Exercises

Digital Signal Processing

Exercise Problems with Solutions

Mikael Swartling, Nedelko Grbic, and Bengt Mandersson

Seyedezahra Chamideh, Navya Sri Garigapati, Johan Isaksson, Guoda Tian, and Jan Eric Larsson

Spring 2020

Department of Electrical and Information Technology
Lund University

Introduction

Problem 1.1

Which of the following signals are periodic? If so, determine the time periods.

a) $\cos(0.01\pi n)$

b) $\cos(\pi\frac{30}{105}n)$

c) $\cos(3\pi n)$

d) $\sin(3n)$

e) $\sin(\pi\frac{62n}{10})$.

Problem 1.5

The analog signal $x_a(t)$ is $x_a(t) = 3\sin(100\pi t)$.

a) Draw $x_a(t)$ for $0\,\text{ms} \leq t \leq 30\,\text{ms}$.

b) Sample the signal with $F_s = 300\,\text{samples/s}$. Determine $x(n) = x_a(nT)$ where $T = 1/F_s$. Determine the frequency f of $x(n)$ and show that $x(n)$ is periodic.

c) Draw $x(n)$. What is the period and what does it correspond to in milliseconds?

d) Determine the minimum F_s so that when $x_a(t)$ is sampled, the max of $x(n)$ assumes the value 3 at $0 \leq n < N$, where N is the time period of $x(n)$.

Problem 1.7

An analog signal contains frequencies up to $10\,\text{kHz}$.

a) Which sampling frequencies can be used if we want to be able to reconstruct the signal?

b) Assume the sampling frequency $F_s = 8\,\text{kHz}$. What happens to the frequency component $F_1 = 5\,\text{kHz}$?

c) Assume the sampling frequency $F_s = 8\,\text{kHz}$. What happens to the frequency component $F_1 = 9\,\text{kHz}$?

Problem 1.8

An analog electrocardiogram (ECG) contains frequencies up to $100\,\text{Hz}$.

a) What is the Nyquist rate for the signal?

b) Which is the highest frequency component that can be uniquely represented by the sampling frequency $F_s = 250\,\text{Hz}$?

Comment: The highest frequency component of the signal is called the Nyquist frequency and the double frequency is called the Nyquist rate. Thus, the sampling frequency must be selected higher than the Nyquist rate to avoid folding distortion.

Problem 1.11

To show the effect of folding, we do the following. We sample the signal $x(t) = 3\cos(100\pi t) + 2\sin(250\pi t)$ with the sampling frequency $F_s = 200\,\text{Hz}$, without using any analog filter before the sampling. We then listen to the signal with the sampling frequency $F_s = 1000\,\text{Hz}$ (ideal reconstruction). How does the signal look after the reconstruction?

Matlab: When recording with a sound card in a PC, the analog signal is automatically filtered with an analog filter switching frequency $F_s/2$, before sampling, to avoid folding distortion. If we want to illustrate the example above, we can, for example, compile with $F_s = 1000\,\text{Hz}$, which sets the limit frequency in the sound card filter to $500\,\text{Hz}$. Then we retain every fifth sample only, and we have reduced the sampling rate to $200\,\text{Hz}$. This corresponds to sampling with $200\,\text{Hz}$ but with the analog filter set at $500\,\text{Hz}$.

Problem 1.13

The discrete signal $x(n) = 6.35\cos((\pi/10)n)$ is quantized with the resolution

a) $\Delta = 0.1$, and

b) $\Delta = 0.02$.

How many bits are needed in the A/D-converter?

Problem 1.14

Determine the bit rate (bits/s) and resolution, if a seismic signal with dynamic 1 volt is sampled with $F_s = 20\,\text{Hz}$ with an 8-bit A/D-converter. Which maximum frequency can be represented in the digital signal?

Problem E1.1

MATLAB: To achieve an echo effect, we use the following link (z^{-D}, which delays a signal by D samples).
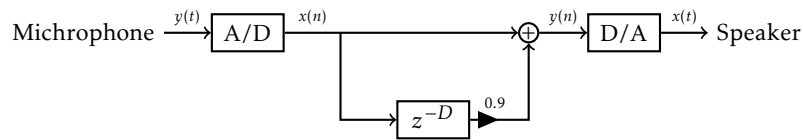
Simulate this in MATLAB and listen to the effect.

```
>> Fs = 10000;
>> t = (0:2*Fs-1)/Fs;          % generate 2s long time vector, FS=1/10000
>> x = chirp(t, 300, 2, 800);  % generate sweep sine between 300 and 800 Hz
>> z = zeros(1, 500);          % generate 500 zeros
>> x1 = [x, z];
>> x2 = [z, 0.9*x];
>> y = x1+x2;                  % generera echo
```

Listen to the signal.

```
>> soundsc(x, Fs)
>> soundsc(y, Fs)
```

Select different values for x and listen to the echo effects.

