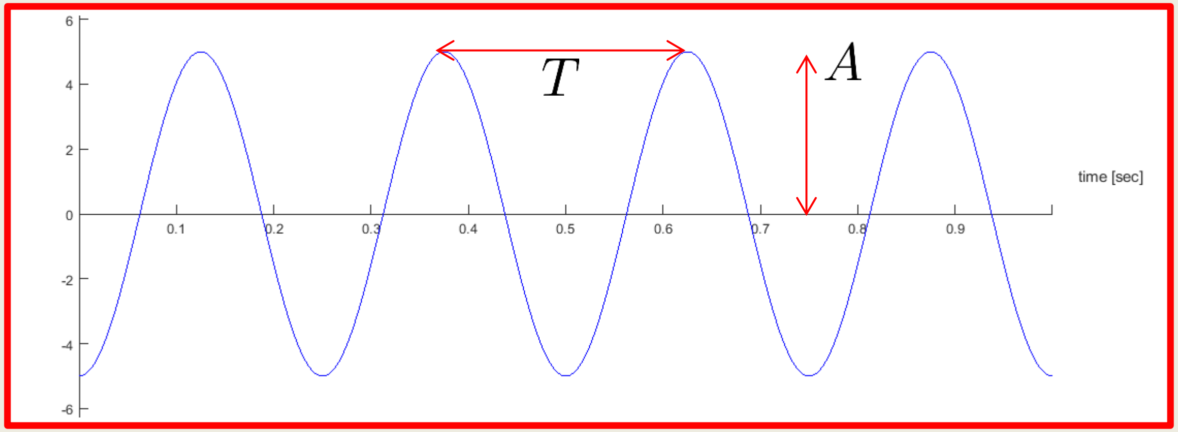
F Frequency [Hz]

Φ Phase [Rad]

EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi)$$



A Amplitude

F Frequency [Hz]

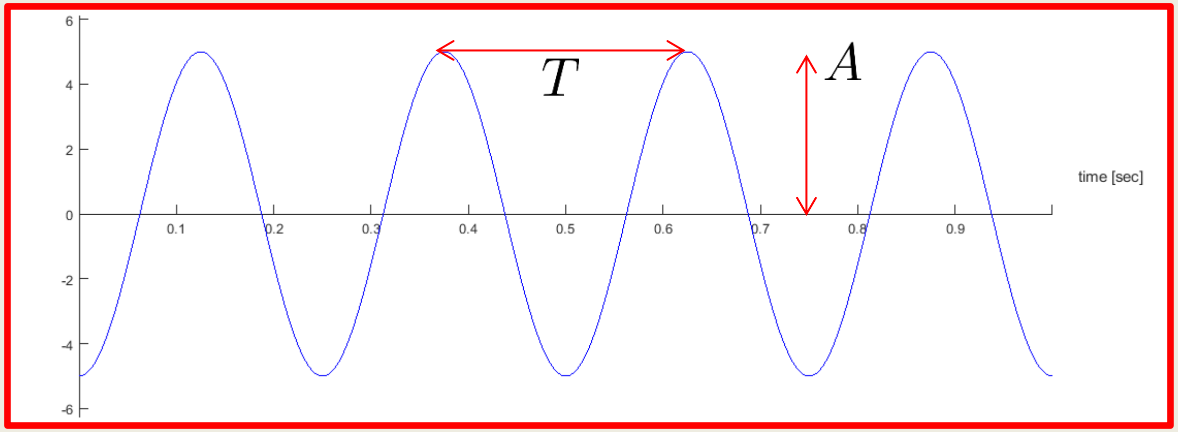
Φ Phase [Rad]

$T = F^{-1}$ Period [s]

EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi) \;= A \cdot \sin(\Omega t - \Phi)$$



A Amplitude

F Frequency [Hz]

Φ Phase [Rad]

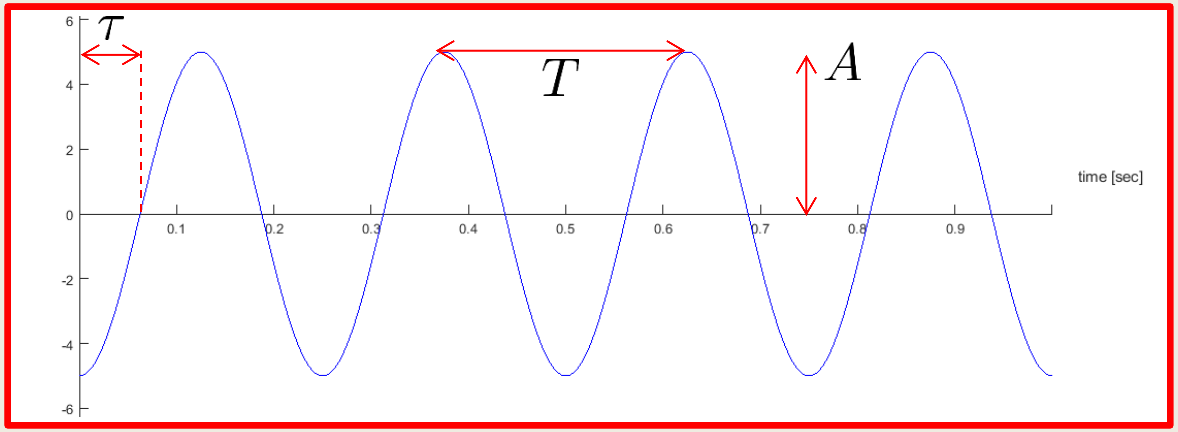
$T = F^{-1}$ Period [s]

$\Omega = 2\pi F$ Freq [Rad/s]

EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi) = A \cdot \sin(\Omega t - \Phi) = A \cdot \sin\left(\Omega\left(t - \frac{\Phi}{\Omega}\right)\right)$$



