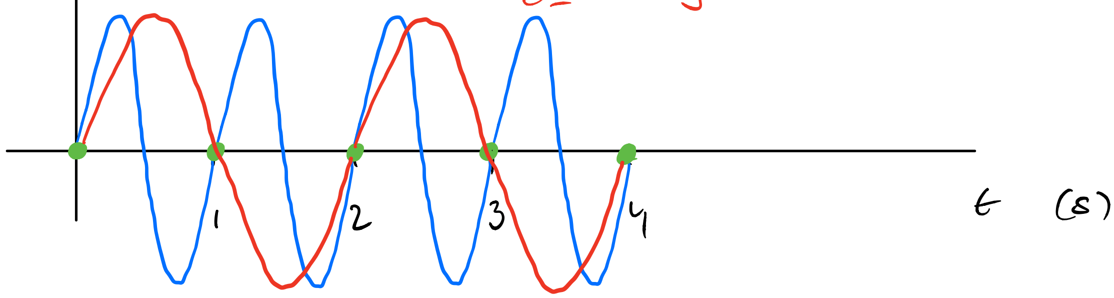
$x(t) = \sin(2\pi t) \quad F = 1.$ $\qquad F_s = 1\ Hz$

$x(n) = [0\ 0\ 0\ 0\ 0]$.

$x(t) = \sin(\pi t) \quad F = \frac{1}{2}$.

$x(n) = [0\ 0\ 0\ 0\ 0]$:



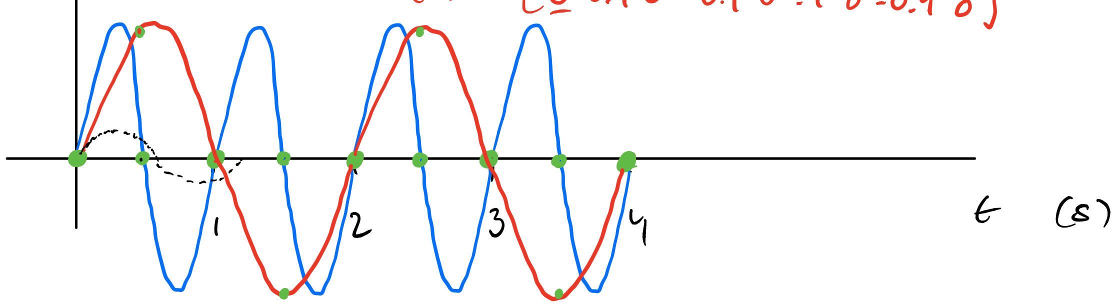
$x(t)$ cont. $F = 1$ and $F = \frac{1}{2}$ $\qquad F_s = 1\ Hz$ is <u>not</u> ok

$x(t) = \sin(2\pi t) \quad F = 1$ $\qquad F_s = 2\ Hz$

$x(n) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$ ←

$x(t) = \sin(\pi t) \quad F = \frac{1}{2} \quad 0 \le F \le 1$

$x(n) = [0\ 0.9\ 0\ -0.9\ 0\ .9\ 0\ -0.9\ 0]$



$x(t)$ cont. $F = 1$ and $F = \frac{1}{2}$ $\qquad F_s = 2\ Hz$ is ok

$\nexists F \le 1 : x(t \mid t = \frac{n}{2}) = 0 \quad \forall n$

$x(t) \qquad X(F) = 0 \quad \forall |F| > B$

$\boxed{F_s = 2B \text{ then it is possible to recover } x(t) \text{ from } x(n)}$

$$X(t) \longrightarrow \boxed{\text{samp}} \xrightarrow{@F_s\ Hz} x(n)$$

$X_A(F)$ $\qquad\qquad$ $\underline{\underline{X(f) = ?}}$ (DTFT)

