Exam in Digital Signal Processing, EITF75

Thursday October 31

1. **Write clearly!** If I cannot read what you write, I **will consider it as not written at all**. My decision on this matter is final, you cannot argue that I should have been able to read it later.
2. It is important to **show the intermediate steps** in arriving at an answer, otherwise you may lose points.
3. When generating problems of the True/False form, I use Matlab's random number generator.
4. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments: If you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that at least something must be correct, points may be deducted for everything that is wrong.
5. Problems are not arranged in an order of ascending difficulty.
6. Allowed tools: Everything that cannot connect to the internet.
7. The exam should be answered in Swedish or English and please keep the language constant for all of the problems. However, if you choose to answer in Swedish, it is *allowed* to use English terminologies.
8. If you think some parameters are missing, state them carefully and then motivate your statement.

Problem 1 (1.0p)

Assume a stable LTI circuit with two distinct real-valued poles, not located at $z = 0$. The system is not at rest, and is turned on at $n = 0$.

a)

Assume that $y(-1) \neq 0$. Further, assume that we have possibility to control the remaining initial conditions $y(-k), k > 1$. An engineer claims that it is possible to control initial conditions in such a way so that the output signal (due to the initial conditions) is 0. In other words, for $x(n) = 0, n \geq 0$ and $y(-1) \neq 0$, we can find $y(-k), k > 1$ such that $y(n) = 0, n \geq 0$. Is this true? If yes, show how to accomplish it. If no, provide derivations/arguments that disproofs the statement.

b)

Another engineer claims that it is always possible to find a *casual stable* input signal $x(n)$, such that the output $y(n) = 0$, $n \geq 0$ no matter what the initial conditions are. Is this true? If yes, show how to accomplish it. If no, provide derivations/arguments that disproofs the statement.

c)

A third engineer claims that for any *casual stable* input signal $x(n)$, it is possible to find initial conditions satisfying $|y(-k)| < \infty$, such that the output signal is unstable. This is doable by adapting the initial conditions to the input signal $x(n)$. Is this true? If yes, show how to accomplish it. If no, provide derivations/arguments that disproofs the statement.

d)

A fourth engineer claims if we are interested in the output signal $y(n)$ at large n, then the initial conditions are not very important, because their influence on $y(n)$ fades out with time. Is this true? If yes, please motivate your reason. If no, provide derivations/arguments that disproofs the statement.

Problem 2 (1.0p)

a)

Determine the response $y(n)$ of the below system. The system is at rest, and turned on at $n = 0$.

$$y(n+2) - 3y(n+1) + 2y(n) = \delta(n).$$

b)

The transfer function of an LTI system is given by

$$H(z) = \frac{0.5z}{(z+0.5)(z-0.5)}.$$

Compute the *steady state output* to the input signal $x(n) = \cos(\pi n/4)$.

c)

For the same transfer function as in part b), compute the *steady state output* to the input signal $x(n) = u(n)\cos(\pi n/4)$. (Hint: There are methods to escape the brute-force calculation.)

d)

