



Digitaltechnik EITF65

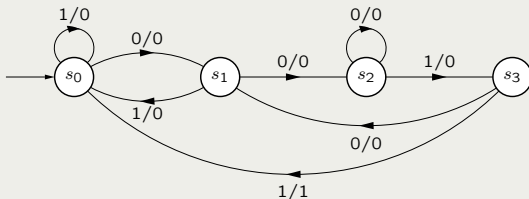
Lecture 10: State Assignment

Detector (Graph)

Example 6.6 (2.3, 2.14)

Detect the sequence 0011:

State transition graph:



Tabular:

s	0	1
s ₀	s ₁ /0	s ₀ /0
s ₁	s ₂ /0	s ₀ /0
s ₂	s ₂ /0	s ₃ /0
s ₃	s ₁ /0	s ₀ /1

Detector (NBCD)

Use **natural binary code** to encode the states:

State $s \ q_1 q_2$	x	
	0	1
$s_0 \ 00$	01/0	00/0
$s_1 \ 01$	10/0	00/0
$s_2 \ 10$	10/0	11/0
$s_3 \ 11$	01/0	00/1

Minimal Functions:

$$q_1^+ = q_1' q_2 x' \vee q_1 q_2'$$

$$q_2^+ = q_1' q_2' x' \vee q_1 q_2 x' \vee q_1 q_2' x$$

$$u = q_1 q_2 x$$

$\Rightarrow$ 6 implicants

Karnaugh maps:

q_1^+	0	1
00	0	0
01	1	0
11	0	0
10	1	1

q_2^+	0	1
00	1	0
01	0	0
11	1	0
10	0	1

u	0	1
00	0	0
01	0	0
11	0	1
10	0	0

Detector (Gray)

Use **Gray code** to encode the states:

State $s \ q_1 q_2$	x	
	0	1
$s_0 \ 00$	01/0	00/0
$s_1 \ 01$	11/0	00/0
$s_2 \ 11$	11/0	10/0
$s_3 \ 10$	01/0	00/1

Minimal Functions:

$$q_1^+ = q_2 x' \vee q_1 q_2$$

$$q_2^+ = x'$$

$$u = x q_1 q_2'$$

$\Rightarrow$ 4 implicants

Karnaugh maps:

q_1^+	0	1
00	0	0
01	1	0
11	1	1
10	0	0

q_2^+	0	1
00	1	0
01	1	0
11	1	0
10	1	0

u	0	1
00	0	0
01	0	0
11	0	0
10	0	1

State assignment

Observation

The number of implicants in a realization of a graph depends on the state assignment.

There are no (known) methods of how to find the optimal state assignment.

Idea

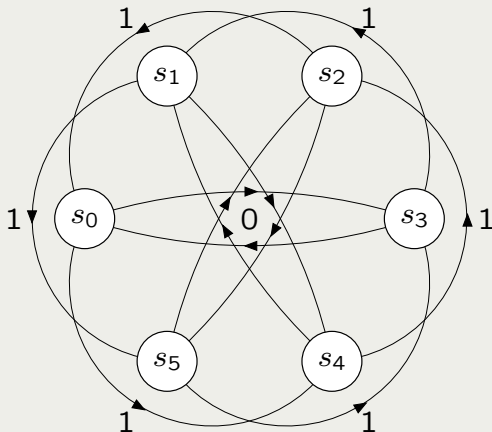
Find a state assignment where few variables change in each transition.

Modulo 6 counter

Example 6.5

A modulo 6 counter ($i = 0 \Rightarrow +3$ and $i = 1 \Rightarrow -2$) has the graph

State transition graph:



Tabular:

s	0	1
s_0	s_3	s_4
s_1	s_4	s_5
s_2	s_5	s_0
s_3	s_0	s_1
s_4	s_1	s_2
s_5	s_2	s_3

Mod 6 (NBCD)

Use natural binary code to get the tabular

s	State. $q_1 q_2 q_3$	x	
		0	1
s_0	000	011	100
s_1	001	100	101
s_2	010	101	000
s_3	011	000	001
s_4	100	001	010
s_5	101	010	011

