1) a/ $j \sum_{n=0}^{\infty} x[n] \sin(\omega n) = 3\sin(2\omega) - 2\sin(3\omega)$

$$\Rightarrow \quad x[n] = x_0\,\delta[n] + 3\,\delta[n-2] - 2\,\delta[n-3]$$

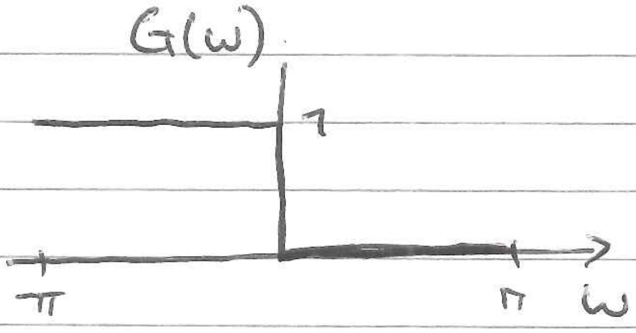
x_0 arbitrary since $x_0 \sin(\omega \cdot 0) = 0$

($2\delta[n+3]$ does not work since $x(n)$ should be causal)

b/

$$y[n] = \underbrace{\left(\delta[n] + j\,h[n]\right)}_{g[n]} * x[n]$$

$$Y(\omega) = G(\omega)\,X(\omega)$$



$$G(\omega) = 1 + j\,H(\omega) \quad \Rightarrow \quad H(\omega) = \begin{cases} 0, & -\pi \le \omega \le 0 \\ j, & 0 < \pi \le \pi \end{cases}$$

(2) a/

$$H(\omega) = H(z)\Big|_{z=e^{i\omega}}$$

$$|H(\omega)|^2 = \frac{(1-ae^{-i\omega})(1-ae^{i\omega})}{(1-\frac{1}{a}e^{-i\omega})(1-\frac{1}{a}e^{i\omega})}$$

$$= \frac{1+a^2-2a\cos(\omega)}{1+a^{-2}-2a^{-1}\cos(\omega)}$$

$$= \frac{a^2(1+a^2-2a\cos(\omega))}{1+a^2-2a\cos(\omega)} = a^2 \not\propto \omega$$

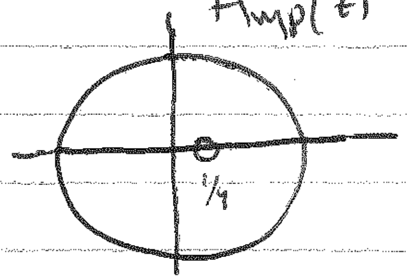
② b/

$$H(z) = \frac{(1-3z^{-1})(1-4z^{-1})}{\left(1-\frac{1}{3}z^{-1}\right)}$$

$$= \underbrace{\frac{(1-3z^{-1})(1-4z^{-1})}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}}_{H_{ap}(z)} \underbrace{\left(1-\frac{1}{4}z^{-1}\right)}_{H_{mp}(z)}$$

$$= 3^2 \cdot 4^2 \cdot \left(1-\frac{1}{4}z^{-1}\right)$$

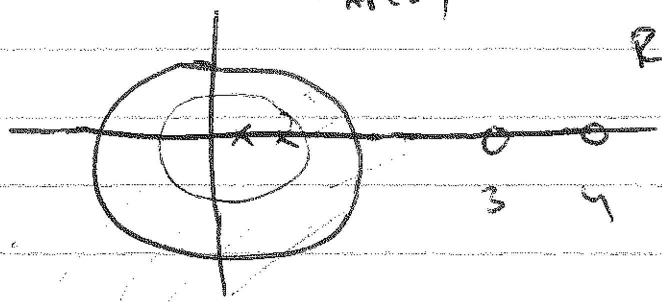
$H_{mp}(z)$



all zeros inside unit circle.

ROC

$H_{AP}(z)$



ROC : $|z| > \frac{1}{3}$

c/ $h[n] = 3^2 \cdot 4^2 \cdot \left[1, -\frac{1}{4}\right]$

③

a/ The effect of $\boxed{\uparrow 2}$ "+" $\boxed{\downarrow 3}$ is to stretch the frequency axis by $3/2$, so this cannot be LTI