A Amplitude

F Frequency [Hz]

Φ Phase [Rad]

$T = F^{-1}$ Period [s]

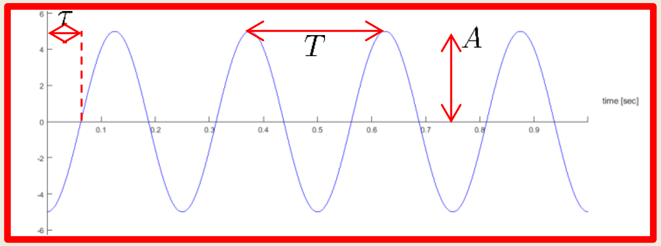
$\Omega = 2\pi F$ Freq [Rad/s]

$\tau = \frac{\Phi}{\Omega}$ Delay [s]

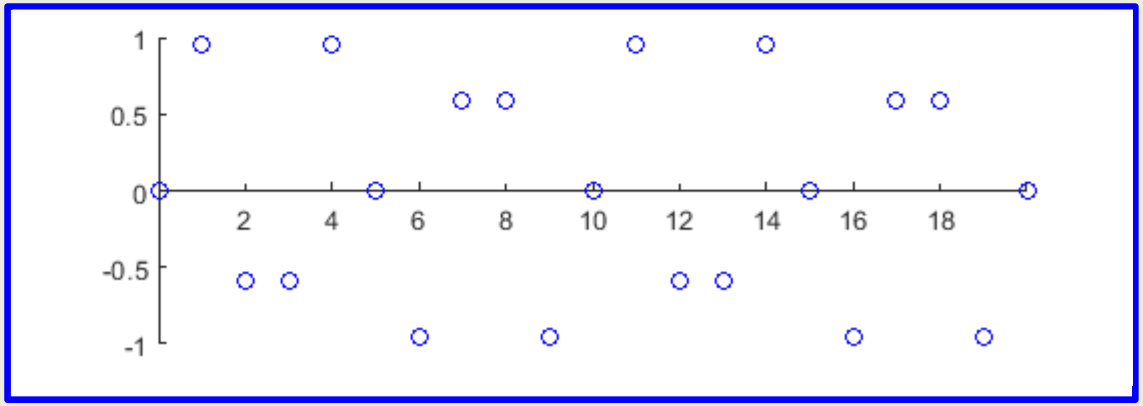
EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi) = A \cdot \sin(\Omega t - \Phi) = A \cdot \sin\left(\Omega\left(t - \frac{\Phi}{\Omega}\right)\right)$$



$$x(n) = A \cdot \sin(2\pi f n - \Phi)$$



f Digital Freq [-]

EITF75, Introduction

Preliminaries of Sinusoids

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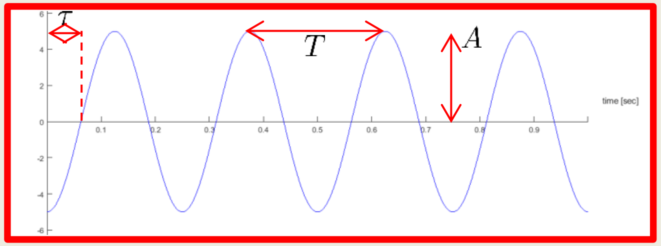
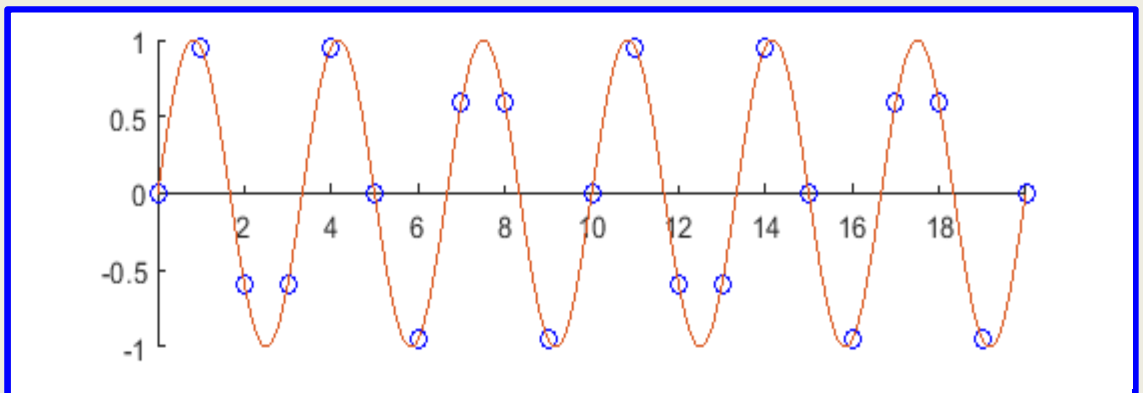


$$x(n) = A \cdot \sin(2\pi f n - \Phi)$$

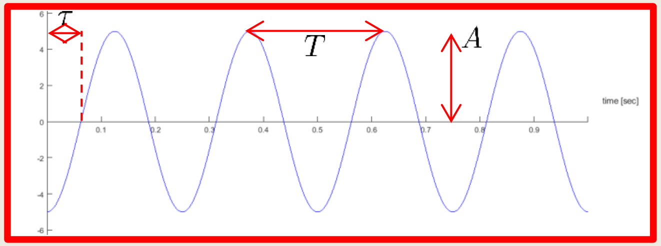


f Digital Freq [-]