1 $$x(t) = \int_{-\infty}^{\infty} X_A(F) e^{i2\pi F t} dt$$

2 $$x(n) = \int_{-\frac{1}{2}}^{\frac{1}{2}} X(f) e^{i2\pi f n} df$$

3 $$x(n) = x(t|t = \tfrac{n}{F_s}) = \int_{-\infty}^{\infty} X_A(F) e^{i2\pi \frac{F n}{F_s}} dF$$

plug 1 into 3

combine 2 and 3

$$\boxed{\int_{-\frac{1}{2}}^{\frac{1}{2}} X(f) e^{i2\pi f n} df = \int_{-\infty}^{\infty} X_A(F) e^{i2\pi \frac{F n}{F_s}} dF}$$

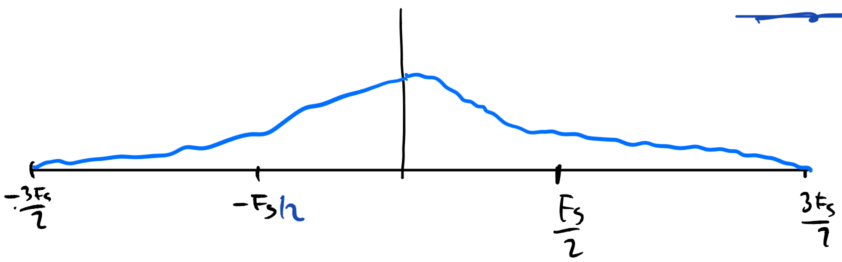
$f = \frac{F}{F_s}$ $\qquad$ $df = \frac{dF}{F_s}$ $\qquad$ $f = \frac{1}{2} \rightarrow F = \frac{F_s}{2}$

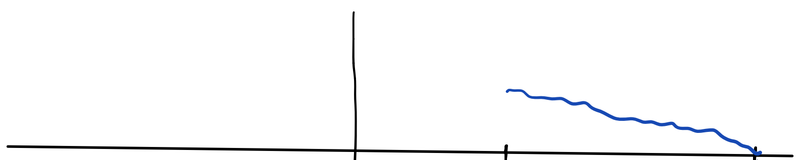
$f = -\frac{1}{2} \rightarrow F = -\frac{F_s}{2}$

$$\frac{1}{F_s}\int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} X\left(\frac{F}{F_s}\right) e^{j2\pi\frac{Fn}{F_s}}\, dF = \int_{-\infty}^{\infty} \underbrace{X_A(F)\, e^{j2\pi\frac{Fn}{F_s}}}_{\text{"}G(x)\text{"}}\, dF$$

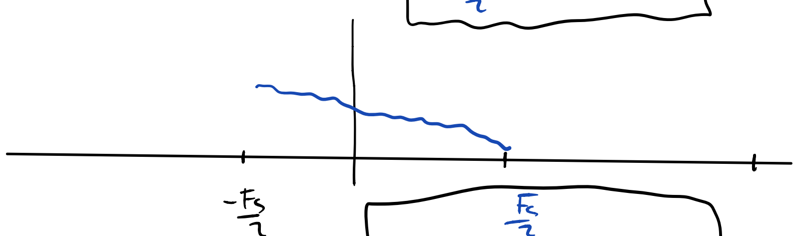
Interlude

$$\int_{-\infty}^{\infty} G(x)\,dx = \int_{-\frac{F_s}{2}}^{F_s/2} G(x)\,dx + \underline{\int_{\frac{F_s}{2}}^{\frac{3F_s}{2}} G(x)\,dx} + \int_{-\frac{3F_s}{2}}^{-F_s/2} G(x)\,dx$$





$$\boxed{\int_{\frac{F_s}{2}}^{\frac{3F_s}{2}} G(x)\,dx}$$



$$\boxed{\int_{\frac{F_s}{2}}^{\frac{F_s}{2}} G(x+F_s)\,dx}$$

$$\int_{-\infty}^{\infty} G(x) = \sum_{k=-\infty}^{\infty} \int_{-\frac{F_s}{2}}^{F_s/2} G(x-kF_s)\,dx$$

