Labs sign-up open

Exercises

E:B Tuesday $08^{15}-100$
Thursday $10^{15}-12^{00}$
15/9

online Tuesday $13^{15}-15$
Thursday $13^{15}-15$

Lecture Thursday 17/9 $15^{15}-?$
solving old (hard) problems

function	z-transform
$h(n) = r^n \cos(\omega n)\, u(n)$	$\dfrac{1 - r\cos(\omega) z^{-1}}{1 - 2r\cos(\omega) z^{-1} + r^2 z^{-2}}$
$h(n) = r^n \sin(\omega n)\, u(n)$	$\dfrac{r \sin(\omega) z^{-1}}{1 - 2r\cos(\omega) z^{-1} + r^2 z^{-2}}$

Ex

$$H(z) = \frac{1 - z^{-1}}{1 - 1.27 z^{-1} + 0.81 z^{-2}}, \qquad h(n) = ?$$

$r^2 = 0.81$

$r = .9$

$2r\cos(\omega) = 1.27$

$\cos(\omega) = \dfrac{1.27}{1.8}$

$\omega = \cos^{-1}\left(\dfrac{1.27}{1.8}\right)$

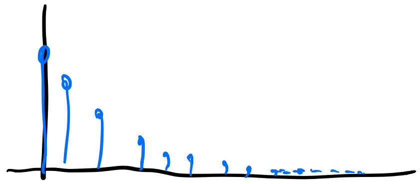
$$= \frac{1 - r\cos(\omega) z^{-1} + \overbrace{r\cos(\omega) z^{-1} - z^{-1}}}{1 - 1.27 z^{-1} + 0.81 z^{-2}}$$

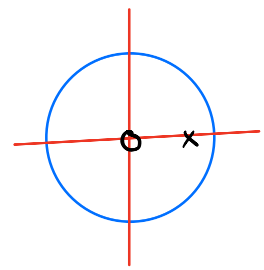
$$= \frac{1 - r\cos(\omega) z^{-1}}{1 - 1.27 z^{-1} + 0.81 z^{-2}} + \frac{z^{-1}\, r\sin(\omega) \cdot \dfrac{r\cos(\omega) - 1}{r\sin(\omega)}}{1 - 1.27 z^{-1} + 0.81 z^{-2}}$$

$\downarrow\ z^{-1}$ $\qquad\qquad \downarrow$

$\rightarrow \quad r^n \cos(\omega n) u(n) + \dfrac{r\cos(\omega) - 1}{r\sin(\omega)} r^n \sin(\omega n) u(n)$

Ex 1

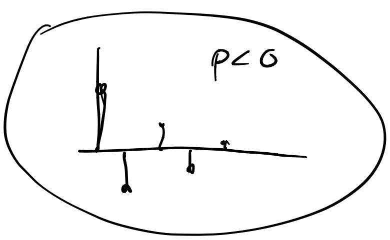




$\dfrac{1}{1-pz^{-1}} \longleftrightarrow p^n u(n)$

No oscillation $\Rightarrow$ real pole

$|p| < 1 \quad p$ is real



$p \leq 0$?

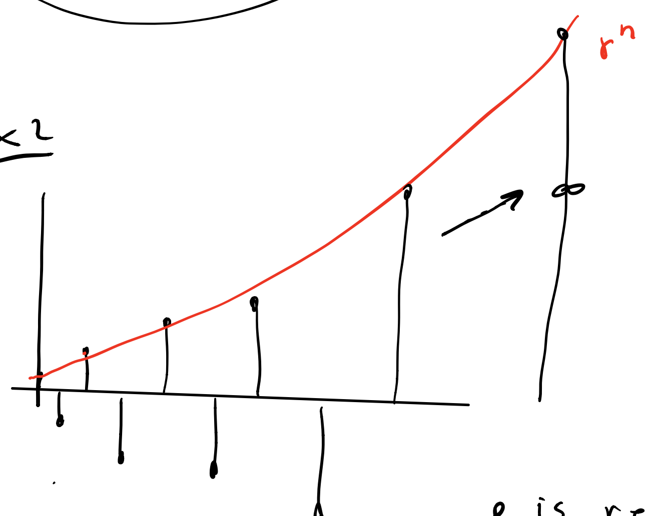
$p \geq 0$!