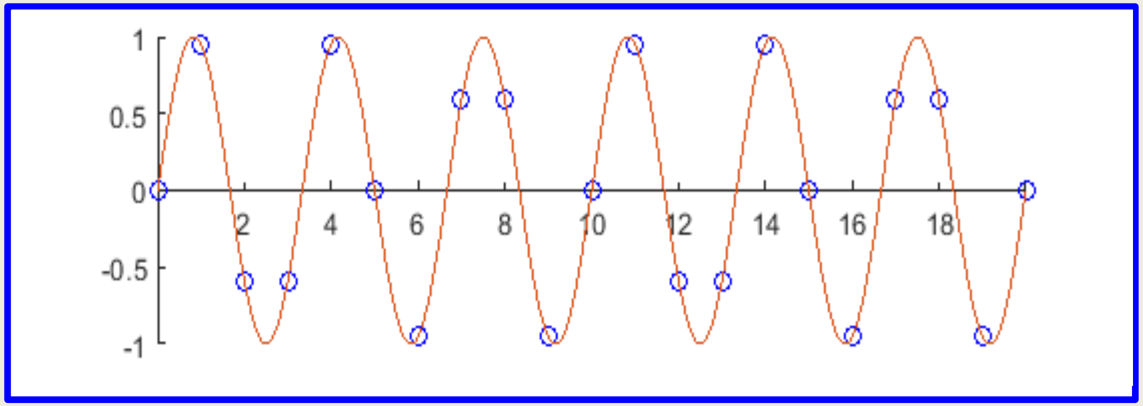
EITF75, Introduction

Preliminaries of Sinusoids

$$x(t) = A \cdot \sin(2\pi F t - \Phi) = A \cdot \sin(\Omega t - \Phi) = A \cdot \sin\left(\Omega\left(t - \frac{\Phi}{\Omega}\right)\right)$$



$$x(n) = A \cdot \sin(2\pi f n - \Phi) = A \cdot \sin(\omega n - \Phi)$$



f Digital Freq [-]

$\omega = 2\pi f$ Digital Freq [-]