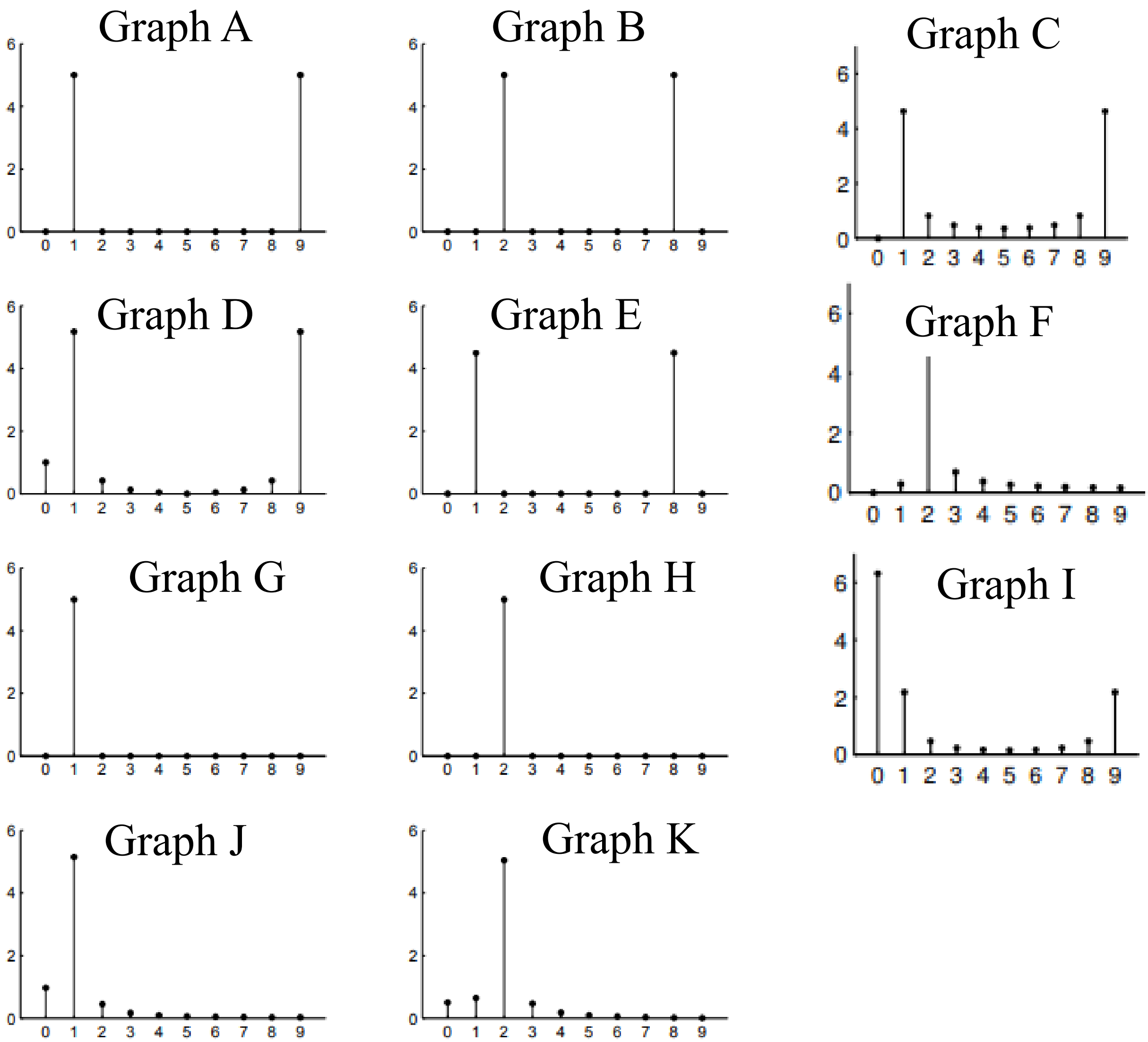
For the same transfer function as in part b), compute the output to the input signal $x(n) = u(n)$.

Problem 3 (1.0p)

Consider the following four signals and index n ranging from $n = 0, 1, \ldots, 9$,

$$x_1(n) = \cos(2\pi n/9) \qquad x_2(n) = \cos(2\pi n/10) \qquad x_3(n) = \sin(2\pi n/10) \qquad x_4(n) = \sin(\pi n/9).$$

Which of the following graphs illustrates the 10-point DFT $|X(k)|$ of each signal?


Graph A
Graph B
Graph C
Graph D
Graph E
Graph F
Graph G
Graph H
Graph I
Graph J
Graph K

Problem 4 (1.0p)

a)

Assume that a continuous signal $x(t)$ with Fourier transform

$$X(F) = \begin{cases} 1 - |F|/1000 & |F| \leq 1000\,\text{Hz} \\ 0 & \text{otherwise} \end{cases}$$

is sampled with sampling rate $F_\text{s} = 1000$ Hz.

Let the resulting discrete samples be the inputs to a digital filter with transfer function

$$H(z) = \frac{z+1}{z^2+0.5}.$$

What is the output of the digital filter?

b)

