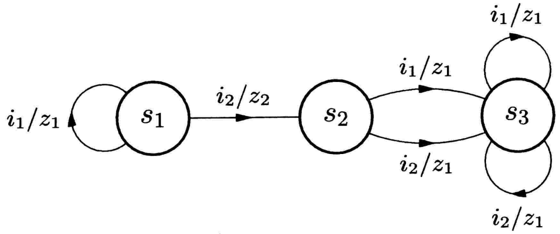
Problems

2.1 Consider a finite-state machine and let s denote the present state and z the present output. Consider the following state-transition graph and assume that we are in state s_1 and have input i_2 at time t_0.



(a) At which time instant is $s = s_2$?

(b) At which time instant is $z = z_2$?

2.2 Consider a state-transition graph for a finite-state machine. Which of the following statements are correct?

(a) The total number of edges entering a state is always equal to the total number of output values.

(b) The total number of edges entering a state is always equal to the total number of input values.

(c) The total number of edges entering a state can never exceed the total number of states.

(d) The total number of edges leaving a state is always equal to the total number of output values.

(e) The total number of edges leaving a state is always equal to the total number of input values.

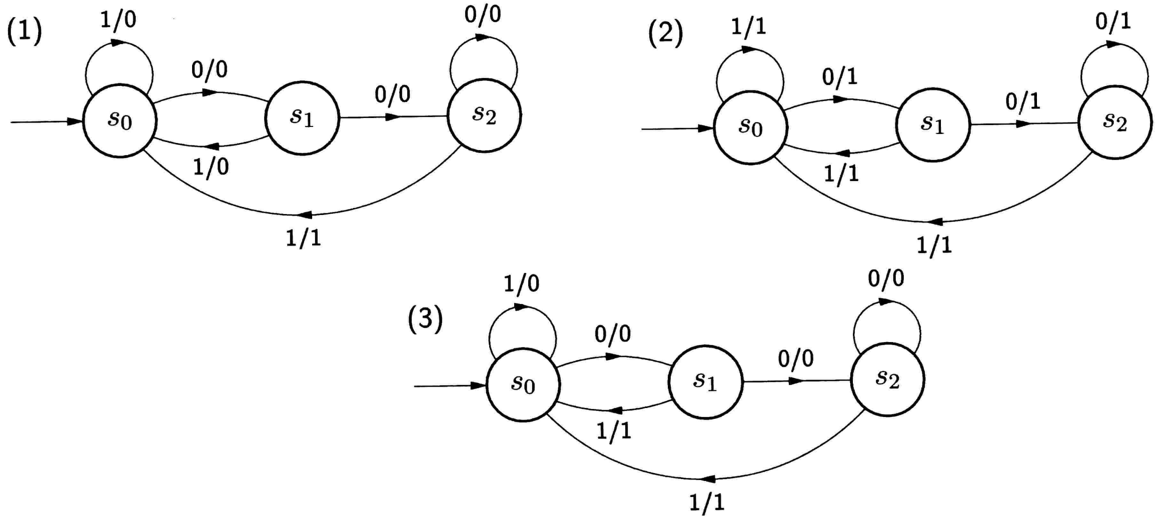
(f) The total number of edges leaving a state is always less than the total number of states.

2.3 For which (if any) of the state-transition graphs given below is the output 1

(a) *if* the three most recent input values are 001?

(b) *only if* the three most recent input values are 001?

(c) *if and only if* the three most recent input values are 001?

