Exam in Multiple Antenna systems, EITF75

Wednesday October 31

Problem 1 (1.0p)

We are given a circuit which we know implements a difference equation having at most 2 poles and an at most 2 zeros. We give an input signal $x(n)$ to the circuit, turn on the circuit at time $n = 0$, and investigate the output signal.

a)

Suppose that $x(n) = 0$, for all n. The output signal is, however, $y(n) = (-1)^n$, $n \geq 0$. Provide as much information about the circuit as you can.

b)

Based on the information provided in a), is it possible to determine the exact input signal $x(n)$ for which the steady state output would become $y(n) = 0$? (With "exact", is meant that it should not be given in terms of some unknown variables).

c)

We apply the input signal $x(n) = u(n)$ (step function). The output signal is, for large values of n, $y(n) = 0$. What information about the circuit does this provide?

d)

We again apply the input signal $x(n) = u(n)$ (step function), but now observes the output signal $y(n)$ at all n. Surprisingly, we find that $x(n) = 0$ for all n. What information about the circuit does this provide?

Problem 2 (1.0p)

Assume an LTI system at rest (zero-state) with system equation

$$H(z) = \frac{2 + z^{-1} + 0.5z^{-2}}{1 + 0.5z^{-1} - 0.25z^{-2}}.$$

Assume that the output signal is

$$y(n) = [(0.5)^n + (-0.5)^n]\, u(n) + (0.5)^{n-1}\, u(n-1).$$

What is the input signal?

Problem 3 (1.0p)

Assume a two-user communication system in which two users are sending signals $x_1(t)$ and $x_2(t)$ to a receiving device. We further assume a noise-free universe. The receiving device is sampling its input signal $x(t) = x_1(t) + x_2(t)$ at a sampling frequency of F_s Hz.

a)

The two users are said to be "separable" if two time-discrete ideal filters $\mathcal{F}_1$ and $\mathcal{F}_2$ can be found such that the output of $\mathcal{F}_1$ is sufficient to reconstruct $x_1(t)$ perfectly - possibly with further processing necessary - without any interference from $x_2(t)$, and similarly for $\mathcal{F}_2$. Suppose that

$$\begin{cases} X_1(F) > 0, & 2000 \leq |F| \leq 4000 \\ X_1(F) = 0, & \text{otherwise} \end{cases}$$

$$\begin{cases} X_2(F) > 0, & 6000 \leq |F| \leq 8000 \\ X_2(F) = 0, & \text{otherwise} \\ X_2(7000 + \delta) = X_2(7000 - \delta) & 0 \leq \delta \leq 1000 \end{cases}$$

What is the smallest possible value for F_s such that the two users are separable?

b)

Assume now that $F_s = 20000$ Hz, and define "separability" as in a). Design, by selecting the zero-pole structure, a practical filter implementation of $\mathcal{F}_1$. (Note: Here, I am mostly interested in checking your understanding on the effect of zero-pole placement and filter characteristics - not to check whether you remember how to get a very good filter. However, your methodology for designing the filter should be clear).

c)

Continuation of b). Suppose now that the filter $\mathcal{F}_1$ has been ideally implemented, and that D/A reconstruction of $x_1(t)$ is desired. Unfortunately, there is no D/A converter available with such a high sampling rate as 20000 Hz. Show how succesful reconstruction can be obtained nonetheless using downsampling and a D/A converter with sampling rate < 20000 Hz.

Problem 4 (1.0)p

An acoustic company would like to implement an echo functionality to its devices. Two options are on the table, circuit I and circuit II. It is assumed that $\alpha > 0$. In both cases, $D = 1000$.

a)

Calculate the system functions for both cases, and compute zeros and poles.

b)

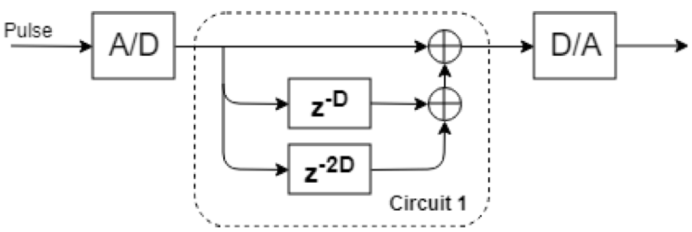
For circuit I, assume that a very short input signal $x(t)$ is used and that the first echo can be heard with a delay of 0.5 seconds. What is the sampling frequency F_s?

C)

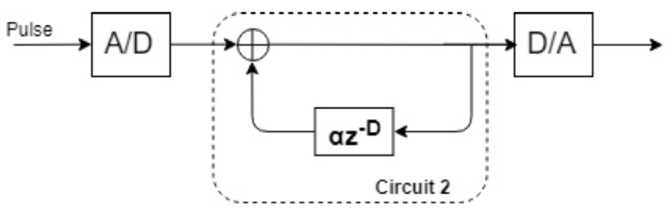
For circuit II, provide a necessary and sufficient condition for the echo to fade out with time for a very short input signal $x(t)$.

D

For circuit II, assume that the echo can be heard by a human as long as the power of the echo is less than 20dB attenuated compared with the original signal. Provide a formula for the number of echos that one can hear for a short input signal $x(t)$.



(a) Circuit 1: FIR filter



(b) Circuit 2: IIR filter

Problem 5 (1.0)p

Assume that you have a signal $x(n)$ that satisfies $x(n) = 0,\ n < 0$ and $x(n) = 0,\ n \geq L$. Let X_k denote the N-DFT of the sequence $x(n)$, $N \geq L$.

a)

Assume that we construct $Y_k = H_k X_k,\ 0 \leq k \leq N-1$. Let $y(n)$ be the IDFT of Y_k, and let $h(n)$ be the IDFT of H_k. Under what conditions is it true that

$$y(n) = x(n) * h(n).$$

b)

Assume that we construct $Y_k = X_k^2,\ 0 \leq k \leq N-1$. Let $y(n)$ be the IDFT of Y_k. Under what conditions is it true that

$$y(n) = x(n) * x(n).$$

c)

Assume that $N = L = 5$, and that a device is provided with the sequence X_k, but that it does not have ability ot compute an IDFT. Can the device compute the value $X(\omega_1)$ for $\omega_1 = 3\pi/5$?

d)

Explain, briefly, why the DFT is important when we can do essentialyl the same using a DTFT. Be concise, but precise - no essays please.

e)

Suppose that we construct $Y_k = H_k X_k,\ 0 \leq k \leq N-1$ where

$$H_k = \frac{1 - \alpha \cos(\omega_0) \exp(-i2\pi k/N)}{1 - 2\alpha \cos(\omega_0) \exp(-i2\pi k/N) + \alpha^2 \exp(-i4\pi k/N)}.$$

Describe the effect on $y(n)$.