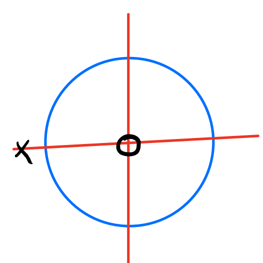
$\dfrac{1}{1-pz^{-1}} = \dfrac{z}{z-p}$

zero $z=0$

pole $z=p$

Ex 2



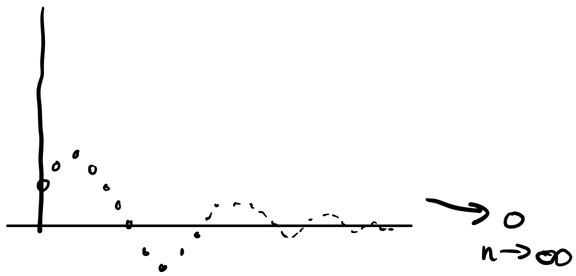


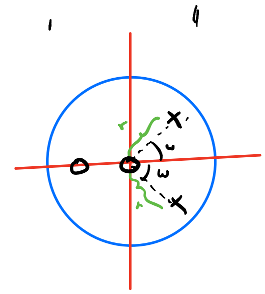
p is real (non-osc.)

$|p| > 1$ (unstable)

$p < 0$

Ex 3





2 poles not real, complex conjugated

$|P_1| = |P_2| < 1$

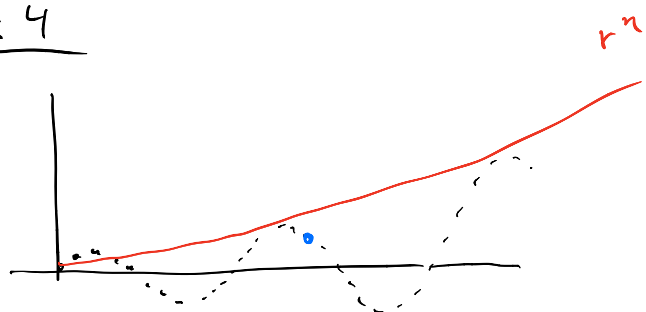
$$\to \frac{A + Bz^{-1}}{1 - 2r\cos(\omega)z^{-1} + r^2 z^{-2}} \quad \frac{z^2}{z^2}$$

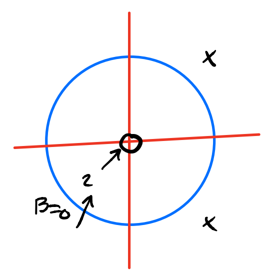
$$\frac{Az^2 + Bz}{\underbrace{z^2 - 2r\cos(\omega)z + r^2}}$$

$\boxed{\begin{array}{l} B \neq 0 \\ z_0 = 0 \\ z_1 \neq 0 \end{array}}$

$B = 0$
$z_0 = 0$
$z_1 = 0$

Ex 4





2 poles

stable ? No $\Rightarrow$ $|P_1| = |P_2| > 1$

$$Y(z) = H(z)X(z) = \frac{U(z)}{V(z)}$$

$\deg(U(z)) = ?$

$\deg(V) = s$.

$\deg(U) = s + g$

ass

$x(n)$ is causal

$$Y(z) \underset{\text{pol. div}}{=} q(z) + \frac{r(z)}{v(z)} \qquad \deg(q) = \delta+1, \quad \deg(r) < \deg(v)$$

$$\underset{\text{PFE}}{=} q(z) + \sum_{k=1}^{s} \frac{A_k}{z-p_k} = q(z) + \underbrace{z^{-1}\underbrace{\sum_{k=1}^{s} \frac{A_k}{1-z_k p_k}}}_{\text{causal}}$$

$= 0 \quad n \leq 0$

$$\begin{cases} x(n) \text{ is causal} \Rightarrow y(n) \text{ is causal} \\ q(z) = a_{\delta+1} z^{\delta+1} + a_s z^s + \dots + a_1 z + a_0 \end{cases}$$

$$\downarrow$$

$$q(n) = \{a_{\delta+1} \cdots a_2 \; a_1 \; \underset{\cdot}{\underline{a_0}}\}$$

$$\Rightarrow \quad \delta = 0$$

$$\left.\begin{cases} Y(z) = H(z)X(z) = \dfrac{u(z)}{v(z)} \\ x(n) \text{ is causal} \end{cases}\right\} \Rightarrow \quad \deg(u) \leq \deg(v)$$

Ex. cont.

$$\begin{cases} \text{save} & 100 \text{ kr/month} \\ 5\,\% & \text{interest} \end{cases}$$

~~start at 0 kr~~

$\boxed{\text{start at } 1000}$

$$y(n) = 1.05\, y(n-1) + x(n) \qquad x(n) = 100 \cdot u(n)$$

$z \downarrow$

$$Y(z) = 1.05\, z^{-1} Y(z) + X(z)$$

$\underline{y(-1) = 0}$

$y(-1) = 1000$

$\underline{y(-1) = 1.05\, y(-2) + x(-1)}$

$\Rightarrow$

$y(n) = 1.05\,y(n-1) + x(n) \qquad n \geq 0 \}$

$\mathcal{Z}$

$Y(z) = 1.05\,z^{-1}Y(z) + X(z)$

Not true

Solution method

One-sided z-transform

$$\boxed{Y^{+}(z) = \sum_{n=0}^{\infty} y(n)\,z^{-n}}$$

all properties of $Y^{+}(z)$ remains except time delay.