A minimal realization:

$$q_1^+ = q_1' q_2' q_3 \vee q_1' q_2' x \vee q_2 q_3' x'$$

$$q_2^+ = q_1 x \vee q_1 q_3 \vee q_1' q_2' q_3' x'$$

$$q_3^+ = q_3' x' \vee q_3 x$$

$\Rightarrow$ 8 implicants

Mod 6 (NBCD) Karnaugh

q_1^+	q_3x			
	00	01	11	10
00	0	1	1	1
01	1	0	0	0
q_1q_2				
11	-	-	-	-
10	0	0	0	0

q_2^+	q_3x			
	00	01	11	10
00	1	0	0	0
01	0	0	0	0
q_1q_2				
11	-	-	-	-
10	0	1	1	1

q_3^+	q_3x			
	00	01	11	10
00	1	0	1	0
01	1	0	1	0
q_1q_2				
11	-	-	-	-
10	1	0	1	0

$$q_1^+ = q_1'q_2'q_3 \vee q_1'q_2'x \vee q_2q_3'x'$$

$$q_2^+ = q_1x \vee q_1q_3 \vee q_1'q_2'q_3'x'$$

$$q_3^+ = q_3'x' \vee q_3x$$

Mod 6 (Gray)

Use Gray code between state transitions:

State.		x	
s	$q_1 q_2 q_3$	0	1
s_0	000	100	001
s_1	101	001	111
s_2	011	111	000
s_3	100	000	101
s_4	001	101	011
s_5	111	011	100

A minimal realization:

$$q_1^+ = q_1' x' \vee q_1 x$$

$$q_2^+ = q_2 x' \vee q_2' q_3 x$$

$$q_3^+ = q_2' x \vee q_3 x'$$

$\Rightarrow$ 6 implicants

Mod 6 (Gray) Karnaugh

q_1^+	q_3x			
	00	01	11	10
00	1	0	0	1
01	-	-	0	1
11	-	-	1	0
10	0	1	1	0

$$q_1^+ = q_1'x' \vee q_1x$$

q_2^+	q_3x			
	00	01	11	10
00	0	0	1	0
01	-	-	0	1
11	-	-	0	1
10	0	0	1	0

$$q_2^+ = q_2x' \vee q_2'q_3x$$

q_3^+	q_3x			
	00	01	11	10
00	0	1	1	1
01	-	-	0	1
11	-	-	0	1
10	0	1	1	1

$$q_3^+ = q_2'x \vee q_3x'$$

Idea of RD

Find blocks of states s.t. their next states also are grouped in blocks

$$s_1, s_2 \in \mathcal{B} \Rightarrow \delta(s_1, i), \delta(s_2, i) \in \tilde{\mathcal{B}}$$

A transition in the graph corresponds to a transition between blocks.

Use the partition to encode the states such that a transition between two blocks corresponds to as few changes in the state variables as possible.

Algorithm

For each pair of states

- ▶ assume the two states are in same block.
- ▶ iteratively, for each input, group the next states of a block into a new block.

The obtained state partitions correspond to dependencies in the graph. Use them to find a state assignment.

Detector (RD)

Example Detector (RD)

Partitions for the detector:

$$P_{01} : \overline{s_0 s_1} \rightarrow \overline{s_0 s_1 s_2} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S}$$

$$P_{02} : \overline{s_0 s_2} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S}$$

$$P_{03} : \overline{s_0 s_3} \rightarrow \overline{s_0 s_3} = \overline{s_0 s_3}; \overline{s_1}; \overline{s_2}$$

$$P_{12} : \overline{s_1 s_2} \rightarrow \overline{s_1 s_2}; \overline{s_0 s_3}$$

$$P_{13} : \overline{s_1 s_3} \rightarrow \overline{s_1 s_2 s_3} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S}$$

$$P_{23} : \overline{s_2 s_3} \rightarrow \overline{s_0 s_1 s_2 s_3} = \mathcal{S}$$

Use q_1 to encode the blocks in $P_{12} = \overline{s_1 s_2}; \overline{s_0 s_3}$.
 $q_1 = \begin{matrix} 0 & 1 \end{matrix}$

Use q_2 and a new partition $P^* = \overline{s_0 s_1}; \overline{s_2 s_3}$
 $q_2 = \begin{matrix} 0 & 1 \end{matrix}$

