Problem 1

$$H(z) = \frac{1}{(1-p_1 z^{-1})(1-p_2 z^{-1})} = \frac{B(z)}{A(z)}$$

a/ $$Y(z) = \frac{N_0(z)}{A(z)}$$

$$N_0(z) = -\left[a_1 y(-1) + a_2 z^{-1} y(-1) + a_2 y(-2)\right]$$

$$a_1 = -(p_1 + p_2) \quad a_2 = p_1 p_2$$

$$- \frac{y(-1) a_1 + a_2 y(-2) + a_2 z^{-1} y(-1)}{A(z)} = 0$$

Not possible if $y(-1) \neq 0$

Answer: Not possible to obtain $y(n) = 0$

b/ $$Y(z) = \frac{N_0(z)}{A(z)} + \frac{X(z)}{A(z)} = 0$$

$$X(z) = -N_0(z)$$

This $X(n)$ is stable

Answer possible

c/ $$Y(z) = \frac{N_0(z)}{A(z)} + \underbrace{\frac{X(z)}{A(z)}}_{\text{Stable}}$$

text implies that $\frac{N_0(z)}{A(z)}$ must be unstable. But $\frac{1}{A(z)}$ is stable and therefore $\frac{N_0(z)}{A(z)}$ is also stable

<u>Answer</u> Not possible

d/ In time domain, $\frac{N_0(z)}{A(z)}$ corresponds to

$$A_1 p_1^n u(n) + A_2 p_2^n u(n)$$ where A_1 and A_2 depend on $y(-1)$ and $y(-2)$

But since $|p_1|, |p_2| < 1$ the signal vanishes at large n

<u>Answer</u>: true

Problem 2

a/

$$y(n+2) - 3y(n+1) + 2y(n) = \delta(n)$$

$$Y(z)\left[z^2 - 3z + 2\right] = 1$$

$$Y(z) = \frac{1}{z^2-3z+2} = \frac{1}{(z-1)(z-2)} = \frac{A_1}{z-1} + \frac{A_2}{z-2}$$

$A_1 = -1 \quad A_2 = 1$

$$Y(z) = -\frac{z^{-1}}{1-z^{-1}} + \frac{z^{-1}}{1-2z^{-1}}$$

$$y(n) = -u(n-1) + 2^{n-1}u(n-1) \qquad \text{(unstable)}$$

Also note that $y(0) = y(1) = 0$

b/

$$H(f) = \frac{\frac{1}{2}e^{-i2\pi f}}{\left(e^{-i2\pi f} + \frac{1}{2}\right)\left(e^{-i2\pi f} - \frac{1}{2}\right)}$$

$$x(n) = \cos\left(2\pi \cdot \tfrac{1}{8} n\right)$$

$$y(n) = \left|H\left(\tfrac{1}{8}\right)\right| \cos\left(2\pi \tfrac{1}{8} n + \phi\left(\tfrac{1}{8}\right)\right)$$

$$H\left(\tfrac{1}{8}\right) = \frac{\frac{1}{2} e^{-i\pi/4}}{\left(e^{-i\pi/4}+\frac{1}{2}\right)\left(e^{-i\pi/4}-\frac{1}{2}\right)} = \frac{\frac{1}{2}\frac{1}{\sqrt{2}}[1-i]}{e^{-\pi/2} - \frac{1}{4}}$$

$$= \frac{\frac{1}{2\sqrt{2}}[1-i]}{-i-\frac{1}{4}} = \frac{1}{2\sqrt{2}} \frac{[1-i][-\frac{1}{4}+i]}{\left(-i-\frac{1}{4}\right)\left(i-\frac{1}{4}\right)}$$

$$= \frac{1}{2\sqrt{2}} \frac{-\frac{1}{4}+i+\frac{1}{4}i+1}{1+\frac{1}{4}i-\frac{1}{4}i+\frac{1}{16}} = \frac{1}{2\sqrt{2}} \frac{\frac{3}{4}+\frac{5}{4}i}{\frac{17}{16}}$$

$$= \frac{16}{8\sqrt{2}} \frac{3+5i}{17} = \sqrt{2}\,\frac{3+5i}{17}$$

$$\left|H\left(\tfrac{1}{8}\right)\right| = \frac{\sqrt{2}}{17}\sqrt{34} = \frac{2}{\sqrt{17}}$$

$$\phi\left(\tfrac{1}{8}\right) = -\tan^{-1}\left(\tfrac{5}{3}\right)$$

c/ Since all poles $\left(z_{1,2} = \pm\frac{1}{2}\right)$ are within the unit-circle, the transient vanishes.