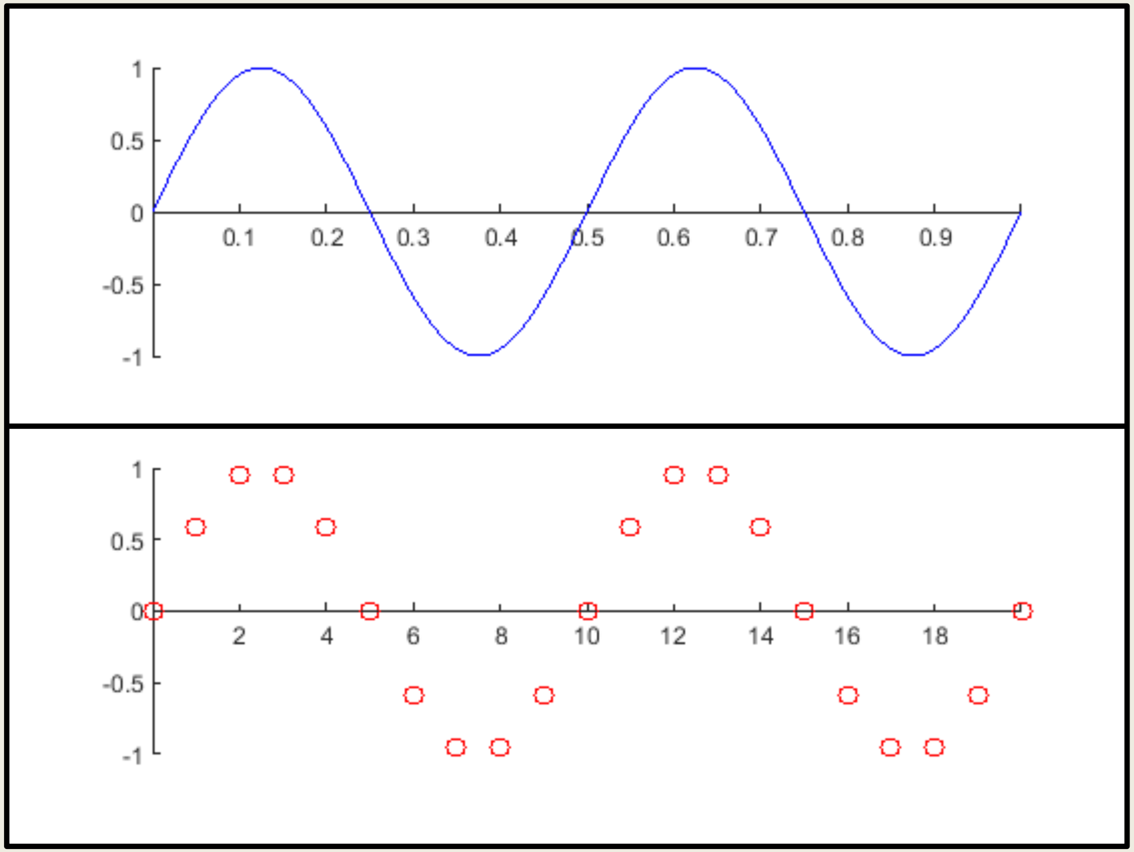
EITF75, Introduction

Preliminaries of Sinusoids

Examples

$$x(t) = A \cdot \sin(2\pi F t)$$
$$F = 2$$

$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 0.1$$



EITF75, Introduction

Preliminaries of Sinusoids

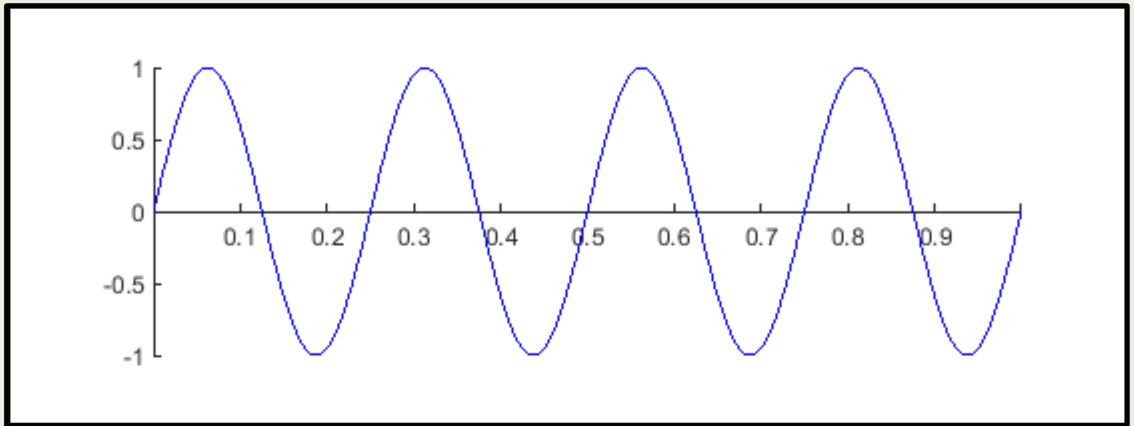
Examples

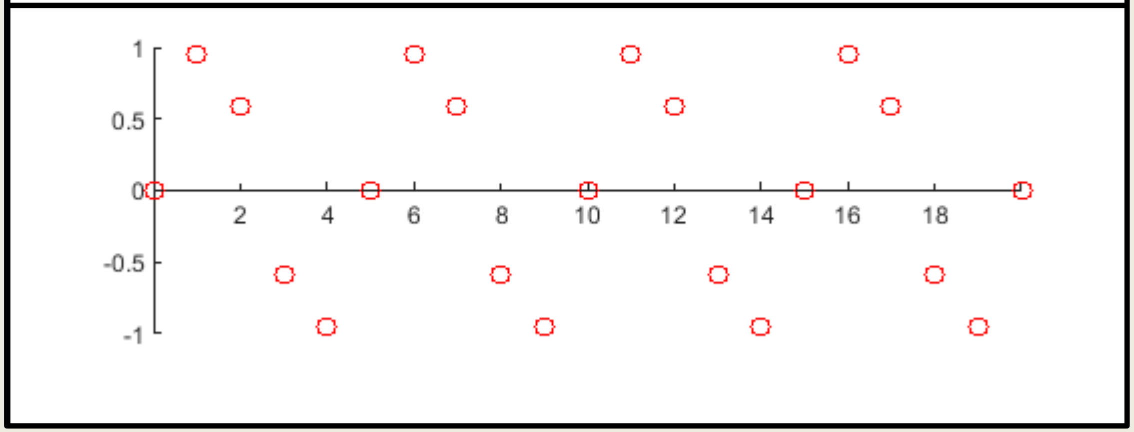
$$x(t) = A \cdot \sin(2\pi F t)$$

$$F = 4$$

$$x(n) = A \cdot \sin(2\pi f n)$$

$$f = 0.2$$





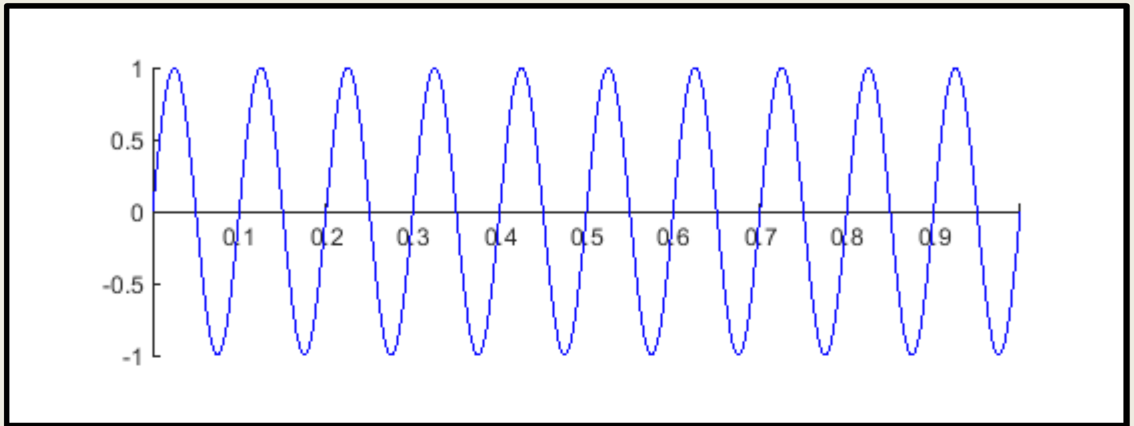
EITF75, Introduction

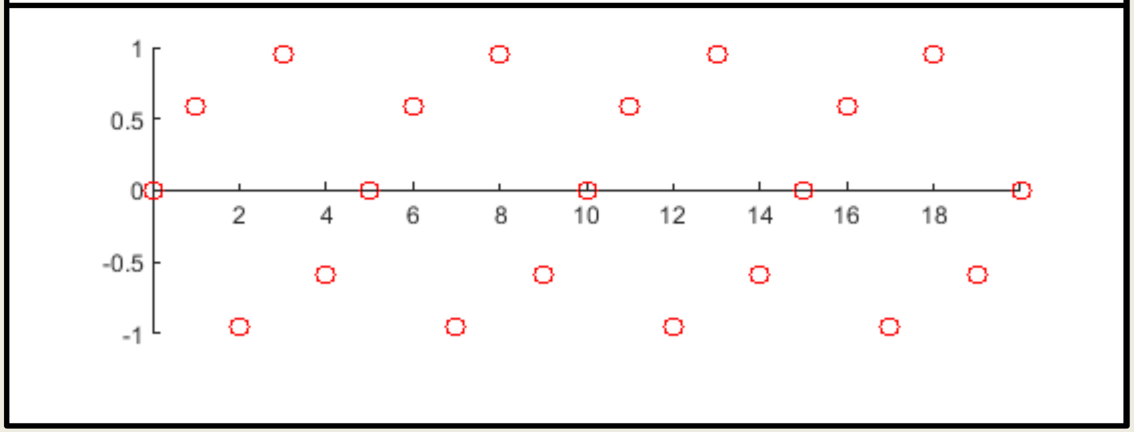
Preliminaries of Sinusoids

Examples

$$x(t) = A \cdot \sin(2\pi F t)$$
$$F = 10$$

$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 0.4$$



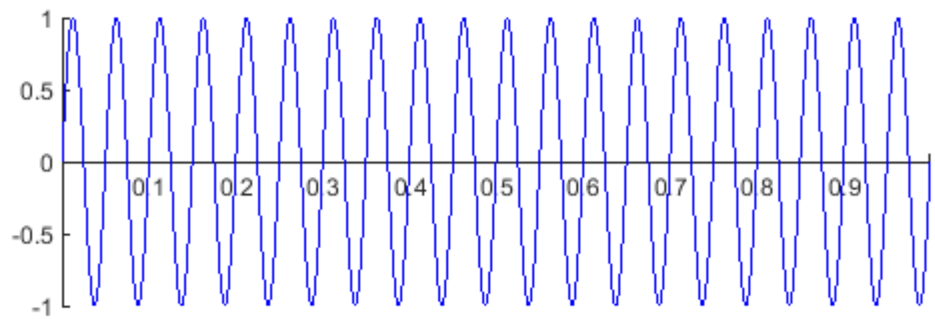