Detector (RD)

Use **RD** to encode the states:

State $s \ q_1 q_2$	x	
	0	1
$s_0 \ 10$	00/0	10/0
$s_1 \ 00$	01/0	10/0
$s_2 \ 01$	01/0	11/0
$s_3 \ 11$	00/0	10/1

Minimal Functions:

$$q_1^+ = x$$

$$q_2^+ = x'q_1' \vee q_1'q_2$$

$$u = xq_1q_2$$

$\Rightarrow$ 4 implicants

Karnaugh maps:

q_1^+	0	1
00	0	1
01	0	1
11	0	1
10	0	1

q_2^+	0	1
00	1	0
01	1	1
11	0	0
10	0	0

u	0	1
00	0	0
01	0	0
11	0	1
10	0	0

Mod 6 (RD-partitions)

Partitions for the modulo 6 counter:

$$\begin{aligned}P_{01} : \overline{s_0 s_1} &\rightarrow \overline{s_0 s_1}; \overline{s_3 s_4 s_5} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S} \\P_{02} : \overline{s_0 s_2} &\rightarrow \overline{s_0 s_2 s_4}; \overline{s_3 s_5} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} \\P_{03} : \overline{s_0 s_3} &\rightarrow \overline{s_0 s_3}; \overline{s_1 s_4} \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} \\P_{04} : \overline{s_0 s_4} &\rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} = P_{02} \\P_{05} : \overline{s_0 s_5} &\rightarrow \overline{s_0 s_5}; \overline{s_2 s_3 s_4} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S} \\P_{12} : \overline{s_1 s_2} &\rightarrow \overline{s_1 s_2}; \overline{s_0 s_4 s_5} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S} \\P_{13} : \overline{s_1 s_3} &\rightarrow \overline{s_0 s_4}; \overline{s_1 s_3 s_5} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} = P_{02} \\P_{14} : \overline{s_1 s_4} &\rightarrow \overline{s_1 s_4}; \overline{s_2 s_5} \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} = P_{03} \\P_{15} : \overline{s_1 s_5} &\rightarrow \overline{s_2 s_4}; \overline{s_1 s_3 s_5} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} = P_{02} \\P_{23} : \overline{s_2 s_3} &\rightarrow \overline{s_0 s_1 s_5}; \overline{s_2 s_3} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S} \\P_{24} : \overline{s_2 s_4} &\rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_5} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} = P_{02} \\P_{25} : \overline{s_2 s_5} &\rightarrow \overline{s_0 s_3}; \overline{s_2 s_5} \rightarrow \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5} = P_{03} \\P_{34} : \overline{s_3 s_4} &\rightarrow \overline{s_3 s_4}; \overline{s_0 s_1 s_2} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S} \\P_{35} : \overline{s_3 s_5} &\rightarrow \overline{s_0 s_2}; \overline{s_1 s_3 s_5} \rightarrow \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} = P_{02} \\P_{45} : \overline{s_4 s_5} &\rightarrow \overline{s_1 s_2 s_3}; \overline{s_4 s_5} \rightarrow \overline{s_0 s_1 s_2 s_3 s_4 s_5} = \mathcal{S}\end{aligned}$$

Mod 6 (RD)

Use P_{02} and P_{03} according to

$$P_{02} = \overline{s_0 s_2 s_4}; \overline{s_1 s_3 s_5} \text{ and } P_{03} = \overline{s_0 s_3}; \overline{s_1 s_4}; \overline{s_2 s_5}$$

$$q_1 = \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix} \quad q_2 q_3 = \begin{matrix} 00 & 11 & 01 \end{matrix}$$

The function table becomes:

State		x	
s	$q_1 q_2 q_3$	0	1
s_0	000	100	011
s_1	111	011	101
s_2	001	101	000
s_3	100	000	111
s_4	011	111	001
s_5	101	001	100

