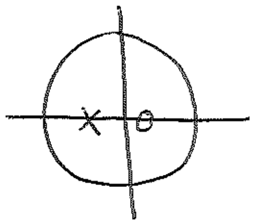
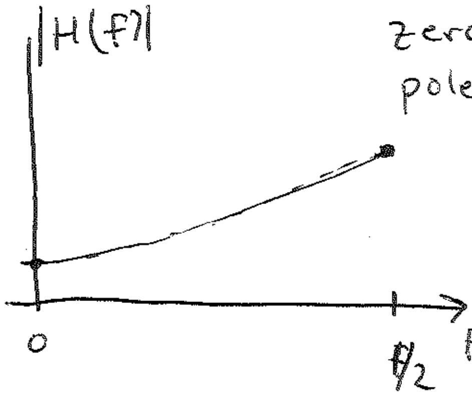
1

a/ $H(z) = \dfrac{3 - z^{-1}}{1 + \frac{1}{2} z^{-1}} = \dfrac{3z - 1}{z + 1/2}$ zero $= \frac{1}{3}$ pole $= -\frac{1}{2}$



b/



zero in $z = \frac{1}{3} \Rightarrow H(0) \approx$ small

pole in $z = -\frac{1}{2} \Rightarrow H(\frac{1}{2}) \approx$ big

c/ $Y(z) = \dfrac{3z - 1}{z + 1/2} \cdot \dfrac{z}{z - 1} = z \left[\dfrac{A}{z + 1/2} + \dfrac{B}{z - 1} \right]$

$A = \frac{5}{3}$ $B = \frac{4}{3}$

$Y(z) = z \dfrac{\frac{5}{3}}{z + 1/2} + \dfrac{z \frac{4}{3}}{z - 1} = \frac{5}{3} \dfrac{1}{1 + z^{-1}/2} + \frac{4}{3} \dfrac{1}{1 - z^{-1}}$

$y(n) = \frac{5}{3} \left(-\frac{1}{2}\right)^n u(n) + \frac{4}{3} u(n)$

d/ $H(z) = \dfrac{B - z^{-1}}{A + \frac{1}{2} z^{-1}}$ $\quad Y(z) = 1 \rightarrow \quad X(z) = \dfrac{A + \frac{1}{2} z^{-1}}{B - z^{-1}}$

$$X(z) = \frac{zA + 1/2}{zB - 1} = \frac{A}{B} \; \frac{z + \frac{1}{2A}}{z - \frac{1}{B}}$$

$X(n)$ stable/bounded $\rightarrow \quad \left|\frac{1}{B}\right| < 1 \quad \Leftrightarrow \quad |B| > 1$

(2) a/ $Y(z) = \dfrac{1}{1 - z^{-1} A} \, X(z) = \dfrac{1}{1 - z^{-1} A}$

$y(n) = A^n \, u(n)$ infinite

b/ $|A| < 1$

c/ $Y(z) = \dfrac{1}{1 + \frac{1}{2} z^{-1}} \; \dfrac{1}{1 - z^{-1}} = z^2 \dfrac{1}{z + 1/2} \cdot \dfrac{1}{z - 1}$

$$= z \left[\frac{A}{z + 1/2} + \frac{B}{z - 1} \right]$$

$$= z \frac{1}{3} \left[\frac{1}{z + 1/2} + \frac{2}{z - 1} \right]$$

$$y(n) = \frac{1}{3} (-1/2)^n \, u(n) + \frac{2}{3} \, u(n)$$

d/

$$y(n) = Ay(n-1) + x(n)$$

$$Y^+(z) = A\left[z^{-1}Y^+(z) + y(-1)\right] + X^+(z)$$

$$Y^+(z)\left[1 - Az^{-1}\right] = X^+(z) + Ay(-1)$$

$$Y^+(z) = \frac{X^+(z)}{1-Az^{-1}} + \frac{Ay(-1)}{1-Az^{-1}}$$

- For $X^+(z) = 1$ we get

$$y(n) = \left[1 + Ay(-1)\right] A^n u(n) = 0 \qquad n \geq 0$$

- For $x(n) = \delta(n-1)$, the input $x(n)$ generates an output at $y(1), y(2)) \ldots$ but $y(0) = 0$ (due to $x(n)$)