Problem N1.1

Write down Euler's formulae for $\cos(2\pi n)$, $\sin(2\pi n)$, and their derivatives.

Problem N1.2

Find the sums of the following geometric series:

a) $\sum_{n=0}^{N} 2^n$

b) $\sum_{n=0}^{\infty} 0.5^n$

Problem N1.3

Find the Nyquist rate for the following signals:

a) $x(t) = 3\cos(150\pi t)$

b) $x(t) = 3\cos(150\pi t) + 2\sin(400\pi t)$

c) $x(t) = 3\sin(100\pi t)\cos(250\pi t)$

Problem N1.4

Express the following signals in terms of impulse signals:

a) $x(n) = nu(n)$

b) $x(n) = u(n+1)$

b) $x(n) = u(n-1)$

Problem N1.5

Which of the following signals are causal?

a) $x(n) = u(n+1)$

b) $x(n) = u(n-1)$

Problem N1.6

Find the following signals for the given: x(n)= $\{1, 2, 1, -3, 5, 8\}$ for n= $\{0, 1, 2, 3, 4, 5\}$

  a) $x(n+1)$

  b) $x(n-1)$

  c) $x(2n)$

  d) $x(0.5n)$

Problem N1.7

Determine whether the following signals are energy or power signals.

  a) $x(n) = u(n), n \geq 0$

  b) $x(n) = nu(n), n \geq 0$

  c) $x(n) = (0.5)^n u(n)$

Problem N1.8

Identify the even or odd signals below.

  a) $x(n) = sin(n)$

  b) $x(n) = cos(n)$

  b) $x(n) = cot(n)$

Problem N1.9

Examine the linearity and time invariance of the following signals.

  a) $y(n) = x(n-1)$

  b) $y(n) = x(n^2)$

  c) $y(n) = x^2(n)$

Problem N1.10

The analog signal $x_a(t) = 3\cos(500\pi t) + 2\sin(1000\pi t)$. Determine the following:

  a) What is the Nyquist rate?

  b) How will the reconstructed signal be with $F_s = 400\,\text{Hz}$, $F_s = 600\,\text{Hz}$, $F_s = 1100\,\text{Hz}$? Please comment on your results.

Impulse Response and Convolution, Chapter 2

Problem 2.1

A time discrete signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3} & -3 \leq n \leq -1 \\ 1 & 0 \leq n \leq 3 \\ 0 & n < -3 \text{ and } n > 3 \end{cases} \tag{1}$$

  a) Draw the signal $x(n)$.

  b) Draw the following alternative signal.

      i) First fold (mirroring around the origin) $x(n)$ and then delay the resulting signal by 4 samples.
      ii) First delay $x(n)$ by 4 samples and then fold the resulting signal.

  c) Draw the signal $x(-n+4)$.

  d) Compare the results in part (b) and (c).

  e) Express the signal $x(n)$ in term of $\delta(n)$ and $u(n)$.

Problem 2.7

Examine the following systems with respect to the properties below.

1) Static or dynamic.

2) Linear or nonlinear.

3) Time invariant or time varying.

4) Causal or non-causal.

5) Stable or unstable.

a) $y(n) = \cos(x(n))$.

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$.

c) $y(n) = x(n)\cos(\omega_0 n)$.

e) $y(n) = \text{trunc}(x(n))$, denotes the integer part of $x(n)$ obtained by rounding.

h) $y(n) = x(n)u(n)$.

j) $y(n) = x(2n)$.

n) An ideal sampling system with input $x_a(t)$ and output $x(n) = x_a(t)$, where $t = nT$ and $-\infty < n < \infty$.

Problem 2.13

a) Show that $\sum_n y(n) = \sum_k x(k) \sum_l h(l)$, where $y(n) = x(n) * h(n)$.

b) Show that the necessary and sufficient conditions for a relaxed LTI system to be BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h .

Problem 2.14

Check if the discrete time systems below are BIBO stable.

1) $y(n) = y(n-1) + x(n)$ for $n > 0$ where y(-1)=0.

4) $y(n) = x(n) + x(n-1) - x(n-4)$.

Problem 2.16

Calculate the convolution $y(n) = x(n) * h(n)$ for the following signals.

1) $x(n) = \left\{ \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{ccc} 1 & 2 & 4 \end{array} \right\}$.

2) $x(n) = \left\{ \begin{array}{ccc} 1 & 2 & -1 \end{array} \right\}$ and $h(n) = x(n)$.

3) $x(n) = \left\{ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{c} 1 \end{array} \right\}$.

4) $x(n) = \left\{ \begin{array}{ccccc} 1 & 1 & \underline{0} & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{cccc} 1 & -2 & -3 & \underline{4} \end{array} \right\}$.

