Task 2

1.

(a) Signal 1 has linear phase because of its symmetry

Signal 2 has also linear phase. If phase is always 0, it is characterized as linear phase.

Signal 3 is not linear phase

(b) Signal 1 is not minimum phase

Signal 2 is minimum phase (it is the only case that a signal can be both linear and minimum phase)

Signal 3 a minimum phase

Solving the equation, all zeros and poles are in the circle

(c) $y(n) = -a_1 y(n-1) - a_2 y(n-2) + x(n) \quad a_2 > 0$

$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) + X(z)$ (~~Ignore the ... they have same poles as the transfer function part~~)

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{z^2}{z^2 + a_1 z + a_2}$$

$z_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2} \quad z_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}$

① decay to 0 both $|z_1|$ and $|z_2| < 1$

② growing unbounded at least one of $|z_1|, |z_2| > 1$

③ ~~oscillating~~ case 1: one of $|z_1|$ or $|z_2|$ equals to 1, another one $|z_1|$ or $|z_2| < 1$ eg. $z_1 = -1$, $z_2 = 0.7$

Case 2: $|z_1| = |z_2| = 1$ eg. $\begin{cases} \cos + \frac{\sqrt{3}}{2} j \\ \cos - \frac{\sqrt{3}}{2} j \end{cases}$

(d) $Y(z)(1 + 2z^{-1} + 4z^{-2}) = (b_0 + b_1 z^{-1} + b_2 z^{-2})X(z)$

$$\Rightarrow H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + 2z^{-1} + 4z^{-2}}$$

$$= \frac{b_0 z^2 + b_1 z + b_2}{z^2 + 2z + 4}$$

we have two poles $z_1 = -1 - j\sqrt{3}$ $z_2 = -1 + j\sqrt{3}$

then the zeros $(b_0 z^2 + b_1 z + b_2)$ has to cancel z_1

$\Rightarrow b_0 z_1^2 + b_1 z_1 + b_2 = 0$ is the condition

(e) all of signals can be the output

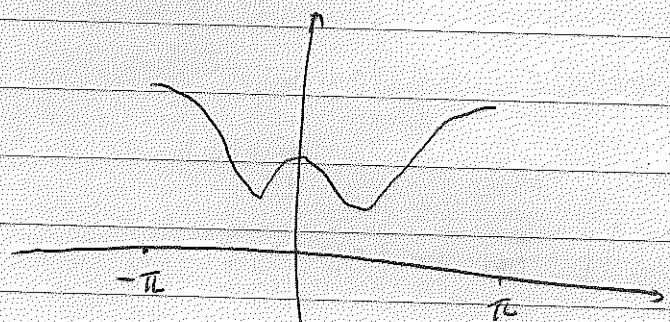
$y_1(n) = 2 \quad -1 \quad 5 \quad -1 \quad 2$ $\quad Y_1(z) = 2 - z^{-1} + 5z^{-2} - z^{-3} + 2z^{-4}$

$H(z)$ is given, then $X(z) = \frac{Y_1(z)}{H(z)}$ as input.

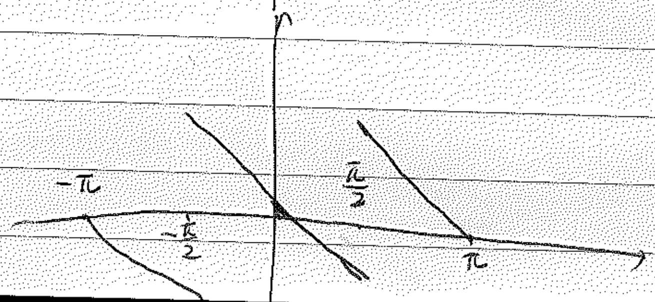
Then same applies for $y_2(n)$ and $y_3(n)$

(f) $y_1(n) * y_3(n) = [\underline{8} \quad 2 \quad 21 \quad 11 \quad 14 \quad 9 \quad 3 \quad 2]$

(g)