Minimal realization:

$$q_1^+ = q_1' x' \vee q_1 x$$

$$q_2^+ = q_2 x' \vee \underline{q_3' x}$$

$$q_3^+ = q_2 \vee \underline{q_3' x} \vee q_3 x'$$

$\Rightarrow$ 6 impliants

Mod 6 (RD) Karnaugh

q_1^+	q_3x			
	00	01	11	10
00	1	0	0	1
01	-	-	0	1
11	-	-	1	0
10	0	1	1	0

$$q_1^+ = q_1'x' \vee q_1x$$

q_2^+	q_3x			
	00	01	11	10
00	0	1	0	0
01	-	-	0	1
11	-	-	0	1
10	0	1	0	0

$$q_2^+ = q_2x' \vee \underline{q_3'x}$$

q_3^+	q_3x			
	00	01	11	10
00	0	1	0	1
01	-	-	1	1
11	-	-	1	1
10	0	1	0	1

$$q_3^+ = q_2 \vee \underline{q_3'x} \vee q_3x'$$

1-Hot

1-Hot state assignment

Use one state variable for each state. Then for *all* transitions two variables are changed.

1-Hot (Detector)

state transition table:

s	State. $q_1q_2q_3q_4$	x	
		0	1
s_0	1000	0100/0	1000/0
s_1	0100	0010/0	1000/0
s_2	0010	0010/0	0001/0
s_3	0001	0100/0	1000/1

Minimal realisation:

$$q_1^+ = xq_3'$$

$$q_2^+ = x'q_2'q_3'$$

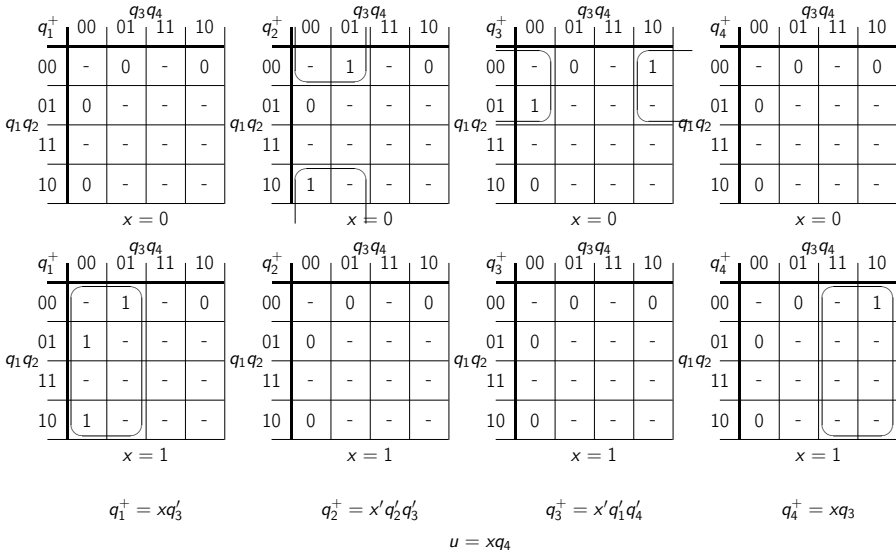
$$q_3^+ = x'q_1'q_4'$$

$$q_4^+ = xq_3$$

$$u = xq_4$$

$\Rightarrow$ 5 implicants

1-hot (Detector) Karnaugh



Mod 6 (1-hot)

A 1-hot assignment for the modulo 6 counter gives:

s	State $q_1 \dots q_6$	x	
		0	1
s_0	100000	000100	000010
s_1	010000	000010	000001
s_2	001000	000001	100000
s_3	000100	100000	010000
s_4	000010	010000	001000
s_5	000001	001000	000100

A minimal realization

$$q_1^+ = q_4 x' \vee q_3 x$$

$$q_2^+ = q_5 x' \vee q_4 x$$

$$q_3^+ = q_6 x' \vee q_5 x$$

$$q_4^+ = q_6 x \vee q_1 x'$$

$$q_5^+ = q_2 x' \vee q_1 x$$

$$q_6^+ = q_3 x' \vee q_2 x$$

$\Rightarrow$ 12 implicants

Summary

Summarising the examples:

Encoding	Detector	Mod 6 counter
Natural	6 impl.	8 impl.
Gray	4 impl.	6 impl.
RD	4 impl.	6 impl.
1-Hot	5 impl.	12 impl.

Conclusion

It is often worth spending some time trying to find a promising state assignment.