5) $x(n) = 0.5^n u(n)$ and $h(n) = 0.25^n u(n)$.

Problem 2.17

Calculate the convolution $y(n) = x(n) * h(n)$ for the following signals.

a) $x(n) = \left\{ \begin{array}{cccc} \underline{1} & 1 & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{cccccccc} \underline{6} & 5 & 4 & 3 & 2 & 1 & 0 & 0 \end{array} \right\}$

b) $x(n) = \left\{ \begin{array}{cccc} \underline{1} & 1 & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{cccccccc} 6 & 5 & 4 & \underline{3} & 2 & 1 & 0 & 0 \end{array} \right\}$

c) $x(n) = \left\{ \begin{array}{ccccccc} \underline{0} & 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{ccccc} 1 & 1 & 0 & 0 & \underline{0} \end{array} \right\}$

d) $x(n) = \left\{ \begin{array}{cccccc} \underline{0} & 0 & 1 & 1 & 1 & 1 \end{array} \right\}$ and $h(n) = \left\{ \begin{array}{ccc} 1 & 1 & \underline{0} \end{array} \right\}$

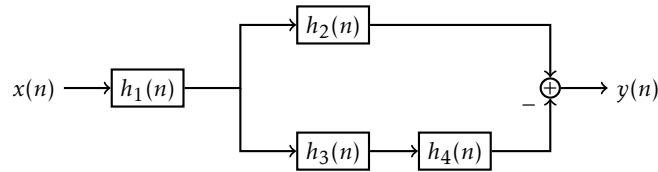**MATLAB:** Calculate the convolutions in MATLAB (conv.m).

Problem 2.21

Calculate the convolution $y(n) = x(n) * h(n)$ for the following signals.

a) $x(n) = a^n u(n)$ and $h(n) = b^n u(n)$ for both $a \neq b$ and $a = b$.

b) $x(n) = \left\{\begin{array}{cccc} 1 & 2 & \underline{1} & 1 \end{array}\right\}$ and $h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$.

Problem 2.35

Consider the interconnection of LTI systems as shown in the following figure.

a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, and $h_4(n)$

b) Determine $h(n)$ when $h_1(n) = \left\{\begin{array}{ccc} 0.5 & 0.25 & 0.5 \end{array}\right\}$, $h_2(n) = h_3(n) = (n+1)u(n)$ and $h_4(n) = \delta(n-2)$.

c) Determine the response of the system in part (b) if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$.

d) Check if the system is BIBO stable.

Problem 2.61

Compute the correlation sequences $r_{xx}(l)$ and cross correlation function $r_{xy}(l)$ for the following signal sequences:

$$x(n) = 1 \quad \text{for } n_0 - N \leq n \leq n_0 + N, \text{ zero otherwise, and} \tag{2}$$

$$y(n) = 1 \quad \text{for } -N \leq n \leq N, \text{ zero otherwise.} \tag{3}$$

Problem 2.62

Calculate the autocorrelation sequences $r_{xx}(l)$ of the following signals.

a) $x(n) = \left\{\begin{array}{cccc} 1 & 2 & 1 & 1 \end{array}\right\}$.

b) $y(n) = \left\{\begin{array}{cccc} 1 & 1 & 2 & 1 \end{array}\right\}$.

What is your conclusion ?

MATLAB: Solve a) and b) above in MATLAB.

Problem 2.64

An audio signal generated by a loudspeaker is reflected at two different walls with the reflection coefficients r_1 and r_2. The signal is recorded by a microphone near the speaker. After sampling, the discrete time signal is $x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$, where k_1 and k_2 are the delays of the echoes. Determine and draw the autocorrelation function $r_{xx}(l)$ for $x(n)$.

The Z Transform, Chapter 3

Problem N3.1

The following first order difference equation is given:

$$y(n) + y(n-1) = x(n) \tag{4}$$

where

$$x(n) = 5u(n) \tag{5}$$

a) Z-transform the equation to express it in the z-domain.

b) Perform an inverse z-transform to express the original equation without $y(n-1)$ and $x(n)$.

Problem N3.2

The following second-order difference equation is given:

$$y(n) - \frac{2}{3}y(n-1) - \frac{1}{3}y(n-2) = x(n) \tag{6}$$

a) Express the equation on the form $Y(z) = H(z)X(z)$ and determine $H(z)$.

b) Determine $h(n)$.