$|A(\omega)| = |4\cos(2\omega) - 2\cos(\omega) + 5|$



Problem 2

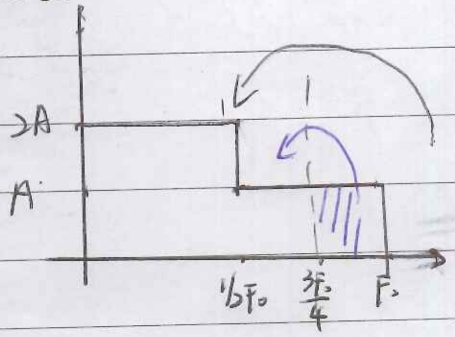
In other to be $\delta(n)$, then $X(f) = const$ $f \in [-\frac{F_s}{2}, \frac{F_s}{2}]$ → Digital

or $\sum_k X(w + kF_s) = const$ $w \in (-\infty, +\infty)$

analog

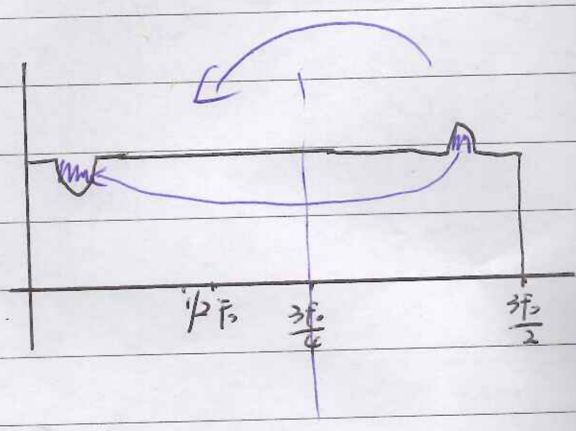
$F_s = 1.5 F_0$, means that spectrum will be folded at $\frac{3F_0}{4}$

possible 1:

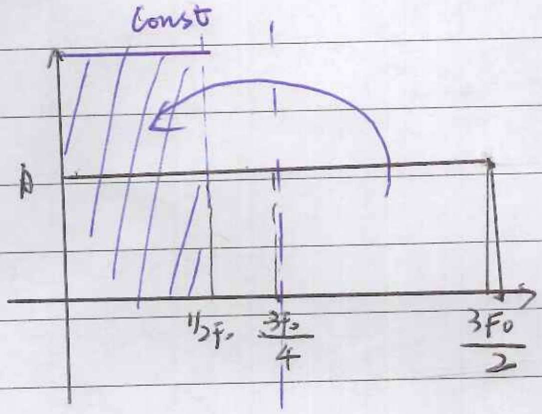


folding

possible 3



possible 2:

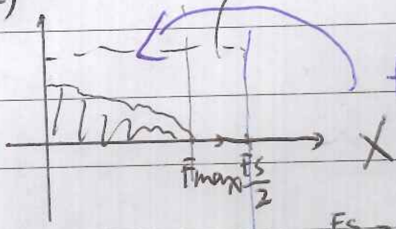


Problem 3:

If $x(n) = \delta(n)$ gap Then from $(0, \frac{F_s}{2})$ we have to be const (no gap)

case 1 (impossible)

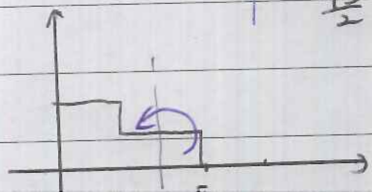
Then $F_{max} > \frac{F_s}{2}$ we have possibility



folding

$\frac{F_s}{2} > F_{max}$ (impossible)

case 2



$\frac{F_s}{2} < F_{max}$ (possible)

Problem 4

$$Y(z) = X_1(z)\, X_2(z)\, X_3(z)$$

$$= \frac{1}{1-0.8z^{-1}} \cdot \frac{1}{1-0.5z^{-1}} \cdot \frac{1}{1-0.2z^{-1}}$$

$$y(n) = \frac{32}{9}(0.8)^n u(n) - \frac{25}{9}(0.5)^n u(n) + \frac{2}{9}(0.2)^n u(n)$$

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