Exam in Systems and Signals, EITF75

Thursday October 29, 2020

1. Write clearly! If I cannot read what you write, I will consider it as not written at all.
2. It is important to show the intermediate steps in arriving at an answer, otherwise you may lose points.
3. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments: If you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that at least something must be correct, points may be deducted for everything that is wrong.
4. Problems are *not* arranged in an order of ascending difficulty.
5. Allowed tools: Everything that cannot connect to the internet (see next bullet as well).
6. You must be connected to one of the zoom-links with your camera on. You may use the computer to scroll the exam, but nothing else. It must be possible to see your working environment. Toilet breaks are of course allowed.
7. You are allowed to answer the exam in English or Swedish
8. Send your solutions (scanned, e.g., by your phone) to fredrik.rusek@eit.lth no later than 1315

Problem 1

Assume the LTI circuit $Ay(n) + 0.5y(n-1) = Bx(n) - x(n-1)$. The system is at rest.

In sub-problems a)-c), assume that A=1, B=3.

a) Make a zero-pole diagram of the circuit

b) Sketch the DTFT of the transfer function

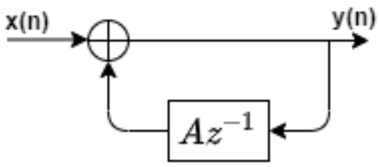
c) Determine the output for the input x(n)=u(n) (u(n) is a step function)

In sub-problem d), we don't make any assumptions on A and B

d) Determine all possible (real-valued) values for A and B so that the unique x(n) for which y(n)=δ(n), is stable.

Problem 2

The block diagram of a system is depicted below, where A is a non-zero real valued constant.



The following sub-problems do <u>not</u> build upon each other.

(a) If $x(n) = \delta(n)$ and the system is at rest, does $y(n)$ have finite or infinite duration?

(b) Please give a sufficient and necessary condition for A so that the system is stable.

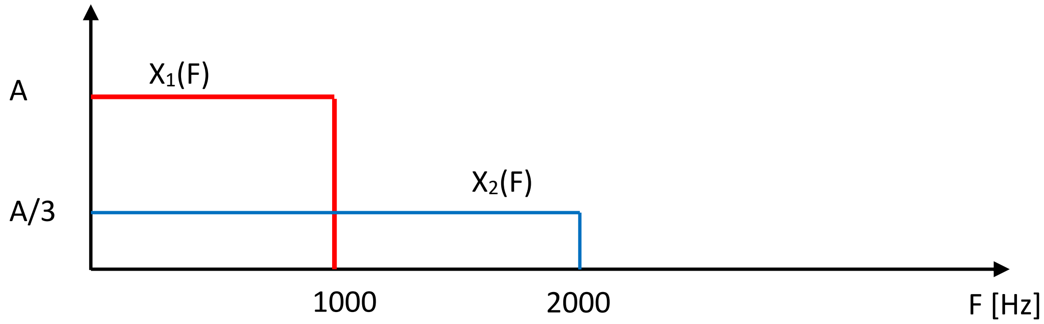
(c) Now assume that $A = -0.5$ and the system is at rest. Calculate y(n) for $x(n) = u(n)$.

(d) Now assume that $A = -0.5$, but the system is not at rest since we know that $y(-1) = 2$. If $x(n) = \delta(n)$, calculate y(n).

Assume the system is not at rest and that the input signal $x(n) = \delta(n-1)$. Is it possible to select A and the initial condition, so that $y(n) = 0, n \geq 0$? Motivate you answer!

Problem 3

Assume two analog signals with Fourier transforms according to the figure below.



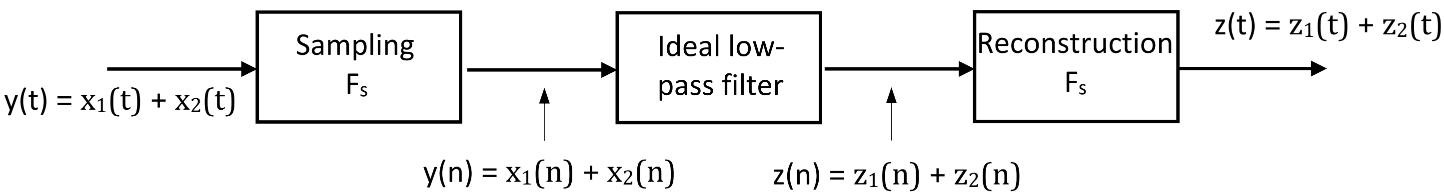
a) Assume that the signal $y(t) = x_1(t) + x_2(t)$ is being sampled with sample frequency F_s to produce $y(n) = x_1(n) + x_2(n)$. Draw the DTFTs of $x_1(n)$ and $x_2(n)$ for the following values of F_s: $F_s \in \{2000, 2500, 3500, 4000, 5000\}$. (Make one drawing for each F_s, where both DTFTs are shown).

