Digitalteknik EITF65

Lecture 10: State Assignment

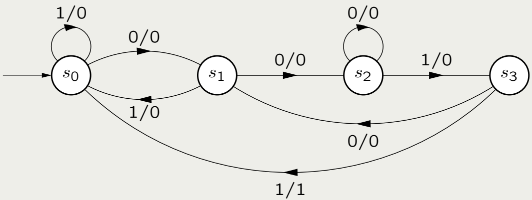
September 26, 2018

Detector (Graph)

Example 6.6 (2.3, 2.14)

Detect the sequence 0011:

State transition graph:



Tabular:

s	0	1
s_0	$s_1/0$	$s_0/0$
s_1	$s_2/0$	$s_0/0$
s_2	$s_2/0$	$s_3/0$
s_3	$s_1/0$	$s_0/1$

Detector (NBCD)

Use natural binary code to encode the states:

State $s\ q_1 q_2$	$x = 0$	$x = 1$
s_0 00	01/0	00/0
s_1 01	10/0	00/0
s_2 10	10/0	11/0
s_3 11	01/0	00/1

Minimal Functions:

$$q_1^+ = q_1' q_2 x' \vee q_1 q_2'$$
$$q_2^+ = q_1' q_2' x' \vee q_1 q_2 x' \vee q_1 q_2' x$$
$$u = q_1 q_2 x$$

$\Rightarrow$ 6 implicants

Karnaugh maps:

q_1^+ ($q_1 q_2$ \ x)	0	1
00	0	0
01	1	0
11	0	0
10	1	1

q_2^+ ($q_1 q_2$ \ x)	0	1
00	1	0
01	0	0
11	1	0
10	0	1

u ($q_1 q_2$ \ x)	0	1
00	0	0
01	0	0
11	0	1
10	0	0

Detector (Gray)

Use Gray code to encode the states:

State s	$q_1 q_2$	$x = 0$	$x = 1$
s_0	00	01/0	00/0
s_1	01	11/0	00/0
s_2	11	11/0	10/0
s_3	10	01/0	00/1

Minimal Functions:

$$q_1^+ = q_2 x' \vee q_1 q_2$$
$$q_2^+ = x'$$
$$u = x q_1 q_2'$$

$\Rightarrow$ 4 implicants

Karnaugh maps:

q_1^+ ($q_1 q_2$ \ x)	0	1
00	0	0
01	1	0
11	1	1
10	0	0

q_2^+ ($q_1 q_2$ \ x)	0	1
00	1	0
01	1	0
11	1	0
10	1	0

u ($q_1 q_2$ \ x)	0	1
00	0	0
01	0	0
11	0	0
10	0	1

State assignment

Observation

The number of implicants in a realization of a graph depends on the state assignment.

There are no (known) methods of how to find the optimal state assignment.

Idea

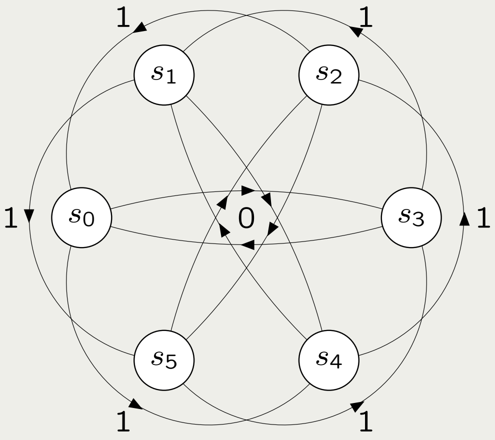
Find a state assignment where few variables change in each transition.

