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Examination, Systems and Signals EITF75

All tools allowed. Send scanned solutions to fredrik.rusek@eit.lth.se no later than 13.15.

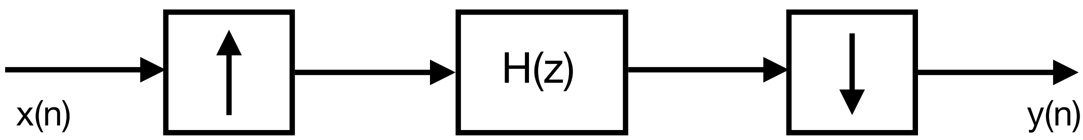
Problem 1 (10p)

For the difference equation $y(n) + \frac{1}{2}y(n-1) = 3x(n) - x(n-1)$, find

1. The system transfer function $H(z)$
2. The pole-zero diagram corresponding to the difference equation
3. The impulse response $h(n)$
4. The Discrete-time Fourier transform of the impulse response $H(f)$
5. The output for the input signal $x(n) = [1 \ \ 1 \ \ -1 \ \ 1]$
6. The steady state response to the input $x(n) = u(n)$ (a step)
7. The transient response to the input $x(n) = u(n)$

Problem 2 (10p)

Consider the following circuit, where an “up-arrow” means upsampling with a factor 2, and a “down-arrow” means downsampling with a factor 2.



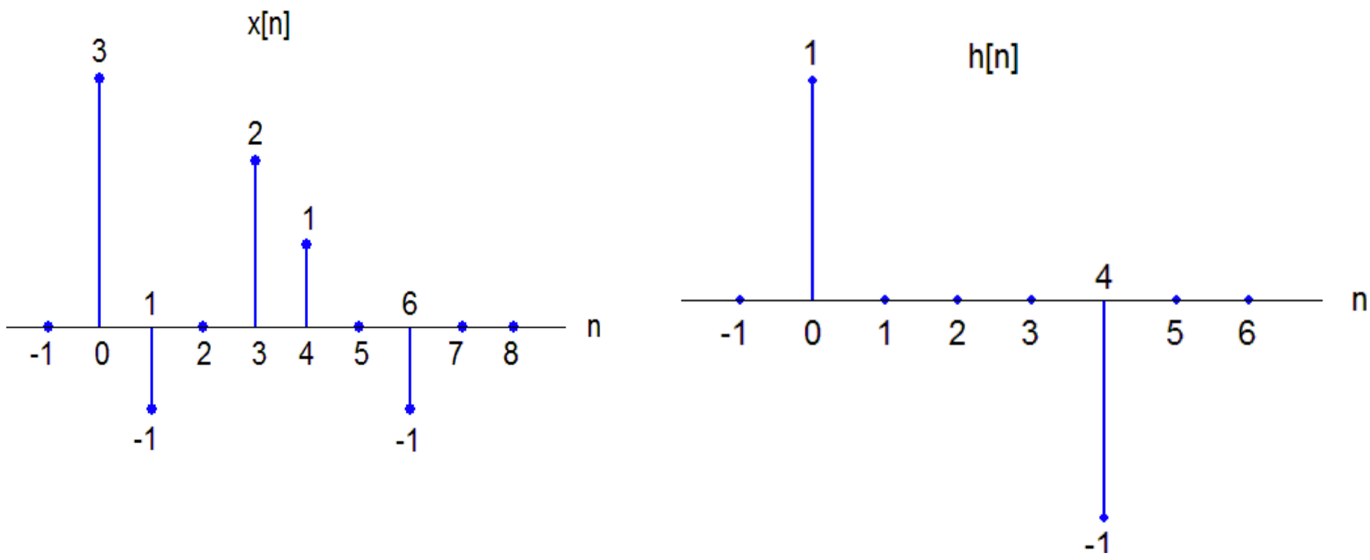
where $H(z) = \dfrac{1}{1 + 0.5z^{-1}}$.

It *is* possible to express the input-output relation between x(n) and y(n) as $y(z) = G(z)x(z)$ where G(z) is found from the upsampling, the filter H(z) and the downsampling. Determine G(z).