No initial cond. / at rest

$\Longleftrightarrow$

normal z-transform

have init. cond. / not at rest

$\Longleftrightarrow$

one-sided z-transf.

Recall: time delay normal z-transform

$$\boxed{\begin{array}{ll} x(n) \longleftrightarrow X(z) \\ x(n-k) \longleftrightarrow z^{-k}X(z) \end{array}}$$

one sided version

$x(n) \longleftrightarrow X^{+}(z)$

$x(n-k) \longleftrightarrow z^{-k}\left[X^{+}(z) + \sum_{n=1}^{k} x(-n)\,z^{n}\right]$

expl.

$X^{+}(z)$

$x(n) = \{x(-2),\ x(-1),\ \boxed{x(0), x(1), \ldots}\}$ $\rightarrow$ $X^{+}(z)$

$x(n-1) = \{x(-2),\ \boxed{x(-1)},\ \boxed{x(0), x(1)}\ \ldots\}$ $\rightarrow$ $z^{-1}X^{+}(z) + x(-1)$

n=0

$x(n-2) = \{\boxed{x(-2)},\ x(-1),\ x(0),\ x(1) \ldots\}$ $\rightarrow$ $z^{-1} \cdot$ $+ x(-2)$

n=0

$= z^{-2}X^{+}(z) + z^{-1}x(-1) + x(-2)$

$y(n) = 1.05\,y(n-1) + x(n)$ Now apply $X^{+}(z)$

$Y^{+}(n) = 1.05\,z^{-1}Y^{+}(z) + 1.05\,y(-1) + X^{+}(z)$

$$Y^+(z)\left[1-1.05z^{-1}\right] = 1.05\,y(-1) + X^+(z)$$

$$Y^+(z) = \frac{1.05}{1-1.05z^{-1}}\,y(-1) + \frac{X^+(z)}{1-1.05z^{-1}}$$

$$= \underbrace{\frac{1050}{1-1.05z^{-1}}}_{1050\cdot(1.05)^n u(n)} + \frac{1}{1-1.05z^{-1}} \cdot \underbrace{\frac{100}{1-z^{-1}}}_{Z(100u(n))}$$

= solution for $y(-1)=0$

$$\boxed{y(n) = \begin{cases} 1000, & n=-1 \\ 1050(1.05)^n + \text{"old solution"} & n \geq 0 \end{cases}}$$

$y(-1)$

$$H(z) = \frac{1}{z-p_1}\,\frac{1}{z-p_2}$$

$\Longleftrightarrow$

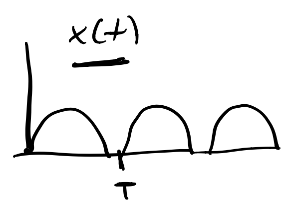
2 init. cond

ex: $y(-1) = 1000$

$y(-2) = 500$

Fourier

Case I: Recap, continuous time, periodic



$$c_k = \frac{1}{T}\int_0^T x(t)\,e^{-i2\pi k \frac{t}{T}}\,dt$$

claim: $\{c_k\}_{-\infty}^{\infty}$ is a representation of $x(t)$

$\{c_k\} \longrightarrow \boxed{} \longrightarrow x(t)$ if possible claim correct

-"- not -"- -"- in -"-

$$\hat{x}(t) = \underbrace{\sum_{k=-\infty}^{\infty} c_k e^{i2\pi k \frac{t}{T}}} = \sum_{k=-\infty}^{\infty} \frac{1}{T}\int_0^T x(\tau)\, \underline{e^{-i2\pi k \frac{\tau}{T}}\, d\tau\, e^{i2\pi k \frac{t}{T}}}$$

change order

$$= \int_0^T x(\tau)\, \underbrace{\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{i2\pi k \frac{(t-\tau)}{T}}}\, d\tau$$

$$= x(t \bmod T) \underset{\substack{x(t) \\ \text{is periodic}}}{=} x(t)$$

claim correct