Modulo 6 counter

Example 6.5

A modulo 6 counter ($i = 0 \Rightarrow +3$ and $i = 1 \Rightarrow -2$) has the graph

State transition graph:



Tabular:

s	0	1
s_0	s_3	s_4
s_1	s_4	s_5
s_2	s_5	s_0
s_3	s_0	s_1
s_4	s_1	s_2
s_5	s_2	s_3

Mod 6 (NBCD)

Use natural binary code to get the tabular

State.		x	
s	$q_1 q_2 q_3$	0	1
s_0	000	011	100
s_1	001	100	101
s_2	010	101	000
s_3	011	000	001
s_4	100	001	010
s_5	101	010	011

A minimal realization:

$$
\begin{aligned}
q_1^+ &= q_1' q_2' q_3 \vee q_1' q_2' x \vee q_2 q_3' x' \\
q_2^+ &= q_1 x \vee q_1 q_3 \vee q_1' q_2' q_3' x' \\
q_3^+ &= q_3' x' \vee q_3 x
\end{aligned}
$$

$\Rightarrow$ 8 implicants

Mod 6 (NBCD) Karnaugh

q_1^+ (q_1q_2 \ q_3x)	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	-	-	-	-
10	0	0	0	0

q_2^+ (q_1q_2 \ q_3x)	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	-	-	-	-
10	0	1	1	1

q_3^+ (q_1q_2 \ q_3x)	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	-	-	-	-
10	1	0	1	0

$q_1^+ = q_1'q_2'q_3 \vee q_1'q_2'x \vee q_2q_3'x'$

$q_2^+ = q_1x \vee q_1q_3 \vee q_1'q_2'q_3'x'$

$q_3^+ = q_3'x' \vee q_3x$

Mod 6 (Gray)

Use Gray code between state transitions:

State.		x	
s	$q_1 q_2 q_3$	0	1
s_0	000	100	001
s_1	101	001	111
s_2	011	111	000
s_3	100	000	101
s_4	001	101	011
s_5	111	011	100

A minimal realization:

$$\begin{aligned} q_1^+ &= q_1' x' \vee q_1 x \\ q_2^+ &= q_2 x' \vee q_2' q_3 x \\ q_3^+ &= q_2' x \vee q_3 x' \end{aligned}$$

$\Rightarrow$ 6 implicants

Mod 6 (Gray) Karnaugh

q_1^+ (q_1q_2 \ q_3x)	00	01	11	10
00	1	0	0	1
01	-	-	0	1
11	-	-	1	0
10	0	1	1	0

q_2^+ (q_1q_2 \ q_3x)	00	01	11	10
00	0	0	1	0
01	-	-	0	1
11	-	-	0	1
10	0	0	1	0

q_3^+ (q_1q_2 \ q_3x)	00	01	11	10
00	0	1	1	1
01	-	-	0	1
11	-	-	0	1
10	0	1	1	1

$q_1^+ = q_1'x' \vee q_1x$

$q_2^+ = q_2x' \vee q_2'q_3x$

$q_3^+ = q_2'x \vee q_3x'$

Reduced dependency

Idea of RD

Find blocks of states s.t. their next states also are grouped in blocks

$$s_1, s_2 \in \mathcal{B} \;\Rightarrow\; \delta(s_1, i), \delta(s_2, i) \in \tilde{\mathcal{B}}$$

A transition in the graph corresponds to a transition between blocks.

Use the partion to encode the states such that a transition between two blocks corresponds to as few changes in the state variables as possible.

Reduced dependency

Algorithm

For each pair of states

- assume the two states are in same block.
- iteratively, for each input, group the next states of a block into a new block.

The obtained state partitions correspond to dependencies in the graph. Use them to find a state assignment.

Detector (RD)

Example Detector (RD)

Partitions for the detector:

$$
\begin{aligned}
P_{01} &: \overline{s_0 s_1} \rightarrow \overline{s_0 s_1 s_2} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S} \\
P_{02} &: \overline{s_0 s_2} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S} \\
P_{03} &: \overline{s_0 s_3} \rightarrow \overline{s_0 s_3} = \overline{s_0 s_3}; \overline{s_1}; \overline{s_2} \\
P_{12} &: \overline{s_1 s_2} \rightarrow \overline{s_1 s_2}; \overline{s_0 s_3} \\
P_{13} &: \overline{s_1 s_3} \rightarrow \overline{s_1 s_2 s_3} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S} \\
P_{23} &: \overline{s_2 s_3} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S}
\end{aligned}
$$