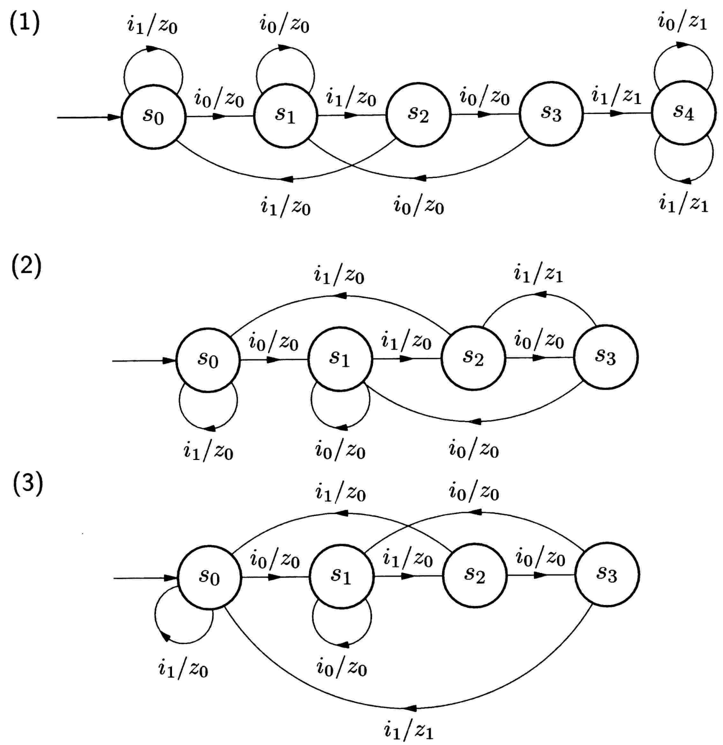


2.4 For which (if any) of the state-transition graphs given below is the output z_1 if and only if

(a) the four most recent input values are $i_0 i_1 i_0 i_1$?

(b) the four-tuple $i_0 i_1 i_0 i_1$ has occurred at least once?

(c) the four-tuple $i_0 i_1 i_0 i_1$ has occurred exactly once?



2.1

(a) $t_0 + 1$; The edge points at the state which the system updates to when it is clocked. This is the *next state* function, $\delta(s, i)$.

(b) t_0; The label on the edge is given directly as output. This is the *output* function, $\lambda(s, i)$.

2.2

a) False. The number of entering edges can be different for different states. There can for example be an edge from every state that resets the system to the starting state.

b) False. See *a*.

c) False. There can be more than one edge from one state to another.

d) False. See *e*.

e) True. The next state function is a function of both the state and the input. In every state, each input value corresponds to an edge leaving the state.

f) False. The statement is not true if the number of states is less than the number of inputs.

2.3

a) The statement means that independent of the current state, the input 001 results in the output 1. This is true for all three graphs since two 0:s drives the machine to state s_2. Then a 1 will output a 1.

b) This means that independent of the current state, the only way to get output 1 is to give 001 as input. In the first graph we can only get to state s_2 by giving two 0:s. The only output 1 is given from state s_2 with input 1. Hence, it is true for the first graph.

The second graph always gives output 1, so the statement is not true. In the third graph we can also get output 1 for input 101, hence, the statement is not true.

c) *If* and *only if* means that both *a* and *b* should be true. That gives graph 1.

2.4

(a) In the first graph we can get stuck in state s_4, which gives a 1 out independent of the input. In the second graph, state s_3 means that the three most resent inputs are $i_0 i_1 i_0$. Then input i_1 will generate output z_1. This is the only way to get z_1 and, hence, the statement is true for the second graph. The third graph will generate a z_1 for every even consecutive occurrence of $i_0 i_1$. Therefore, $i_0 i_1 i_0 i_1 i_0 i_1$ (where the four most resent inputs are $i_0 i_1 i_0 i_1$ will not generate z_1.

Concluding the above, we see that it is only the second graph that fulfills the statement.

(b) In the first graph input $i_0 i_1 i_0 i_1$ will lead to state s_4, which will generate z_1 for ever. Hence, the statement is true for the first graph. For the other two it is false.

(c) The statement means that the output becomes z_1 after the first occurrence of $i_0 i_1 i_0 i_1$, and stays that way until the next occurrence when it is reset to z_0. This is not true for any of the three graphs.

2.5