EITF75, Introduction

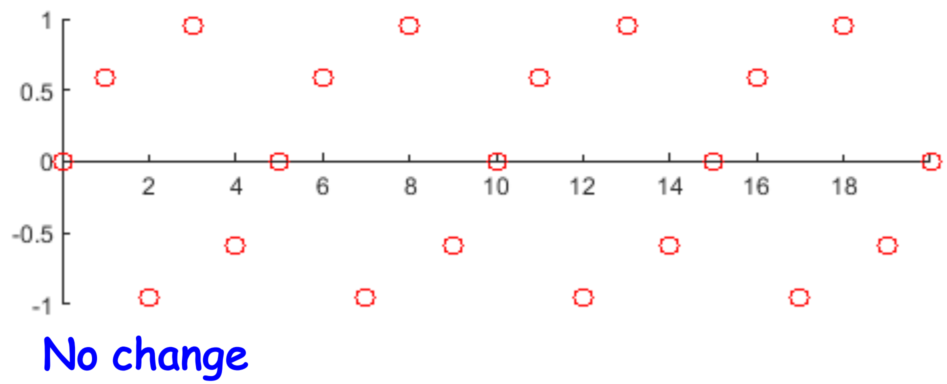
Preliminaries of Sinusoids

Examples

$$x(t) = A \cdot \sin(2\pi F t)$$
$$F = 20$$



$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 1.4$$



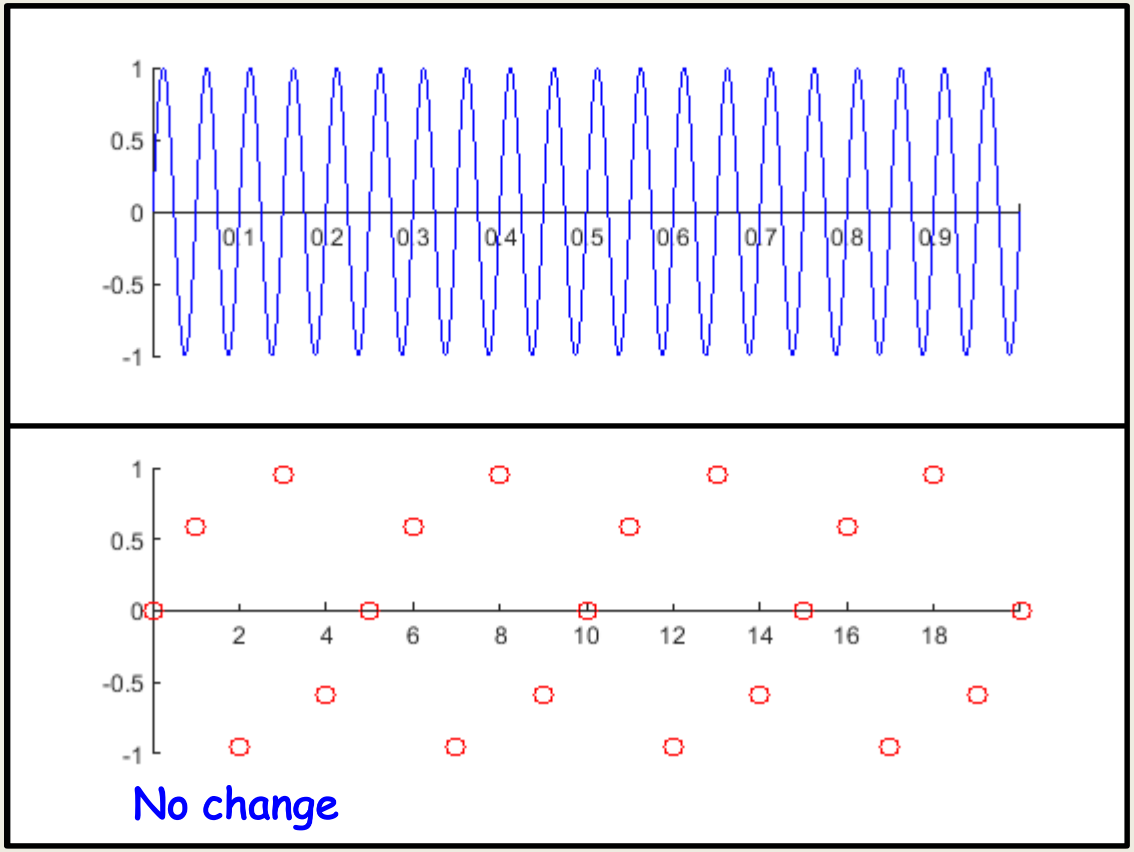
EITF75, Introduction

Preliminaries of Sinusoids

Examples

$$x(t) = A \cdot \sin(2\pi F t)$$
$$F = 20$$

$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 8.4$$



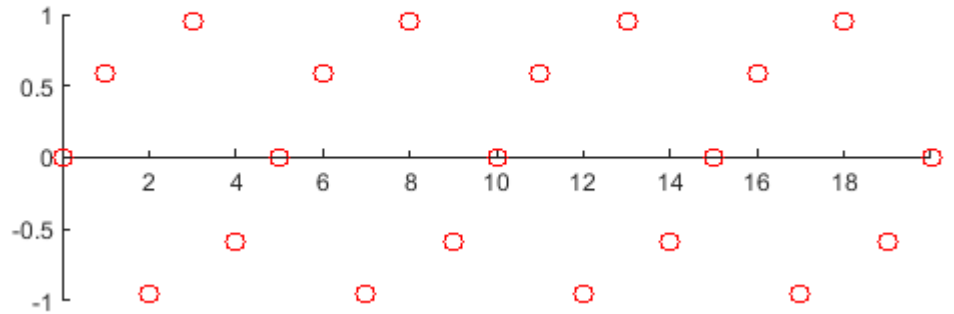
EITF75, Introduction

Preliminaries of Sinusoids

Explanation

$$x(n) = A \cdot \sin(2\pi f n)$$

$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 8.4$$



No change

EITF75, Introduction

Preliminaries of Sinusoids

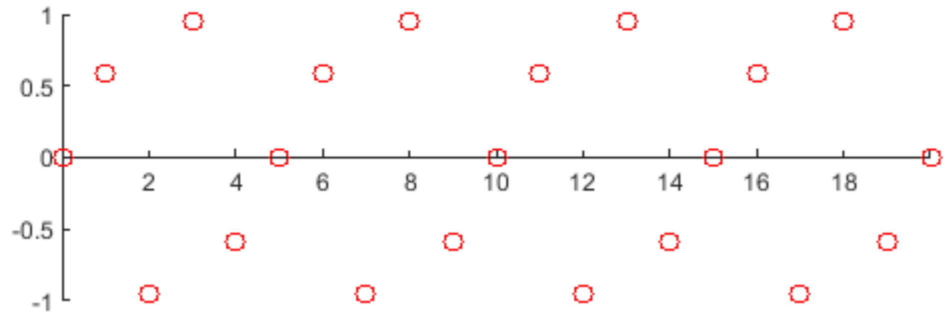
