3.34

$$\begin{aligned}
H(z) &= \sum_{n=-1}^{-\infty} 3^n z^{-n} + \sum_{n=0}^{\infty} (\frac{2}{5})^n z^{-n} \\
&= \frac{-1}{1-3z^{-1}} + \frac{1}{1-\frac{2}{5}z^{-1}}, \text{ ROC: } \frac{2}{5} < |z| < 3 \\
X(z) &= \frac{1}{1-z^{-1}} \\
Y(z) &= H(z)X(z) \\
&= \frac{-\frac{13}{5}z^{-1}}{(1-z^{-1})(1-3z^{-1})(1-\frac{2}{5}z^{-1})}, \text{ ROC: } 1 < |z| < 2 \\
&= \frac{\frac{13}{6}}{1-z^{-1}} - \frac{\frac{3}{2}}{1-3z^{-1}} - \frac{\frac{2}{3}}{1-\frac{2}{5}z^{-1}}
\end{aligned}$$

Therefore,

$$y(n) = \frac{3}{2}3^n u(-n-1) + \left[\frac{13}{6} - \frac{2}{3}(\frac{2}{5})^n\right] u(n)$$

3.35

(a)

$$\begin{aligned}
h(n) &= (\frac{1}{3})^n u(n) \\
H(z) &= \frac{1}{1-\frac{1}{3}z^{-1}} \\
x(n) &= (\frac{1}{2})^n cos\frac{\pi n}{3} u(n) \\
X(z) &= \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} \\
Y(z) &= H(z)X(z) \\
&= \frac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})} \\
&= \frac{\frac{1}{7}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{6}{7}(1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} + \frac{3\sqrt{3}}{7}\frac{\frac{\sqrt{3}}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}
\end{aligned}$$

Therefore,

$$y(n) = \left[\frac{1}{7}(\frac{1}{3})^n + \frac{6}{7}(\frac{1}{2})^n cos\frac{\pi n}{3} + \frac{3\sqrt{3}}{7}(\frac{1}{2})^n sin\frac{\pi n}{3}\right] u(n)$$

(b)

$$\begin{aligned}
h(n) &= (\frac{1}{2})^n u(n) \\
H(z) &= \frac{1}{1-\frac{1}{2}z^{-1}} \\
x(n) &= (\frac{1}{3})^n u(n) + (\frac{1}{2})^{-n} u(-n-1)
\end{aligned}$$

$$
\begin{aligned}
X(z) &= \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} \\
Y(z) &= H(z)X(z) \\
&= \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})(1-2z^{-1})} \\
&= \frac{\frac{10}{3}}{1-\frac{1}{2}z^{-1}} + \frac{-2}{1-\frac{1}{3}z^{-1}} + \frac{\frac{-4}{3}}{1-2z^{-1}}
\end{aligned}
$$

Therefore,

$$
y(n) = \left[\frac{10}{3}(\frac{1}{2})^n - 2(\frac{1}{3})^n\right]u(n) + \frac{4}{3}2^n u(-n-1)
$$

(c)

$$
\begin{aligned}
y(n) &= -0.1y(n-1) + 0.2y(n-2) + x(n) + x(n-1) \\
H(z) &= \frac{1+z^{-1}}{1+0.1z^{-1}-0.2z^{-2}} \\
x(n) &= (\frac{1}{3})^n u(n) \\
X(z) &= \frac{1}{1-\frac{1}{3}z^{-1}} \\
Y(z) &= H(z)X(z) \\
&= \frac{1+z^{-1}}{(1-\frac{1}{3}z^{-1})(1+0.1z^{-1}-0.2z^{-2})} \\
&= \frac{-8}{1-\frac{1}{3}z^{-1}} + \frac{\frac{28}{3}}{1-0.4z^{-1}} + \frac{\frac{-1}{3}}{1+0.5z^{-1}}
\end{aligned}
$$

Therefore,

$$
y(n) = \left[-8(\frac{1}{3})^n + \frac{28}{3}(\frac{2}{5})^n - \frac{1}{3}(\frac{1}{2})^n\right]u(n)
$$

(d)

$$
\begin{aligned}
y(n) &= \frac{1}{2}x(n) - \frac{1}{2}x(n-1) \\
\Rightarrow Y(z) &= \frac{1}{2}(1-z^{-1})X(z) \\
X(z) &= \frac{10}{1+z^{-2}} \\
\text{Hence, } Y(z) &= 10\frac{(1-z^{-1})/2}{1+z^{-2}} \\
y(n) &= 5cos\frac{\pi n}{2}u(n) - 5cos\frac{\pi(n-1)}{2}u(n-1) \\
&= \left[5cos\frac{\pi n}{2} - 5sin\frac{\pi n}{2}\right]u(n-1) + 5\delta(n) \\
&= 5\delta(n) + \frac{10}{\sqrt{2}}sin(\frac{\pi n}{2} + \frac{\pi}{4})u(n-1) \\
&= \frac{10}{\sqrt{2}}sin(\frac{\pi n}{2} + \frac{\pi}{4})u(n)
\end{aligned}
$$