b) Assume that $x_1(t)$ is a sound signal and that $x_2(t)$ is interference/noise. We define the input SNR as the signal-energy divided by the interfering-energy, i.e.,

$$\mathrm{SNR}_{\mathrm{input}} = \frac{\int_{-\infty}^{\infty} |X_1(F)|^2 dF}{\int_{-\infty}^{\infty} |X_2(F)|^2\, dF}.$$

Calculate the input SNR.

c) To improve the SNR, we would like to suppress the interference as much as possible. To do this, we use the following structure



The cutoff-frequency of the low-pass filter should be selected such that nothing in $x_1(n)$ is lost, but as much as possible of the interference is removed. This selection of cutoff frequency maximizes output SNR, defined as

$$\mathrm{SNR}_{\mathrm{output}} = \frac{\int_{-\infty}^{\infty} |Z_1(F)|^2 dF}{\int_{-\infty}^{\infty} |Z_2(F)|^2\, dF}.$$

Calculate the output-SNRs for the different sampling frequencies given in a). Comment, briefly, on the results.

Problem 4

Assume a (real-valued) discrete filter h(n) of length 20, i.e.,

$$h(n) = 0 \quad \text{if} \quad n < 0 \quad \text{or} \quad n \geq 20$$

a) Assume that we would like to perform spectral DTFT analysis of the filter, but that we are only interested in the frequency contents in the range 0.1 < f < 0.2. At our disposal we have an N-point DFT calculator, where N is arbitrary. In this problem we do NOT have access to any DTFT calculator.

   Explain, as detailed as you can,
   i) how to perform the spectral analysis in the given range.
   ii) how N impacts the resolution of the analysis.

b) Assume that we use a signal x(n) of length 200 as input to the filter (which results in a linear convolution). Explain how to determine the output signal by means of the N-point DFT calculator mentioned in a). You also have access to an N-point IDFT in this sub-problem. (IDFT = inverse DFT).

c) Assume that we use an input signal x(n) of the form

   $$x(n) = \cos(2\pi 0.12 n)\, u(n),$$

   where $u(n)$ represents the unit step signal, and that we are only interested in determining the steady-state response. Again, explain/show how to make use of the N-point DFT calculator to determine the steady-state response.
   Hint: The form of the steady-state response should be known to you in advance, so it is a question about determining a few parameters.

Problem 5

Below you find three ”rows” with plots, each ”row” contains three plots.

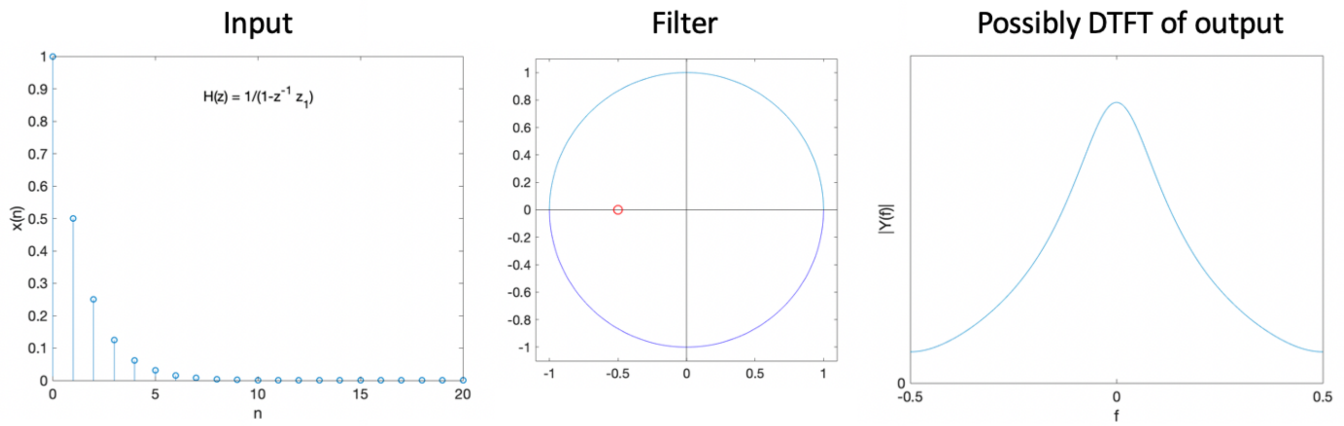
The leftmost plot within each row, illustrates the input signal (truncated to 21 taps, the true input signals are of infinite duration).