Explanation

$$x(n) = A \cdot \sin(2\pi f n) = A \cdot \sin(2\pi (f' + k) n)$$

$$f = (f' + k), \; -\frac{1}{2} \leq f' < \frac{1}{2}, \; k \in \mathbb{Z}$$

$$x(n) = A \cdot \sin(2\pi f n)$$

$$f = 8.4$$



No change

EITF75, Introduction

Preliminaries of Sinusoids

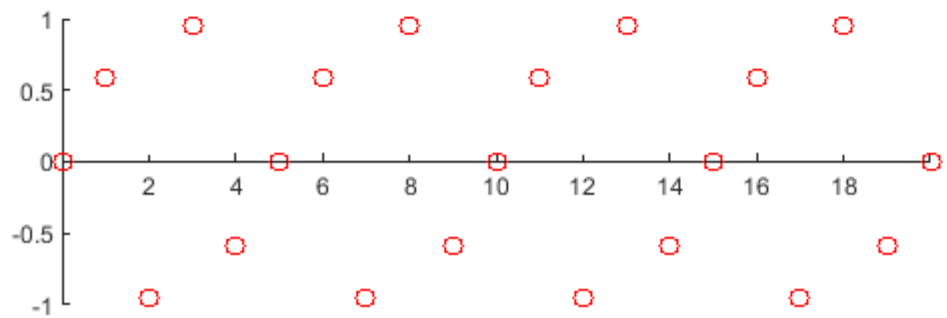
Explanation

$$x(n) = A \cdot \sin(2\pi f n) = A \cdot \sin(2\pi (f' + k)n) = A \cdot \sin(2\pi f' n)$$

$$f = (f' + k),\ -\frac{1}{2} \leq f' < \frac{1}{2},\ k \in \mathbb{Z}$$

$$x(n) = A \cdot \sin(2\pi f n)$$

$$f = 8.4$$



No change

EITF75, Introduction

Preliminaries of Sinusoids

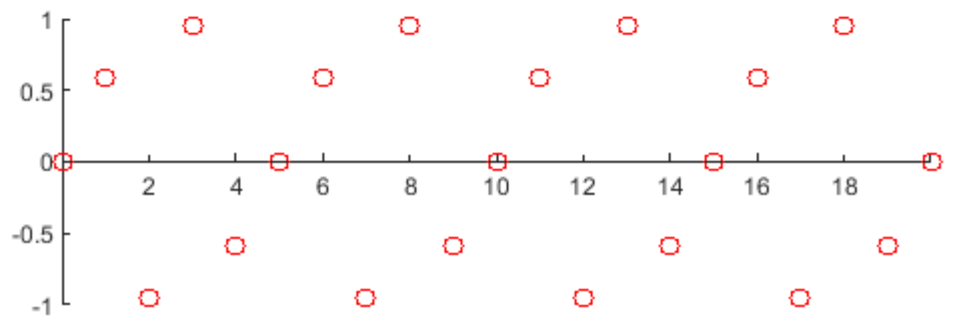
$$x(n) = A \cdot \sin(2\pi f' n)$$
$$-\frac{1}{2} \leq f' < \frac{1}{2}$$