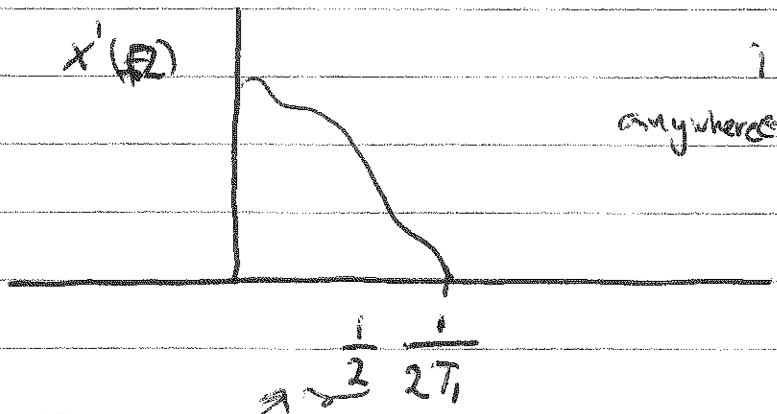
b/ To undo the frequency stretch of $3/2$ we must put $T_2 = \frac{2}{3} T_1$

c/

Consider

$$x[n] \to \boxed{\uparrow 2} \to \boxed{D/A} \to x'(t)$$

$$\uparrow T$$



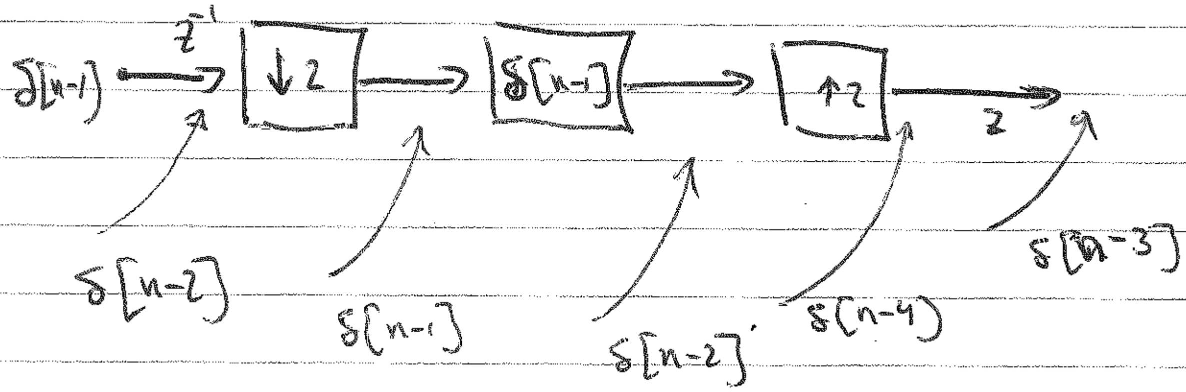
if $H(\Omega)$ is ~~not~~ zero anywhere in $|\Omega| \le \frac{1}{4T_1}$ then one cannot recover that piece

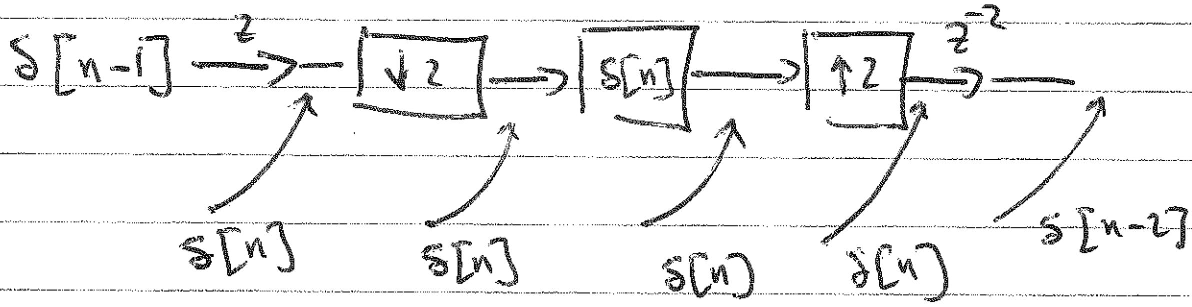
due to $\boxed{\uparrow 2}$

④

A/ multiplication with a constant is linear, so A is valid

B/ consider input $\delta[n-1]$ and $h[n] = \delta[n-1]$





Not valid

(4) C + D

Valid, see text around Figures 11.5.3 and 11.5.4 in the textbook

(5)

$x_c(t)$ is not a cosine since a cosine has two peaks $\Rightarrow$ $x_1(t)$, $x_2(t)$, $x_7(t)$, $x_8(t)$ are gone

$|V[k]|^2 \approx 60\text{ dB} \rightarrow |V[k]| \approx 1000$

$x_3(t)$, $x_5(t)$, $x_9(t)$ are gone since they are too weak

carrier frequency of $x_c(t)$ is not exactly a frequency $\omega_k = \frac{2\pi k}{32}$ sampled by the DFT. Otherwise, the DFT would be non zero at exactly 1 value of k.

$x_6(t)$ and $x_0(t)$ are gone

$250\pi \rightarrow \frac{\pi}{4} \rightarrow k=4$

$187.5\pi \rightarrow \frac{3\pi}{16} \rightarrow k=3$

$x_4(t)$ is the only remaining signal