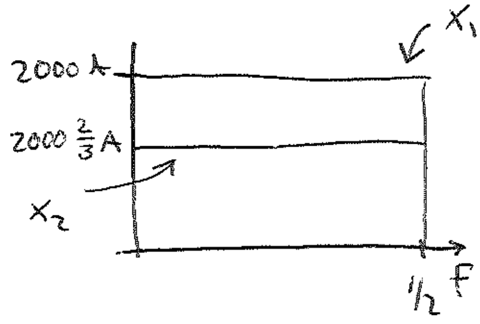
However, $y(-1)$ generates an output at $y(0) = Ay(-1)$

So, either A or $y(-1)$ must be 0.

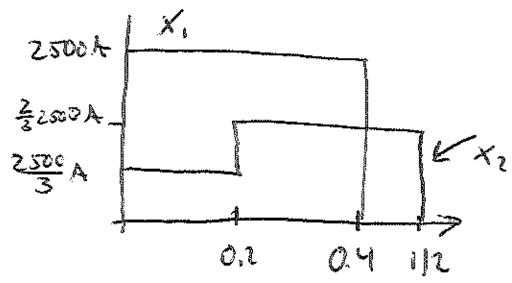
(3)

a/

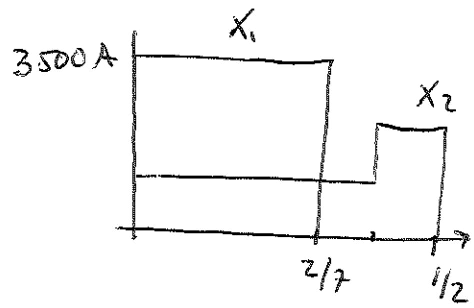
$F_s = 2000$



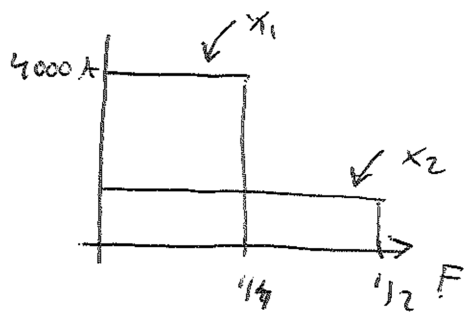
$F_s = 2500$



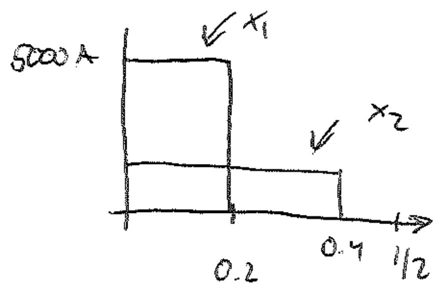
$F_s = 3500$



$F_s = 4000$



$F_s = 5000$



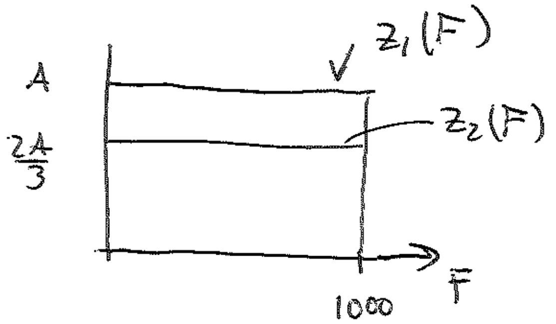
b/

$$SNR_{INPUT} = \frac{2 \cdot 1000 A^2}{2 \cdot 2000 \left(\frac{A}{3}\right)^2} = \frac{1}{2} \frac{1}{1/9} = \frac{9}{2}$$

c/

cutoff	F_s
1/2	2000
0.4	2500
2/7	3500
1/4	4000
0.2	5000

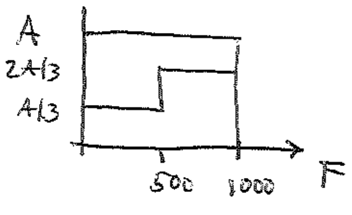
$F_s = 2000$



$$SNR_{output} = \frac{2 \cdot 1000 \, A^2}{2 \cdot 1000 \left(\frac{2A}{3}\right)^2} = \frac{9}{4}$$