Important

Discrete sinusoid defined with a frequency of at most $\frac{1}{2}$ (or π rad) in magnitude

(Since higher frequencies than this don't make any sense)

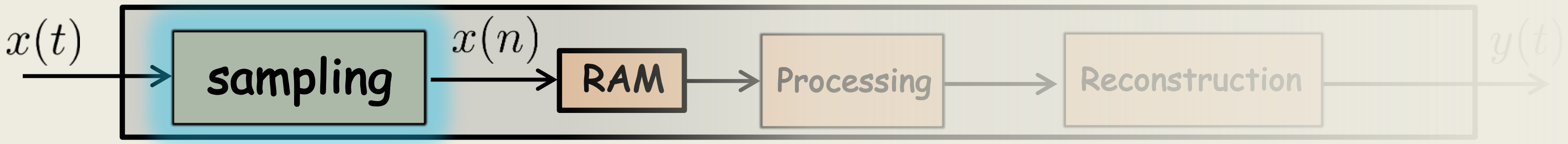
$$x(n) = A \cdot \sin(2\pi f n)$$
$$f = 8.4$$



No change

EITF75, Introduction

Preliminaries of Sampling

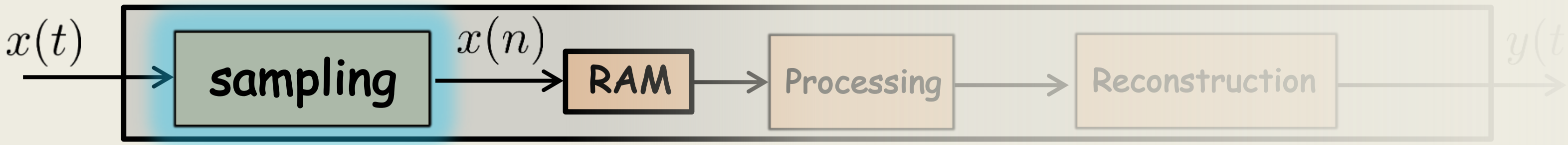


$$x(t) = \sin(2\pi 440t)$$

Sample with 1000 samples/sec

EITF75, Introduction

Preliminaries of Sampling



$$x(t) = \sin(2\pi 440 t)$$

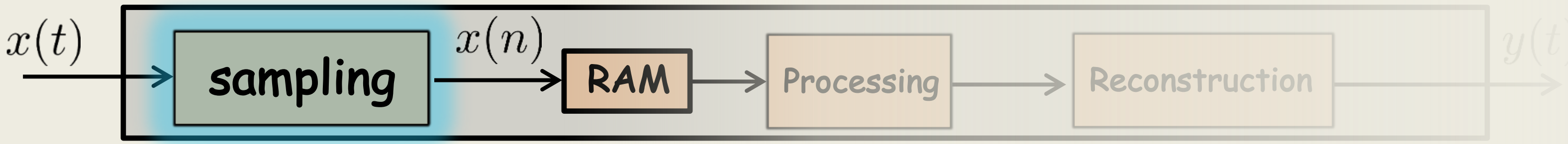
Sample with 1000 samples/sec

$$x(n) = x(t|t = nT_s = n/F_s)$$

symbol	meaning	Ex. value
F_s	# Samples/sec [Hz]	10^3
T_s	Time between samples [s]	$1/F_s = 10^{-3}$

EITF75, Introduction

Preliminaries of Sampling



$$x(t) = \sin(2\pi 440 t)$$

Sample with 1000 samples/sec

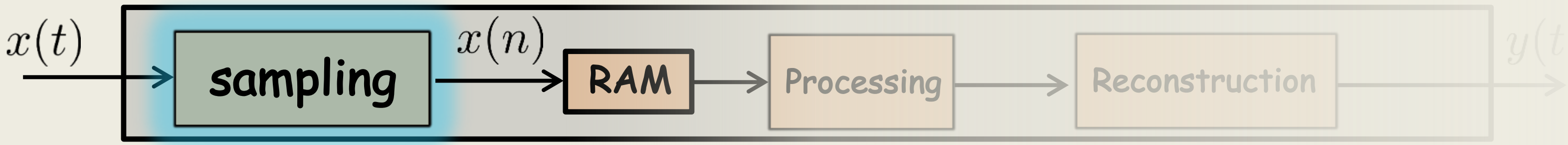
$$x(n) = x(t|t = nT_s = n/F_s)$$

$$= \sin\left(2\pi \frac{440}{1000} n\right)$$

symbol	meaning	Ex. value
F_s	# Samples/sec [Hz]	10^3
T_s	Time between samples [s]	$1/F_s = 10^{-3}$

EITF75, Introduction

Preliminaries of Sampling



$$x(t) = \sin(2\pi 440 t)$$

Sample with 1000 samples/sec

$$\boxed{\begin{aligned} x(n) &= x(t|t = nT_s = n/F_s) \\ &= \sin\left(2\pi \frac{440}{1000} n\right) \\ &= \sin\left(2\pi \cdot 0.44 n\right) \end{aligned}}$$

$$f = f' = 0.44$$

symbol	meaning	Ex. value
F_s	# Samples/sec [Hz]	10^3
T_s	Time between samples [s]	$1/F_s = 10^{-3}$