Therefore, same answer as in b/

d/ $$Y(z) = \frac{1}{2} z^2 \frac{1}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)(z-1)}$$

$$\frac{1}{(z-\frac{1}{2})(z+\frac{1}{2})(z-1)} = -\frac{2}{z-\frac{1}{2}} + \frac{2/3}{z+\frac{1}{2}} + \frac{4/3}{z-1}$$

$$y(n) = -\frac{1}{2}^{(n+1)} u(n+1) + \frac{1}{3}\left(-\frac{1}{2}\right)^{n+1} u(n\ 1) + \frac{2}{3} u(n+1)$$

Problem 3

Fact 1 A signal of form $\cos(2\pi \frac{k_0 n}{N})$

$n = 0, \ldots, N-1$

has DFT $X(k) = \begin{cases} N, & k = k_0 \text{ and } N-k_0 \\ 0, & \text{otherwise} \end{cases}$

Fact 2 $x(n) = \sin(\frac{2\pi k_0 n}{N})$ $\quad y(n) = \cos(\frac{2\pi k_0 n}{N})$

$\Rightarrow \quad |X(k)| = |Y(k)| \quad \forall k$

Fact 2 $\Rightarrow$ $x_2(n)$ and $x_3(n)$ are indistinguishable

Fact 3 A cosine/sine has ≥ 2 peaks

Fact 3 $\Rightarrow$ F, G, H, J, K

not possible

$\underline{\underline{x_2(n)}}$ $x_2(n) = \cos\left(\frac{2\pi n}{10}\right) = \cos\left(2\pi \frac{k_0}{N} n\right)$

with $k_0 = 1$

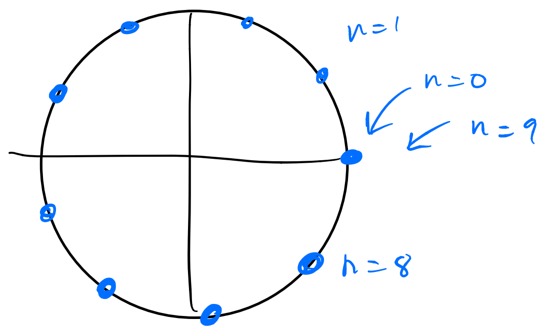
Therefore $x_2(n)$ & $x_3(n) \Leftrightarrow$ A

$\underline{\underline{x_1(n)}}$ $x_1(n) = \cos\left(2\pi \frac{1}{9} n\right) = \cos\left(2\pi \frac{10/9}{10} n\right)$

Not deltas, but something happens around $k \approx 1$

possible graphs: C, D, I

$$X_1(0) = \sum_{n=0}^{9} \cos\left(\frac{2\pi n}{9}\right) = 1$$



only $\underline{D}$ remains

$\underline{\underline{X_4(n)}}$ $\quad x_4(n) = \sin\left(\frac{\pi n}{9}\right) = \sin\left(2\pi \frac{5/9}{10} n\right)$

