Exam in systems and signals (digital signalbehandling), EITF75

Thursday April 25

1. **Write clearly!** If I cannot read what you write, I **will consider it as not written at all**. My decision on this matter is final, you cannot argue that I should have been able to read it later.
2. It is important to **show the intermediate steps** in arriving at an answer, otherwise you may lose points.
3. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments if you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that at least something must be correct, points may be deducted for everything that is wrong.
4. Problems are not arranged in an order of ascending difficulty.
5. Each problem carries 1 point.
6. Allowed tools: Pocket calculator, Course book, Lecture slides, printed versions of Nedo's slides.
7. The exam should be answered in Swedish or English
8. If you think there are some parameters missing, please state clearly those parameters by your own.

Problem 1

a.) (0.5p) $x[n]$ is a real-valued, casual sequnece with discrete-time Fourier transform $X(\omega)$. Determine a choice for $x[n]$ if the imaginary part of $X(\omega)$ is given by:

$$\text{Im}\{X(\omega)\} = 3\sin(2\omega) - 2\sin(3\omega).$$

b.) (0.5p) Let $y[n]$ be created as:

$$y[n] = x[n] + \jmath h[n] \star x[n],$$

where $\jmath = \sqrt{-1}$ and "$\star$" denotes convolution. Determine a choice for $H(\omega)$ so that

$$\begin{cases} Y(\omega) = X(\omega), & -\pi < \omega < 0 \\ Y(\omega) = 0, & 0 < \omega < \pi \end{cases}$$

Problem 2

a.) (0.2p) Assume that $H(z)$ is of the form

$$H(z) = \frac{1 - az^{-1}}{1 - \frac{1}{a}z^{-1}}$$

for some real-valued a. Show that $|H(\omega)|$ is independent of ω.

b.) (0.5p) Consider the transfer function

$$H(z) = \frac{1 - 7z^{-1} + 12z^{-2}}{1 - \frac{1}{3}z^{-1}}.$$

Define $H_{\text{mp}}(z)$ as a FIR minimum-phase filter, and $H_{\text{ap}}(z)$ as an all-pass filter with the property

$$|H_{\text{ap}}(\omega)| = K$$

for some constant K. Determine $H_{\text{mp}}(z)$ and $H_{\text{ap}}(z)$ such that $H(z) = H_{\text{mp}}(z)H_{\text{ap}}(z)$. Also, indicate the region of convergence for $H_{\text{ap}}(z)$ and $H_{\text{mp}}(z)$.

c.) (0.3p) Find a casual $h[n]$ for the $H(z)$ given in problem b).

Problem 3

Consider the generation of $y[n]$ from $x[n]$ in Figure 1. The parameters T_1 and T_2 are the sampling rates of the D/A and A/D converters, respectively. The boxes with arrows are up-sampling and down-sampling, respectively, and $H(\Omega)$ represents a time-continuous LTI filter.
Hint: You may consider this problem at frequency domain.

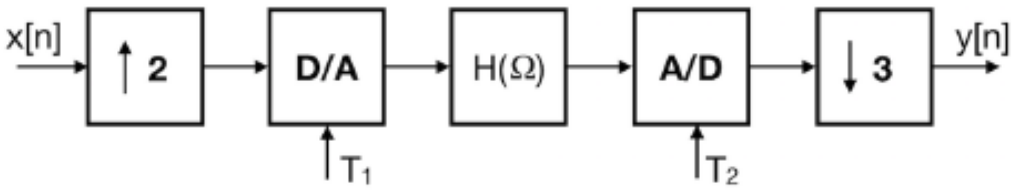

Figure 1: System model for Problem 3

a.) (0.4p) For $T_1 = T_2 = 10^{-4}$, is the system LTI? Motivate your answer.

b.) (0.4p) Determine a necessary condition on T_1 and T_2 for the system to always be LTI.

c.) (0.2p) Assume that the condition in problem b) is fulfilled. Determine necessary condition(s) for the inverse system to exist. In other words, what condition(s) is needed in order to guarantee that $x[n]$ can be found from $y[n]$.