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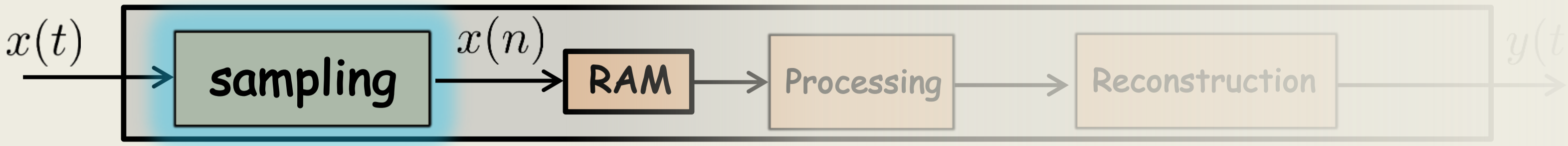
Preliminaries of Sampling



$$x(t) = \sin(2\pi 440 t)$$

Sample with 1000 samples/sec

$$x(n) = x(t|t = nT_s = n/F_s)$$

$$= \sin\left(2\pi \frac{440}{1000} n\right)$$

$$= \sin\left(2\pi \cdot 0.44 n\right)$$

$$f = f' = 0.44$$

$$x(t) = \sin(2\pi 440 t) - \sin(2\pi 1440 t)$$

Sample with 1000 samples/sec

$$x(n) =$$

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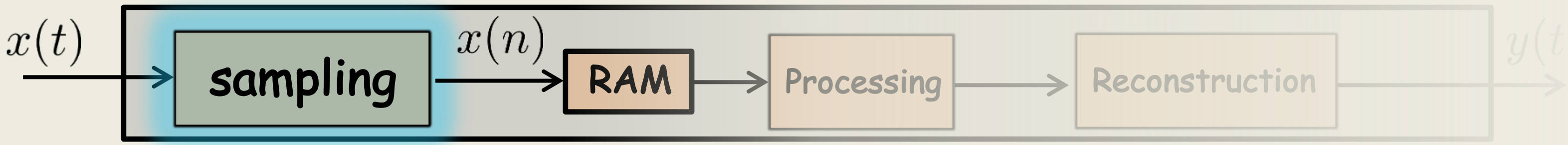
Preliminaries of Sampling



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Sample with 1000 samples/sec

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Sample with 1000 samples/sec

$$x(n) = \sin\left(2\pi \cdot 0.44 n\right) - \sin\left(2\pi \cdot 1.44 n\right)$$

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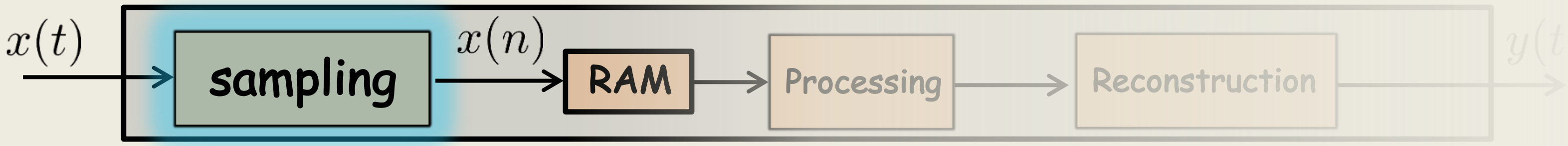

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Sample with 1000 samples/sec

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$$f = f' = 0.44$$

$$x(t) = \sin(2\pi 440 t) - \sin(2\pi 1440 t)$$

Sample with 1000 samples/sec

$$\begin{aligned} x(n) &= \sin(2\pi \cdot 0.44 n) \\ &\quad - \sin(2\pi \cdot 1.44 n) \\ &= \sin(2\pi \cdot 0.44 n) \\ &\quad - \sin(2\pi \cdot 0.44 n) \\ &= 0 \end{aligned}$$

Sampling rate too low

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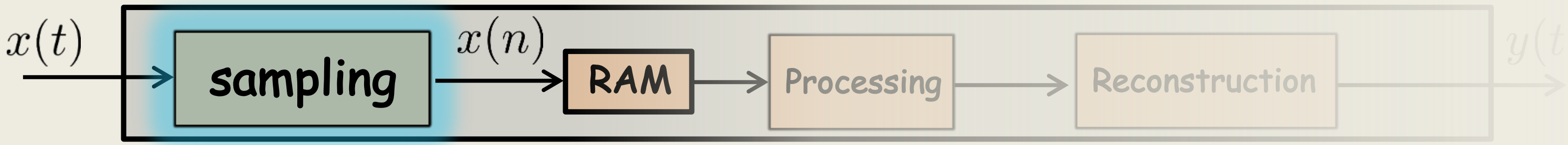
Preliminaries of Sampling



Conclusion (verify this at home)

If $x(t) = \sum_{\ell} \sin(2\pi F_\ell t), \quad |F_\ell| \leq F_{\max}$

and we want a sampling Frequency such that we can reconstruct $x(t)$ from $x(n)$, then F_s must satisfy

$$F_s \geq \dots$$

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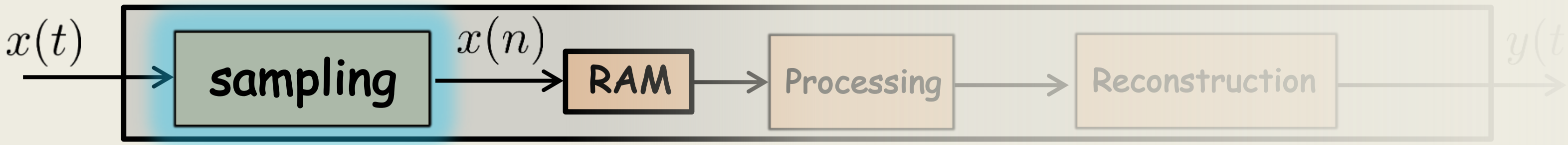
Preliminaries of Sampling



Conclusion (verify this at home)

If $x(t) = \sum_{\ell} \sin(2\pi F_\ell t), \quad |F_\ell| \leq F_{\max}$

and we want a sampling Frequency such that we can reconstruct $x(t)$ from $x(n)$, then F_s must satisfy

$$F_s \geq 2F_{\max}$$