Summary

- LTI system
  - ↳ LTI $\Leftrightarrow$ $y(n) = h(n) * x(n)$ $\Leftrightarrow$ $\overbrace{\sum_{k=0}^{N} y(n-k)a_k = \sum_{k=0}^{M} x(n-k)b_k}^{(*)}$

- Z-transform / one-sided Z
  - ↳ solve/analyze (*)

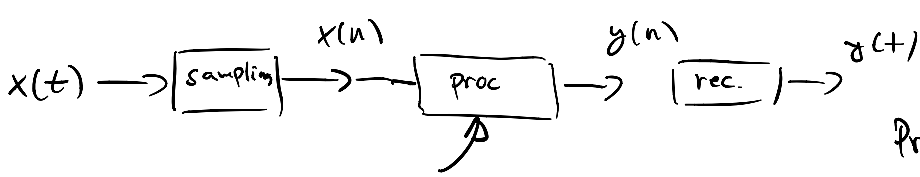
- DTFT (Discrete Time Fourier Transform)
  - ↳ $x(n) = \cos(2\pi f_0 n)$ $\rightarrow$ $y(n) = |H(f_0)| \cos(2\pi f_0 n + \phi(f_0))$

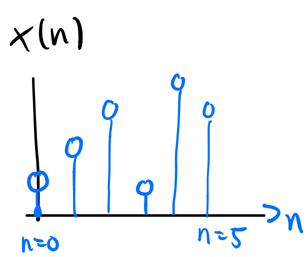
- Sampling - reconstructions
  - ↳ connection to the real world

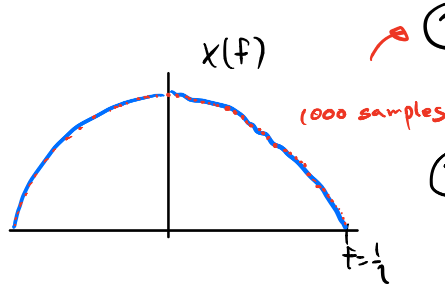
↑ Insights

↓ implem.

- DFT Discrete Fourier Transform
  - ↳ Implementation





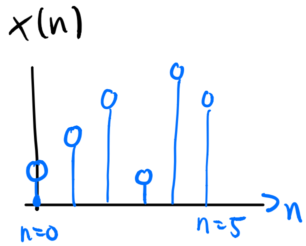


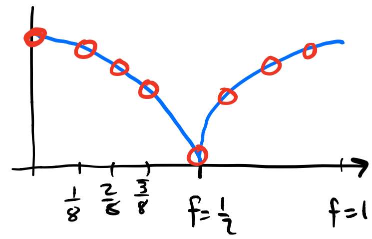
Problem

1. Continuos! computer has problems
2. Inefficiency! 6 numbers $\longleftrightarrow$ cont. time Fourier

"DFT is a 6-number representation of X(f)"
(= DTFT)

Experiment: take N samples of $X(f)$





ex: $N=8$

DTFT

$$X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-i2\pi f n}$$

samples located at

$$f = \frac{k}{N}$$

$$k = 0, 1, \ldots, N-1$$

$$X(k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-i2\pi \frac{kn}{N}}$$

Inversion

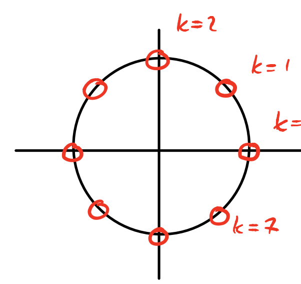
Not true ?

$$x(n) \neq \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{i2\pi \frac{n}{N} k} = \frac{1}{N}\sum_{k=0}^{N-1} \overbrace{\sum_{m=-\infty}^{\infty} x(m)\, e^{-i2\pi \frac{m}{N} k}}^{X(k)} \cdot e^{i2\pi \frac{n}{N} k}$$

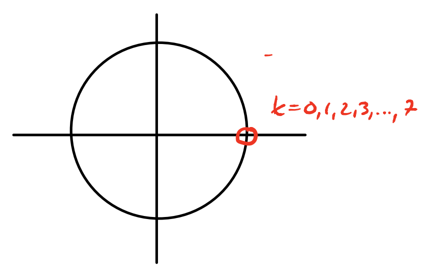
$$= \frac{1}{N}\sum_{m=-\infty}^{\infty} x(m) \underbrace{\sum_{k=0}^{N-1} e^{i2\pi \frac{n-m}{N} k}}$$

$= 0$ if $n \neq m$

$= N$ if $n = m$



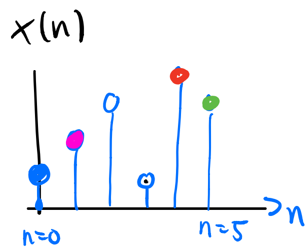
$n \neq m$ ($N=8$)

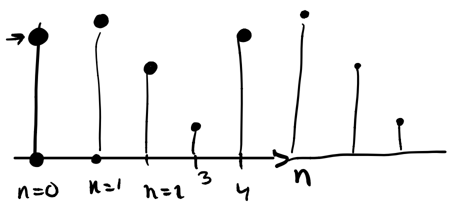


$n = m$ ($N=8$)

$$= \sum_{m=-\infty}^{\infty} x(m)\, \delta(n-m \mod N)$$

$$= \sum_{m=-\infty}^{\infty} x(n-mN)$$





$N = 4$

$$\sum_{m=-\infty}^{\infty} x(n-mN) = \underset{\neq 0}{\overset{m=-1}{x(4)}} + \overset{m=0}{x(0)} + \underset{=0}{\overset{m=1}{x(-4)}}$$

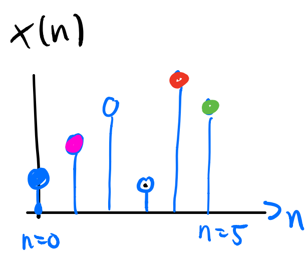
$$= \underline{x(0)} + \underline{x(4)}$$

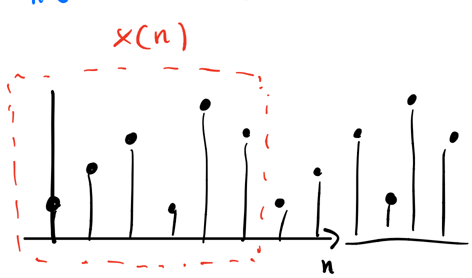
$n=1$ $\qquad \underset{\neq 0}{\underline{x(5)}} + \underline{x(1)} + \underset{=0}{x(-3)}$

$n=2$ $\qquad \underset{=0}{x(6)} \quad x(2) + \underset{=0}{x(-2)}$

$n=3$ $\qquad \underset{=0}{x(7)} + x(3) + \underset{=0}{x(-1)}$

$n=4$ $\qquad x(8) + x(4) + x(0)$





repeat for
$N=6$