Use q_1 to encode the blocks in $\begin{array}{lcc} P_{12} = & \overline{s_1 s_2}; & \overline{s_0 s_3}. \\ q_1 = & 0 & 1 \end{array}$

Use q_2 and a new partition $\begin{array}{lcc} P^* = & \overline{s_0 s_1}; & \overline{s_2 s_3} \\ q_2 = & 0 & 1 \end{array}$

Detector (RD)

Use RD to encode the states:

State	x	
s $q_1 q_2$	0	1
s_0 10	00/0	10/0
s_1 00	01/0	10/0
s_2 01	01/0	11/0
s_3 11	00/0	10/1

Minimal Functions:

$$q_1^+ = x$$
$$q_2^+ = x' q_1' \vee q_1' q_2$$
$$u = x q_1 q_2$$

$\Rightarrow$ 4 implicants

Karnaugh maps:

q_1^+, $q_1 q_2$ \ x	0	1
00	0	1
01	0	1
11	0	1
10	0	1

q_2^+, $q_1 q_2$ \ x	0	1
00	1	0
01	1	1
11	0	0
10	0	0

u, $q_1 q_2$ \ x	0	1
00	0	0
01	0	0
11	0	1
10	0	0

Mod 6 (RD-partitions)

Partitions for the modulo 6 counter:

$$
\begin{array}{llllll}
P_{01} & : \overline{s_0 s_1} & \rightarrow \overline{s_0 s_1}; \overline{s_3 s_4 s_5} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S} \\
P_{02} & : \overline{s_0 s_2} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_3 s_5} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & \\
P_{03} & : \overline{s_0 s_3} & \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4} & \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} & \\
P_{04} & : \overline{s_0 s_4} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & = P_{02} \\
P_{05} & : \overline{s_0 s_5} & \rightarrow \overline{s_0 s_5}; \overline{s_2 s_3 s_4} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S} \\
P_{12} & : \overline{s_1 s_2} & \rightarrow \overline{s_1 s_2}; \overline{s_0 s_4 s_5} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S} \\
P_{13} & : \overline{s_1 s_3} & \rightarrow \overline{s_0 s_4}; \overline{s_1 s_3 s_5} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & = P_{02} \\
P_{14} & : \overline{s_1 s_4} & \rightarrow \overline{s_1 s_4}; \overline{s_2 s_5} & \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} & = P_{03} \\
P_{15} & : \overline{s_1 s_5} & \rightarrow \overline{s_2 s_4}; \overline{s_1 s_3 s_5} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & = P_{02} \\
P_{23} & : \overline{s_2 s_3} & \rightarrow \overline{s_0 s_1 s_5}; \overline{s_2 s_3} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S} \\
P_{24} & : \overline{s_2 s_4} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_5} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & = P_{02} \\
P_{25} & : \overline{s_2 s_5} & \rightarrow \overline{s_0 s_3}; \overline{s_2 s_5} & \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} & = P_{03} \\
P_{34} & : \overline{s_3 s_4} & \rightarrow \overline{s_3 s_4}; \overline{s_0 s_1 s_2} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S} \\
P_{35} & : \overline{s_3 s_5} & \rightarrow \overline{s_0 s_2}; \overline{s_1 s_3 s_5} & \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} & = P_{02} \\
P_{45} & : \overline{s_4 s_5} & \rightarrow \overline{s_1 s_2 s_3}; \overline{s_4 s_5} & \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} & = \mathcal{S}
\end{array}
$$

Mod 6 (RD)

Use P_{02} and P_{03} according to

$$P_{02} = \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} \text{ and } P_{03} = \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5}$$
$$q_1 = \quad 0 \qquad\quad 1 \qquad\qquad q_2 q_3 = 00 \quad 11 \quad 01$$

The function table becomes:

State		x	
s	$q_1 q_2 q_3$	0	1
s_0	000	100	011
s_1	111	011	101
s_2	001	101	000
s_3	100	000	111
s_4	011	111	001
s_5	101	001	100

Minimal realization:

$$\begin{aligned} q_1^+ &= q_1' x' \vee q_1 x \\ q_2^+ &= q_2 x' \vee \underline{q_3' x} \\ q_3^+ &= q_2 \vee \underline{q_3' x} \vee q_3 x' \end{aligned}$$

$\Rightarrow$ 6 impliants

Mod 6 (RD) Karnaugh

q_1^+ (q_1q_2 \ q_3x)	00	01	11	10
00	1	0	0	1
01	-	-	0	1
11	-	-	1	0
10	0	1	1	0

q_2^+ (q_1q_2 \ q_3x)	00	01	11	10
00	0	1	0	0
01	-	-	0	1
11	-	-	0	1
10	0	1	0	0

q_3^+ (q_1q_2 \ q_3x)	00	01	11	10
00	0	1	0	1
01	-	-	1	1
11	-	-	1	1
10	0	1	0	1

$q_1^+ = q_1'x' \vee q_1x$

$q_2^+ = q_2x' \vee \underline{q_3'x}$

$q_3^+ = q_2 \vee \underline{q_3'x} \vee q_3x'$

1-Hot

1-Hot state assignment

Use one state variable for each state. Then for *all* transitions two variables are changed.

1-Hot (Detector)

state transition table:

	State.	x	
s	$q_1 q_2 q_3 q_4$	0	1
s_0	1000	0100/0	1000/0
s_1	0100	0010/0	1000/0
s_2	0010	0010/0	0001/0
s_3	0001	0100/0	1000/1

Minimal realisation:

$$q_1^+ = xq_3'$$
$$q_2^+ = x'q_2'q_3'$$
$$q_3^+ = x'q_1'q_4'$$
$$q_4^+ = xq_3$$
$$u = xq_4$$

$\Rightarrow$ 5 implicants

1-hot (Detector) Karnaugh

q_1^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	0
01	0	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 0$

q_2^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	1	-	0
01	0	-	-	-
11	-	-	-	-
10	1	-	-	-

$x = 0$

q_3^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	1
01	1	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 0$

q_4^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	0
01	0	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 0$

q_1^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	1	-	0
01	1	-	-	-
11	-	-	-	-
10	1	-	-	-

$x = 1$

q_2^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	0
01	0	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 1$

q_3^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	0
01	0	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 1$

q_4^+ (q_1q_2 \ q_3q_4)	00	01	11	10
00	-	0	-	1
01	0	-	-	-
11	-	-	-	-
10	0	-	-	-

$x = 1$

$$q_1^+ = xq_3' \qquad q_2^+ = x'q_2'q_3' \qquad q_3^+ = x'q_1'q_4' \qquad q_4^+ = xq_3$$

$$u = xq_4$$

Mod 6 (1-hot)

A *1-hot* assignment for the modulo 6 counter gives:

State		x	
s	$q_1 \dots q_6$	0	1
s_0	100000	000100	000010
s_1	010000	000010	000001
s_2	001000	000001	100000
s_3	000100	100000	010000
s_4	000010	010000	001000
s_5	000001	001000	000100

A minimal realization

$$
\begin{aligned}
q_1^+ &= q_4x' \vee q_3x \\
q_2^+ &= q_5x' \vee q_4x \\
q_3^+ &= q_6x' \vee q_5x \\
q_4^+ &= q_6x \vee q_1x' \\
q_5^+ &= q_2x' \vee q_1x \\
q_6^+ &= q_3x' \vee q_2x
\end{aligned}
$$

$\Rightarrow$ 12 implicants

Summary

Summarising the examples:

Encoding	Detector	Mod 6 counter
Natural	6 impl.	8 impl.
Gray	4 impl.	6 impl.
RD	4 impl.	6 impl.
1-Hot	5 impl.	12 impl.

Conclusion

It is often worth spending some time trying to find a promising state assignment.