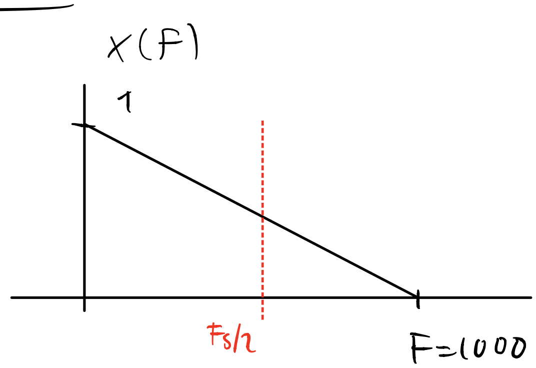
$$k_0 = \frac{5}{9}$$

C or I

$$X_4(0) = \sum x_4(n) \neq 0 \Rightarrow \underline{\underline{I}}$$

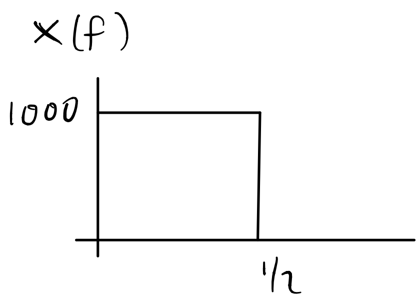
Problem 4

a/



$F_s = 1000$ Hz

$\rightarrow$

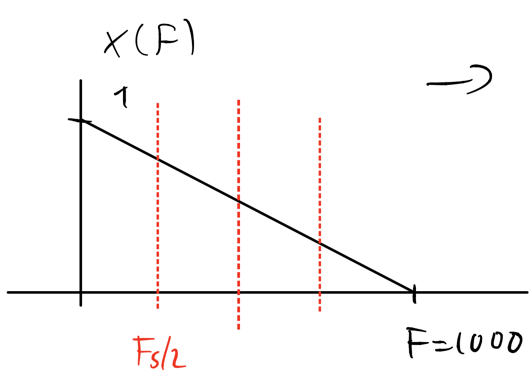


$\rightarrow \quad x(n) = \delta(n)$

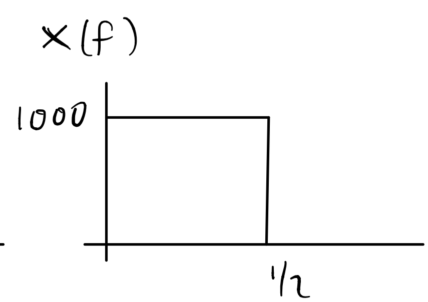
$x(n) * h(n) = h(n)$

remains to invert $H(z)$ [standard problem]

b/



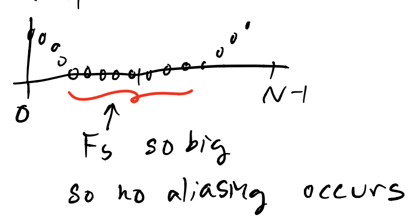
$\rightarrow$



(same result)

c/ No, since output in a and b are identical, but sampling of a yields a different output.

d/ For given F_s, the accuracy is adjusted by zero-padding. Then F_s should be selected such that we get $|X(k)|$



Problem 5

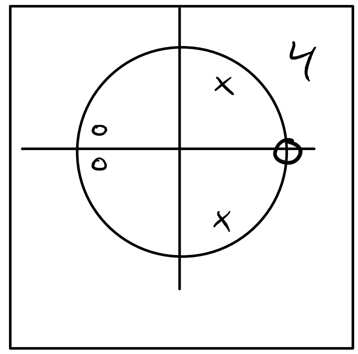
a/ see book

b/ A and B has zeros in their DTFTs due to zeros at the unit circle.

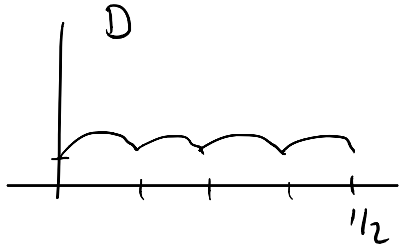
There is only 1 zero/pole plot with this property, so we know that the missing zero/pole is for either A or B.

Of A and B, 2 fits with A

B would be



H(f) corresponding to 3 would be periodic so this must be D



Therefore $C \Leftrightarrow 1$

<u>Summary</u>

$$\boxed{\begin{array}{l} 1 \Leftrightarrow C \\ 2 \Leftrightarrow A \\ 3 \Leftrightarrow D \\ 4 \Leftrightarrow B \end{array}}$$