Problem 4

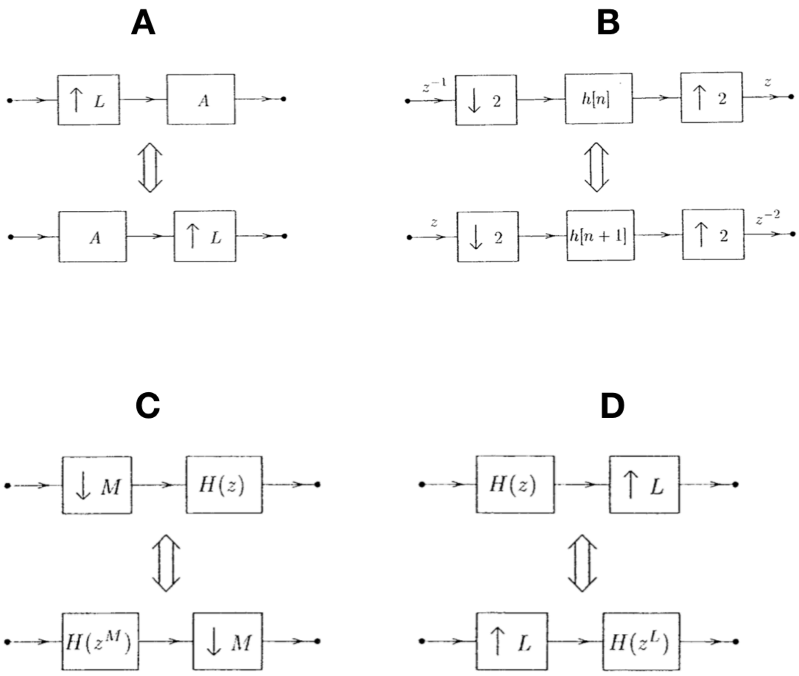

Figure 2: Proposed identities for Problem 4

Four proposed identities are shown in Figure 2, (A-D). Please motivate for each of the identity if they are valid or not? (An arrow with z^{-k} on top of it marks a k-step delay.)

Hint: To disprove a statement, it is often easist to disprove it by means of a counterexample.

Problem 5

A system for discrete-time spectral analysis of time-continuous signals is shown in Figure 3, where $w[n]$ is a rectangular window of length 32.

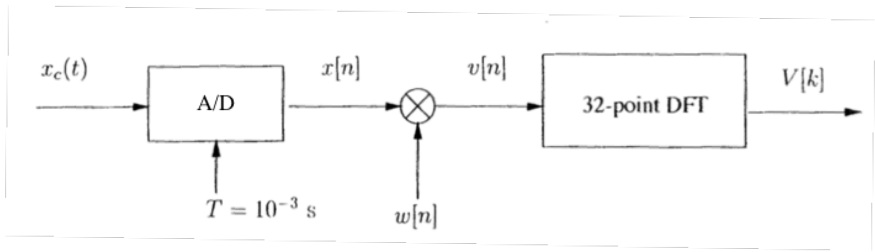

Figure 3: System model for Problem 5

$$w[n] = \begin{cases} 1/32, & 0 \leq n \leq 31 \\ 0, & \text{otherwise} \end{cases}$$

The output $|V[k]|$ is shown, in dBs, in Figure 4.

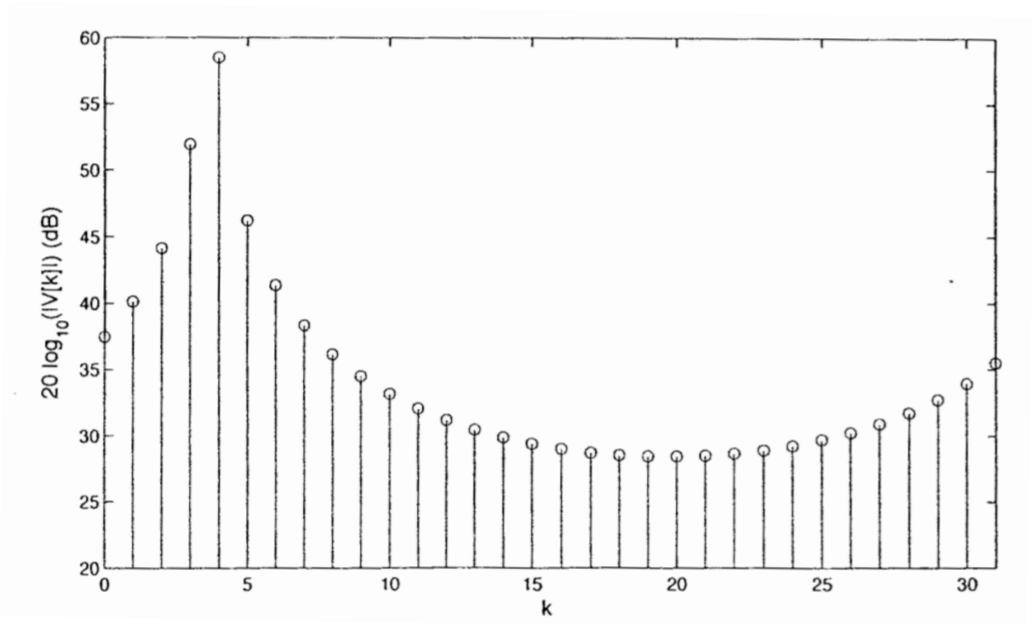

Figure 4: Output $|V[k]|$ in dB

Listed below are ten signals, at least one of which was the input $x_c(t)$. Indicate which signal(s) could have been the input $x_c(t)$ which produced the plot of $|V[k]|$ shown in dB units in Figure 4. Motivate your answer.
Hint: Do not brute calculate, think carefully and deeply of the properties of DFT, sampling and windowing and then answer the question.

$$\begin{aligned} x_1(t) &= 1000\cos(230\pi t) \\ x_2(t) &= 1000\cos(115\pi t) \\ x_3(t) &= 10e^{\jmath 460\pi t} \\ x_4(t) &= 1000e^{\jmath 230\pi t} \\ x_5(t) &= 10e^{\jmath 230\pi t} \end{aligned} \qquad \begin{aligned} x_6(t) &= 1000e^{\jmath 250\pi t} \\ x_7(t) &= 10\cos(250\pi t) \\ x_8(t) &= 1000\cos(218.75\pi t) \\ x_9(t) &= 10e^{\jmath 200\pi t} \\ x_{10}(t) &= 1000e^{\jmath 187.5\pi t} \end{aligned}$$