$$\frac{1}{F_s}\int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} X\left(\frac{F}{F_s}\right) e^{i2\pi\frac{Fn}{F_s}}\, dF = \int_{-\infty}^{\infty} X_A(F)\, e^{i2\pi\frac{Fn}{F_s}}\, dF$$

$$= \sum_{k=-\infty}^{\infty}\int_{-\frac{F_s}{2}}^{F_s/2} X_A(F-kF_s)\, e^{\frac{i2\pi n}{F_s}(F-kF_s)}\, dF$$

$$= \sum_{k=-\infty}^{\infty}\int_{-\frac{F_s}{2}}^{F_s/2} X_A(F-kF_s)\, e^{\frac{i2\pi n}{F_s}\cdot F}\, dF$$

$$= \int_{-\frac{F_s}{2}}^{F_s/2} \sum X_A(F-kF_s)\, e^{i2\pi\frac{F}{F_s}n}\, dF$$

$$X\left(\frac{F}{F_s}\right) = F_s\sum_{k=-\infty}^{\infty} X_A(F-kF_s)$$

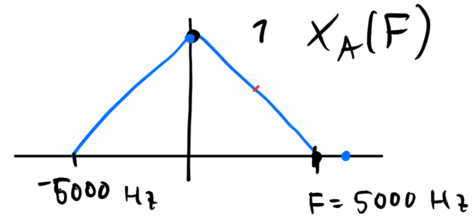
$$\frac{F}{F_s} = f \rightarrow \boxed{X(f) = F_s\sum_{k=-\infty}^{\infty} X_A((f-k)F_s)}$$

DTFT

$X_A(F)$ F_s Hz $X(F)$

$x(t)$ — $\boxed{\text{sampling}}$ — $x(n)$

$$\boxed{x(f) = F_s \sum_{k=-\infty}^{\infty} x_A((f-k)F_s)}$$



$F_s = 20000$ Hz

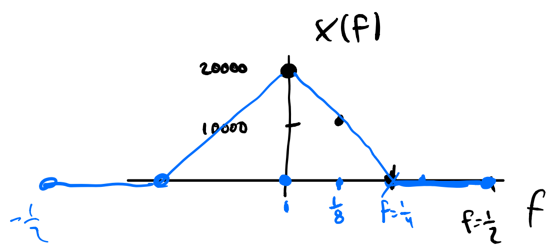
$x(f=0)$: $x(0) = 20000\Big[\underbrace{\overset{k=-1}{X_A(F_s)}}_{=0} + \underbrace{\overset{k=0}{X_A(0)}}_{1} + \underbrace{\overset{k=1}{X_A(-F_s)}}_{=0}\Big] = 20000$

$x(f=\frac{1}{8})$: $x(\frac{1}{8}) = 20000\Big[\underbrace{\overset{k=-1}{X_A(\frac{9}{8}F_s)}}_{=0} + \underbrace{\overset{k=0}{X_A(\frac{1}{8}F_s)}}_{=X_A(2500) = \frac{1}{2}} + \underbrace{\overset{k=1}{X_A(-\frac{7}{8}F_s)}}_{=0}\Big] = 10000$

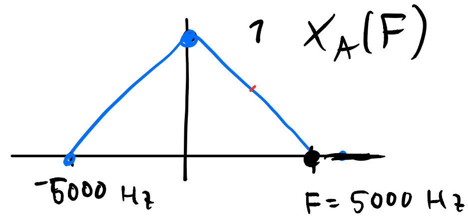
$x(f=\frac{1}{4})$: $x(\frac{1}{4}) = 20000\Big[\underbrace{X_A(\frac{10}{8}F_s)}_{=0} + \underbrace{X_A(\frac{F_s}{4})}_{\frac{F_s}{4}=5000,\ =0} + \underbrace{X_A(-\frac{3}{4}F_s)}_{-15000,\ =0}\Big] = 0$