(e)

$$
y(n) = -y(n-2) + 10x(n)
$$

$$
\begin{aligned}
Y(z) &= \frac{10}{1+z^{-2}}X(z) \\
X(z) &= \frac{10}{1+z^{-2}} \\
Y(z) &= \frac{100}{(1+z^{-2})^2} \\
&= \frac{50}{1+jz^{-1}} + \frac{50}{1-jz^{-1}} + \frac{-25jz^{-1}}{(1+jz^{-1})^2} + \frac{25jz^{-1}}{(1-jz^{-1})^2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(n) &= \{50\left[j^n + (-j)^n\right] - 25n\left[j^n + (-j)^n\right]\} u(n) \\
&= (50-25n)(j^n + (-j)^n)u(n) \\
&= (50-25n)2cos\frac{\pi n}{2}u(n)
\end{aligned}
$$

(f)

$$
\begin{aligned}
h(n) &= (\frac{2}{5})^n u(n) \\
H(z) &= \frac{1}{1-\frac{2}{5}z^{-1}} \\
x(n) &= u(n) - u(n-7) \\
X(z) &= \frac{1-z^{-n}}{1-z^{-1}} \\
Y(z) &= H(z)X(z) \\
&= \frac{1-z^{-n}}{(1-\frac{2}{5}z^{-1})(1-z^{-1})} \\
&= \frac{\frac{5}{3}}{1-z^{-1}} + \frac{\frac{-2}{3}}{1-\frac{2}{5}z^{-1}} - \left[\frac{\frac{5}{3}}{1-z^{-1}} + \frac{\frac{-2}{3}}{1-\frac{2}{5}z^{-1}}\right] z^{-7}
\end{aligned}
$$

Therefore,

$$
y(n) = \frac{1}{3}\left[5 - 2(\frac{2}{5})^n\right] u(n) - \frac{1}{3}\left[5 - 2(\frac{2}{5})^{n-7}\right] u(n-7)
$$

(g)

$$
\begin{aligned}
h(n) &= (\frac{1}{2})^n u(n) \\
H(z) &= \frac{1}{1-\frac{1}{2}z^{-1}} \\
x(n) &= (-1)^n, \qquad -\infty < n < \infty \\
&= cos\pi n, \qquad -\infty < n < \infty
\end{aligned}
$$

$x(n)$ is periodic sequence and its z-transform does not exist.

$$
\begin{aligned}
y(n) &= |H(w_0)|cos[\pi n + \Theta(w_0)], w_0 = \pi \\
H(z) &= \frac{1}{1-\frac{1}{2}e^{-jw}} \\
H(\pi) &= \frac{1}{1+\frac{1}{2}} \\
&= \frac{2}{3}, \qquad \Theta = 0. \\
\text{Hence, } y(n) &= \frac{2}{3}cos\pi n, \qquad -\infty < n < \infty
\end{aligned}
$$

(h)

$$\begin{aligned}
h(n) &= (\frac{1}{2})^n u(n) \\
H(z) &= \frac{1}{1-\frac{1}{2}z^{-1}} \\
x(n) &= (n+1)(\frac{1}{4})^n u(n) \\
X(z) &= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})^2} \\
Y(z) &= H(z)X(z) \\
&= \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})^2} \\
&= \frac{4}{1-\frac{1}{2}z^{-1}} + \frac{\frac{-1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})^2} + \frac{-3}{1-\frac{1}{4}z^{-1}}
\end{aligned}$$

Therefore,

$$y(n) = \left[4(\frac{1}{2})^n - n(\frac{1}{4})^n - 3(\frac{1}{4})^n\right] u(n)$$

3.36

$$\begin{aligned}
H(z) &= \frac{1-2z^{-1}+2z^{-2}-z^{-3}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})(1-\frac{1}{5}z^{-1})}, \qquad \frac{1}{2} < |z| < 1 \\
&= \frac{1-z^{-1}+z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{5}z^{-1})}, \qquad \frac{1}{2} < |z| < 1 \\
\text{(a) } Z_{1,2} &= \frac{1 \pm j\sqrt{3}}{2},\ p_1 = \frac{1}{2},\ p_2 = \frac{1}{5} \\
\text{(b) } H(z) &= 1 + \left[\frac{\frac{5}{2}}{1-\frac{1}{2}z^{-1}} + \frac{-2.8}{1-\frac{1}{5}z^{-1}}\right] z^{-1} \\
h(n) &= \delta(n) + \left[5(\frac{1}{2})^n - 14(\frac{1}{5})^n\right] u(n)
\end{aligned}$$

3.37

$$\begin{aligned}
y(n) &= 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2) \\
Y(z) &= \frac{z^{-1}+z^{-2}}{1-0.7z^{-1}+0.12z^{-2}} X(z) \\
x(n) &= nu(n) \\
X(z) &= \frac{z^{-1}}{(1-z^{-1})^2} \\
Y(z) &= \frac{z^{-2}+z^{-3}}{(1-z^{-1})^2(1-\frac{3}{10}z^{-1})(1-\frac{2}{50}z^{-2})}
\end{aligned}$$

$\Rightarrow$ System is stable

$$Y(z) = \frac{4.76z^{-1}}{(1-z^{-1})^2} + \frac{-12.36}{(1-z^{-1})} + \frac{-26.5}{(1-\frac{3}{10}z^{-1})} + \frac{38.9}{(1-\frac{2}{5}z^{-1})}$$