Handin Assignment 1.

1. a. False.

Supposing that solving differential equation, signal sometimes is not causal, but we still use one-side Z-transform.

b. True Refering to Textbook P186 Example 4.2.

c. True

$$H(z) = \sum_{i=0}^{N} a_i z^{-i}$$

Then all poles are at $z=0$

d. True

Taking sinc function as example

e. false should be "LTI" system

f. False. DTFS is the counter example.

2. (a) No, since the coefficient before $x(n)$ is n, which changes over time.

or, solving the equation

$$y(n) = \left(\tfrac{1}{2}\right)^{n+1} y(-1) + \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^{k} (n-k)\, x(n-k)$$

if $x(n-D)$ inputs to system, output $y \neq y(n-D)$

(b) $Z[Y(n)] = Z[\tfrac{1}{2} y(n-1)] + Z[n\, \delta(n)]$

$$Y(z) = \tfrac{1}{2}[Y(z) z^{-1} + y(-1)] + 0$$

$$\Rightarrow \quad Y(z) = \frac{\tfrac{1}{2} y(-1)}{1 - \tfrac{1}{2} z^{-1}} \quad \Rightarrow \quad y(n) = \tfrac{1}{2} y(-1) \left(\tfrac{1}{2}\right)^{n}$$

In this problem, I give full point if $y(-1)=0$ is assumed and stated clearly, otherwise only half point is given.

In addition, I ignore the error propagation to (c) and (d)

(c) $x(n) = \left(\frac{1}{5}\right)^n u(n) \Rightarrow X(z) = \frac{1}{1-0.2z^{-1}}$

$$Y(z) = \frac{1}{2}\left[z^{-1}Y(z) + y(-1)\right] - z\frac{d}{dz}\frac{1}{1-0.2z^{-1}}$$

$$= \frac{1}{2}z^{-1}Y(z) + \frac{1}{2}y(-1) + z\cdot\frac{z^{-2}}{(1-0.2z^{-1})^2 5}$$

$$\Rightarrow Y(z) = \frac{\frac{1}{2}y(-1)}{1-0.2z^{-1}} + \frac{z^{-1}}{5(1-0.2z^{-1})^2(1-0.5z^{-1})}$$

(d) $H(z) = 1 - \frac{1}{5}z^{-1}$

$$Y_2(z) = H(z)Y(z) = \frac{1}{2}y(-1) + \frac{z^{-1}}{5(1-0.2z^{-1})(1-0.5z^{-1})}$$

$$y_2(n) = \frac{1}{2}\delta(n)\cdot y(-1) + \frac{2}{3}\left[(0.5)^n - (0.2)^n\right]u(n)$$