$x(f=0.3) = \ldots = 0$

$x(f=\frac{1}{2}) = \ldots = 0$



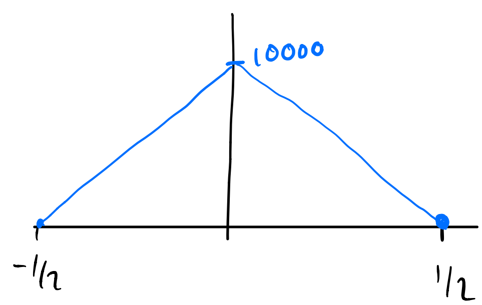
$$\boxed{X(f) = F_s \sum_{k=-\infty}^{\infty} X_A((f-k)F_s)}$$



$F_s = 10000$ Hz

$$X(f) = F_s \Big[\underbrace{\cancel{X_A(F_s + fF_s)}}_{k=-1} + \underbrace{X_A(fF_s)}_{k=0} + \underbrace{\cancel{X_A(-F_s + fF_s)}}_{k=1} \Big]$$

$$= F_s X_A(f F_s)$$

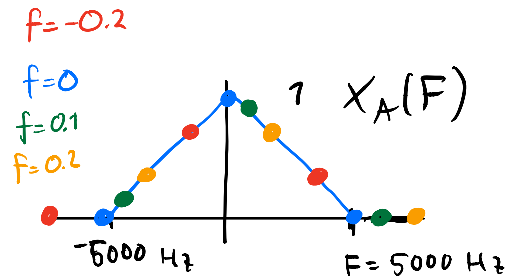


$X(\frac{1}{2}) = 10000 X_A(\frac{F_s}{2})$

$X(0) = 10000\ X_A(0)$

$X(-\frac{1}{2}) = 10000 \cdot X_A(-\frac{F_s}{2})$

$$\boxed{x(f) = F_s \sum_{k=-\infty}^{\infty} x_A((f-k)F_s)}$$



$F_s = 5000$ Hz

$$X(f) = 5000\Big[\underset{k=-1}{X_A(5000 + f\cdot 5000)} + \underset{k=0}{X_A(f\cdot 5000)} + \underset{k=1}{X_A(-5000 + f\cdot 5000)}\Big]$$

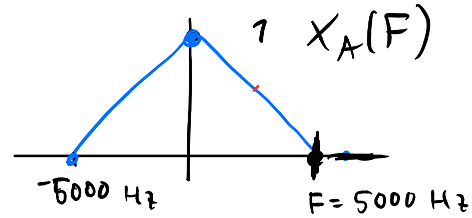
$$X(0) = 5000\Big[\underset{=0}{X_A(5000)} + \underset{=1}{X_A(0)} + \underset{=0}{X_A(-5000)}\Big]$$

$$= 5000$$

$$X(f=0.1) = 5000\Big[\overset{=0}{X_A(5000+500)} + \underset{\neq 0}{X_A(500)} + \underset{\neq 0}{X_A(-5000+500)}\Big] = 1$$