$F_s = 2500$

- remove everything to the right of cutoff
- Divide Amplitude with F_s
- replace frequency $\frac{1}{2}$ with $F_s/2$

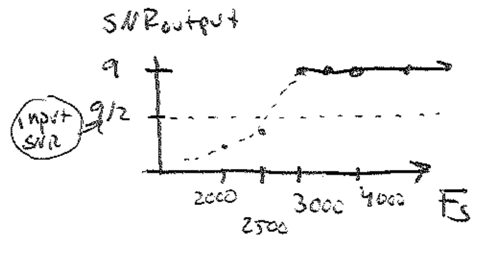


$$SNR = \frac{2000 \, A^2}{\left(\frac{A}{3}\right)^2 \cdot 1000 + \left(\frac{2A}{3}\right)^2 \cdot 1000} =$$

$$= \frac{2}{\frac{1}{3^2} + \frac{4}{9}} = \frac{2}{\frac{5}{9}} = \frac{2 \cdot 9}{5}$$

$F_s = 3500, 4000, 5000$: repeat the above 3 bullets

$SNR = 9$ for all cases



④ a/ Let $\tilde{h}^N$ denote $h(n)$, but zero-padded to length N, i.e.,

$$\tilde{h}^N = [h(0)\ h(1) \ldots h(19)\ \underbrace{00\cdots0}_{N-20\text{ zeros}}]$$

The N-DFT of $\tilde{h}^N$, $\tilde{H}(k)$, computes the DTFT $H(f)$ at frequencies $f = \frac{k}{N}$ $k=0,1,\ldots$

If we are interested in $H(f)$ in $0.1<f<0.$

we should calculate the DFT $\tilde{H}(k)$ and then consider the values $\tilde{H}(k)$ where

$$k = \frac{N}{10}, \frac{N}{10}+1, \ldots, \frac{N}{5}$$

The resolution is controlled by N in the sense that we compute samples of $H(f)$ with a stepsize $\frac{1}{N}$ in f.

b/ $h(n) * x(n)$ is of length 219

To convert a circular convolution into a linear one, we zero-pad both $x(n)$ and $h(n)$ to length 219.

$h(n) * x(n)$ is then $\mathrm{IDFT}\left(\mathrm{DFT}(\tilde{h}^N) \cdot \mathrm{DFT}(\tilde{x}^N)\right)$

c/ we know that

$$y_{steady}(n) = |H(0.12)| \cos(2\pi 0.12 n + \angle H(0.12))$$

where $H(0.12) = |H(0.12)| e^{i \angle H(0.12)}$

So, we must use the DFT to calculate $H(0.12)$.

Now, an N-DFT calculats $f = \frac{k}{N}$

So we can for example zero-pad $h(n)$ to 100 taps, calculate the DFT and pick up the value $\underset{\uparrow \text{DFT}}{H(12)}$

5

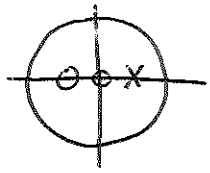
Row 1

The first signal is of the form

$$x(n) = \frac{1}{2}^{n} u(n)$$

and the filter has one zero at $z = -\frac{1}{2}$

Therefore, $Y(z) = z \cdot \frac{z + \frac{1}{2}}{z - \frac{1}{2}}$



The DTFT is large around $f = 0$ and small around $f = \frac{1}{2}$

This is in agreement with the figure

so, yes, true

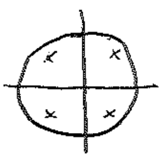
Row 2

The filter forces the output DTFT to be zero at $f = \frac{1}{4}$ which is not the case, so, no, false.

Row 3

examination of the input reveals two poles at $0.9e^{\pm i 2\pi \frac{1}{8}}$

So



This is in perfect agreement with the figure, so yes, true