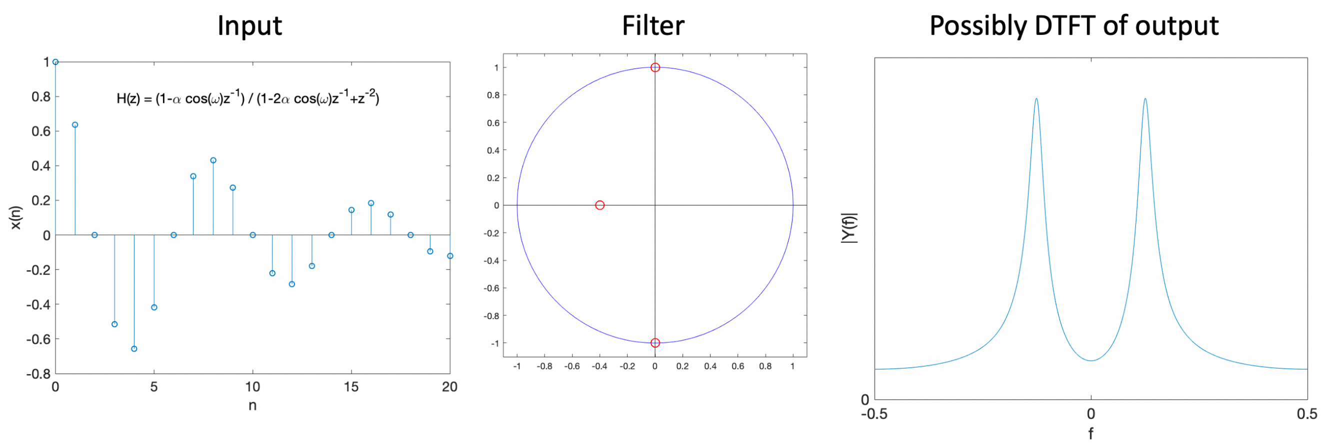
The middle plot shows the zero-pole configuration of a filter to which the input signal is applied.

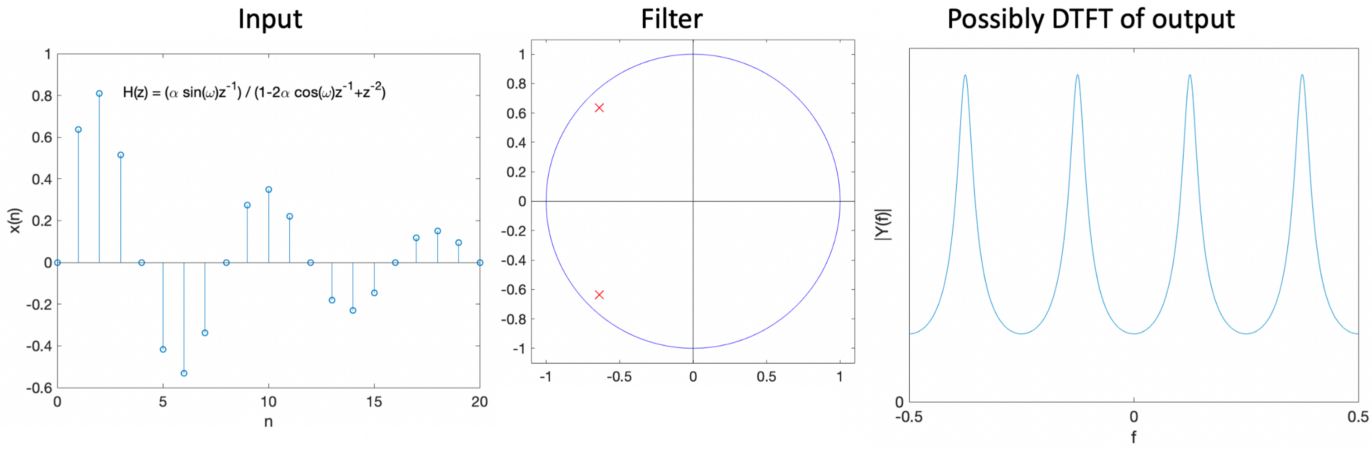
The rightmost plot shows either the DTFT of the corresponding output signal, <u>or</u> some other DTFT that could not possibly have been created by using the input signal to said filter.

For each one of the three ”rows”, determine if the rightmost DTFT is the true DTFT of the output signal. Provide motivations.

Hiint: For the one(s) (if any) DTFTs that are not the true output signal DTFT, there are important errors, not just minor ones that requires very detailed analysis.






Input
H(z) = (α sin(ω)z^-1) / (1-2α cos(ω)z^-1+z^-2)
x(n)
n
Filter
Possibly DTFT of output
|Y(f)|
f