Problem 3 (5+5p)

This problem deals with the DFT.
HINT: It is possible to solve the entire problem without calculating any DFT, its inverse, or the DTFT. This is possible by using properties of DFTs.

Let x(n) and h(n) be the two signals shown below.



PART A)
Take the 8-point DFT of both x(n) and h(n) to obtain X[k] and H[k]. Multiply to obtain Y[k]=H[k]X[k]. Now take the inverse 8-point IDFT of Y[k] to obtain y(n). What is y(n)?

RECALL THE HINT: there is a quicker way to determine y(n) than to numerically calculate the DFTs and then calculating the IDFT.

PART B)
Calculate the DTFT of x(n), i.e., obtain X(f), 0 < f < 1. Sample X(f) at sample points f={0,0.25,0.5,0.75} to obtain
$R[0] = X(0)$, $R[1] = X(0.25)$, $R[2] = X(0.5)$, $R[3] = X(0.75)$.

Now, compute the 4-point IDFT of R[0], R[1], R[2], R[3]. Sketch your result.

RECALL THE HINT: there is a quicker way to determine y(n) than to numerically calculate the DTFT, sample, and then calculating the IDFT.

Problem 4 (10p)

Assume that we have a continuous (analog) time signal x(t) of length 100ms, and assume further that it is band-limited to 10000Hz, i.e., if X(F) is the analog Fourier transform, then |X(F)|=0, |F|>10000.

The goal of this problem is to compute samples of X(F) with 5Hz sample spacing in the range 0 < F < 10000. This can be done with a 4000-point DFT. Specifically, we want to obtain a 4000-point sequence x(n) for which the 4000-point DFT X[k] is related to X(F) by:

$$X[k] = \alpha X(F = 5k), \;\; k = 0...1999 \tag{1}$$

where α is a scale factor that is not important to us. Note that there are more than 2000 values of X[k], but we only care about the first 2000.

The following method to obtain a 4000-point sequence is proposed. First the signal x(t) is sampled with a sample period of $T = 50\mu s$. The resulting 2000 samples are used to form z(n) as follows:

$$z(n) = \begin{cases} x(kT), & 0 \leq n \leq 1999 \\ x((k-2000)T), & 2000 \leq n \leq 3999 \\ 0, & \text{otherwise}. \end{cases}$$

The 4000-point DFT Z[k] is computed.

For this method, determine how Z[k] is related to X(F). Indicate the relationship using a “typical” Fourier transform X(F). Explicitly state whether or not Z[k] is the desired result, i.e., if Z[k] equals X[k] as specified in Equation 1.

Problem 5 (10p)

Consider the difference equation:

$$y(n) + (2 \cdot 0.7 \cdot 0.8)y(n-1) + 0.8^2 y(n-2) = x(n) + \frac{1}{2}x(n-1)$$

where " · " means normal multiplication, i.e., the coefficient of y(n-1) is 1.12.

We have the following 3 input signals:

$$x_1(n) = u(n) \quad \text{(a step)} \qquad x_2(n) = u(n)\cos(2\pi n/4) \qquad x_3(n) = u(n)\cos(2\pi n/2)$$

We have the following 5 steady-state output signals:

$$y_1(n) = A_1 u(n) \qquad y_2(n) = A_2 u(n)\cos(2\pi n/4 - \alpha_2) \qquad y_3(n) = A_3 u(n)\cos(2\pi n/2)$$

$$y_4(n) = A_4 u(n)\cos(2\pi n/2 - \pi/2) \qquad y_5(n) = A_5 u(n)\cos(2\pi n - \alpha_5).$$

Of the 5 output signals, only 3 can occur for the 3 input signals. Or, in other words, each input signal corresponds to one steady-state output signal, so there are two outputs with no matching input.

You should:

1. Determine the output signals that cannot occur.
2. For the 3 outputs that can occur, determine their amplitude values, i.e., the A_k variables.

Observe: You do not have to determine the values α_2 and α_5 in case $y_2(n)$ or $y_5(n)$ are valid output signals.