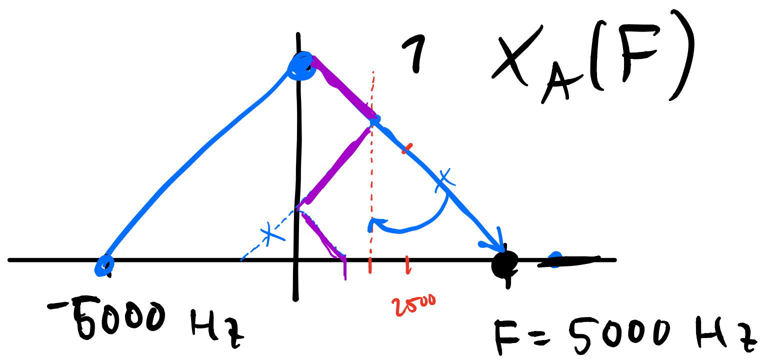
f=-0.2

$$\boxed{x(f) = F_s \sum_{k=-\infty}^{\infty} x_A((f-k)F_s)}$$



$F_s = 4000$ Hz

By goldMg



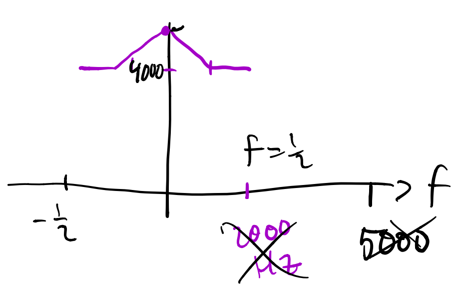
Step 1 $\frac{F_s}{2} = 2000$

step 2 Fold at 2000 Hz

step 3 if end up at negative side : Fold again at F=0

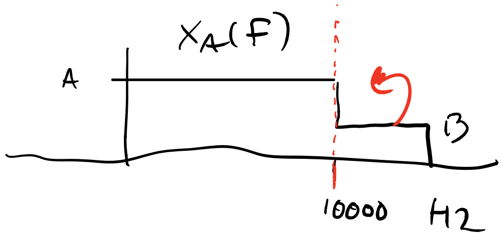
Step 4 Add

step 5 multiply by F_s



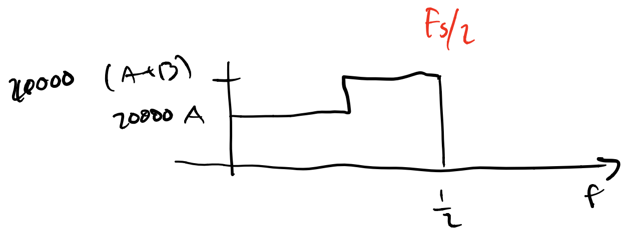
step 6 change $\frac{F_s}{2} \rightarrow f = \frac{1}{2}$

Ex



$F_s = 20000$

find $X(f)$



Reconstruction

$x(t) \xrightarrow{F_s} x(n) \longrightarrow \boxed{\;?\;}_{F_s} \longrightarrow x(t)$

$$x(t) = \int_{-\infty}^{\infty} X_A(F)\, e^{j2\pi F t}\, dF = \int_{-F_s/2}^{F_s/2} X_A(F)\, e^{j2\pi F t}\, dF$$

$$= \left[\begin{array}{llll} F = F_s f & dF = F_s\, df & & \\ F = F_s/2 \rightarrow f = \frac{1}{2} & & F = -\frac{F_s}{2} & f = -\frac{1}{2} \end{array}\right] =$$

$$= F_s \int_{-\frac{1}{2}}^{\frac{1}{2}} X_A(F_s f)\, e^{j2\pi F_s f t}\, df =$$

$$\left[\; X(f) = \underline{F_s X_A(f F_s)} \quad \text{holds if no aliasing } (k=0 \text{ is only } k \text{ alive}) \;\right]$$

$$= \int_{-1/2}^{1/2} X(f)\, e^{i2\pi F_s f t}\, df = \int_{-1/2}^{1/2} \sum x(n)\, e^{-i2\pi n f}\, e^{i2\pi F_s f t}\, df$$

$$X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-i2\pi n f}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \underbrace{\int_{-1/2}^{1/2} e^{i2\pi f (F_s t - n)}\, df}$$

$$\int_{-1/2}^{1/2} e^{i2\pi f (F_s t - n)}\, df = \frac{\sin\left(\pi F_s \left(t - n/F_s\right)\right)}{\pi F_s \left(t - \frac{n}{F_s}\right)}$$

$$\left[\frac{\sin(x\pi)}{\pi x} \triangleq \operatorname{sinc}(x) \right]$$

$$= \operatorname{sinc}\left(F_s \left(t - \frac{n}{F_s}\right)\right)$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} x(n)\, \operatorname{sinc}\left(F_s \left(t - \frac{n}{F_s}\right)\right)}$$