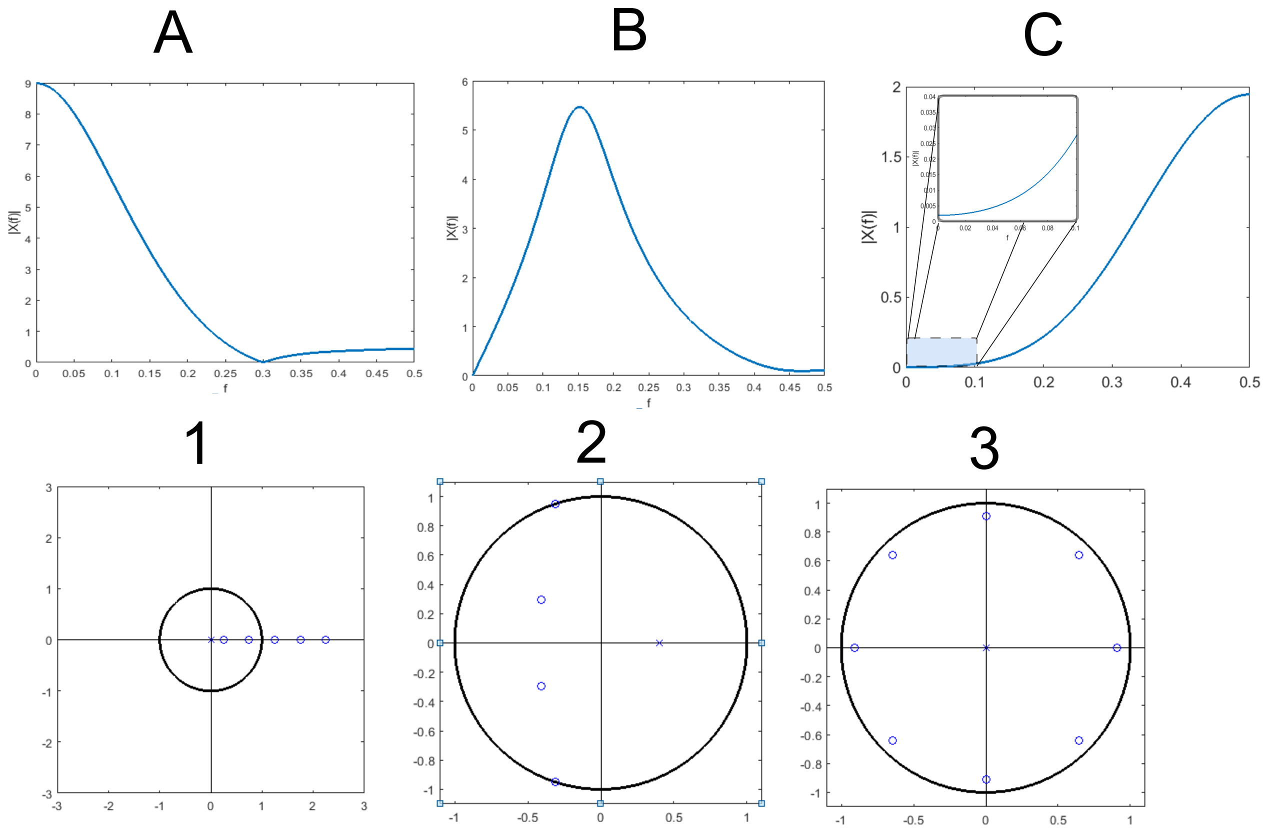
Repeat part a) but with sampling rate $F_\text{s} = 500$ Hz.

c)

Is it true that the result in part b) can be obtained by down-sampling the result in part a) by a factor of 2? (That is, down-sampling of the output of the digital filter in a) would result in the digital filter output in b)). You can solve this problem even if you have not solved a) and b).

d)

Assume that we would like to carry out Computer/Matlab-based spectral analysis of the continuous pulse $x_\text{analog}(t) = 1$, $0 \leq t \leq 1$. At our disposal, we have samples $x(n) = x_\text{analog}(n/F_\text{s})$ where F_s can be chosen at will, and a routine for computing DFTs. Explain how to obtain a plot of the Fourier-transform of $x_\text{analog}(t)$. Comment on our ability to increase the accuracy of the plot.



Problem 5 (1.0p)

a)

Compute the DTFT of the signal $x(n) = (0.5)^{|n-3|}$.

b)

Below there are 3 pole-zero diagrams (1-3) shown and 3 DTFTs (A-C). Regarding to diagram 2, there are two zeros located on the circle, at around points $(-\frac{1}{3}, \pm\sqrt{\frac{8}{9}})$, which may not show very clearly in the printed paper. Two of the DTFTs corresponds to two of the DTFTs, but there is one DTFT whose pole-zero diagram is not shown, and there is one pole-zero diagram whose DTFT is not shown. Determine which pole-zero diagram that corresponds to which DTFT, and sketch the missing DTFT (call it D) and pole-zero diagram (call it 4).