Problem 3.1

Determine the Z-transform of:

a) $x(n) = \left\{\begin{array}{cccccccc} 3 & 0 & 0 & 0 & 0 & \underline{6} & 1 & -4 \end{array}\right\}$

b) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 5 \\ 0 & n \leq 4 \end{cases}$

Problem 3.2

Determine the Z-transform of the signals below, and draw their pole-zero patterns:

a) $x(n) = (1 + n)u(n)$.

c) $x(n) = (-1)^n \cdot 2^{-n} \cdot u(n)$.

f) $x(n) = Ar^n \cos(\omega_0 n + \phi)u(n) \quad 0 < r < 1$.

h) $x(n) = \left(\frac{1}{2}\right)^n \cdot [u(n) - u(n - 10)]$.

Problem 3.8

Use the convolution property of the Z-transform to determine the following Z-transforms:

a) Determine $Y(z)$ expressed in $X(z)$ for $y(n) = \sum_{k=-\infty}^{n} x(k)$.

b) Determine $X(z)$ for $x(n) = (n + 1)u(n)$. Hint: show that $x(n) = u(n) * u(n)$.

Problem 3.9

The Z-transform $X(z)$ of the real signal $x(n)$ includes a pair of complex-conjugate zeros and a pair of complex-conjugate poles. What happens to these pairs if we multiply $x(n)$ by $e^{j\omega_0 n}$?
Hint: Use the scaling theorem in the z-domain.

Problem 3.14

Determine the inverse Z-transform of $x(n)$, $x(n)$ causal, of the following signals:

a) $X(z) = \dfrac{1 + 3z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$

b) $X(z) = \dfrac{1}{1 - z^{-1} + 0.5z^{-2}}$

c) $X(z) = \dfrac{z^{-6} + z^{-7}}{1 - z^{-1}}$

d) $X(z) = \dfrac{1 + 2z^{-2}}{1 + z^{-2}}$

g) $X(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 + 4z^{-1} + 4z^{-2}}$

MATLAB: Plot some of these functions for $z = e^{j\omega}$ using MATLAB.

Problem 3.16

Determine the convolution of $x_1(n)$ and $x_2(n)$ below using the z-transform.

a) $x_1(n) = \left(\frac{1}{4}\right)^n \cdot u(n-1)$ and $x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right]u(n)$.

c) $x_1(n) = 0.5^n u(n)$ and $x_2(n) = \cos(\pi n)u(n)$.

Problem 3.14

Compute the inverse Z-transform $x(n)$, $x(n)$ causal, of the following signals:

a) $X(z) = \dfrac{1 + 3z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$

b) $X(z) = \dfrac{1}{1 - z^{-1} + 0.5z^{-2}}$

c) $X(z) = \dfrac{z^{-6} + z^{-7}}{1 - z^{-1}}$

d) $X(z) = \dfrac{1 + 2z^{-2}}{1 + z^{-2}}$

g) $X(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 + 4z^{-1} + 4z^{-2}}$

MATLAB: Plot some of these functions for $z = e^{j\omega}$ using MATLAB.

Problem 3.16

Compute the convolution between $x_1(n)$ and $x_2(n)$ below using the Z-transform:

a) $x_1(n) = \left(\frac{1}{4}\right)^n \cdot u(n-1)$ and $x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right]u(n)$.

c) $x_1(n) = 0.5^n u(n)$ and $x_2(n) = \cos(\pi n)u(n)$.

Problem 3.35

Compute the output signal $y(n) = x(n) * h(n)$ for:

a) $h(n) = \left(\frac{1}{3}\right)^n u(n)$ and $x(n) = (1/2)^n \cdot \cos(\pi/3n)u(n)$.

d) $y(n) = 0.5x(n) - 0.5x(n-1)$ and $x(n) = 10\cos(\pi/2\, n)\, u(n)$.

Problem 3.40

The input and output signals for an LTI system are given by $x(n) = 0.5^n u(n) - 0.25(0.5)^{n-1} \cdot u(n-1)$ and $y(n) = \left(\frac{1}{3}\right)^n u(n)$.

a) Compute the impulse response $h(n)$ and the system function $H(z)$.

b) Compute the difference equation.

c) Compute a realization of minimal order.

d) Is the system stable?

Problem 3.49

Use the single-sided Z-transform to compute $y(n)$, where $n \geq 0$ for the following cases:

b) $y(n) - 1.5y(n-1) + 0.5y(n-2) = 0$, $y(-1) = 1$, $y(-2) = 0$.

c) $y(n) = 0.5y(n-1) + x(n)$, $x(n) = (1/3)^n u(n)$, $y(-1) = 1$.

d) $y(n) = 0.25(y(n-2) + x(n)$, $x(n) = u(n)$, $y(-1) = 0$, $y(-2) = 1$.

Problem E3.1

Combine each pole zero plot with the corresponding impulse response. How is the impulse response affected by the distance beween the pole and the unit circle? How is the impulse response affected by double poles?

Pole zero A — Impulse response I

Pole zero B — Impulse response II

Pole zero C — Impulse response III
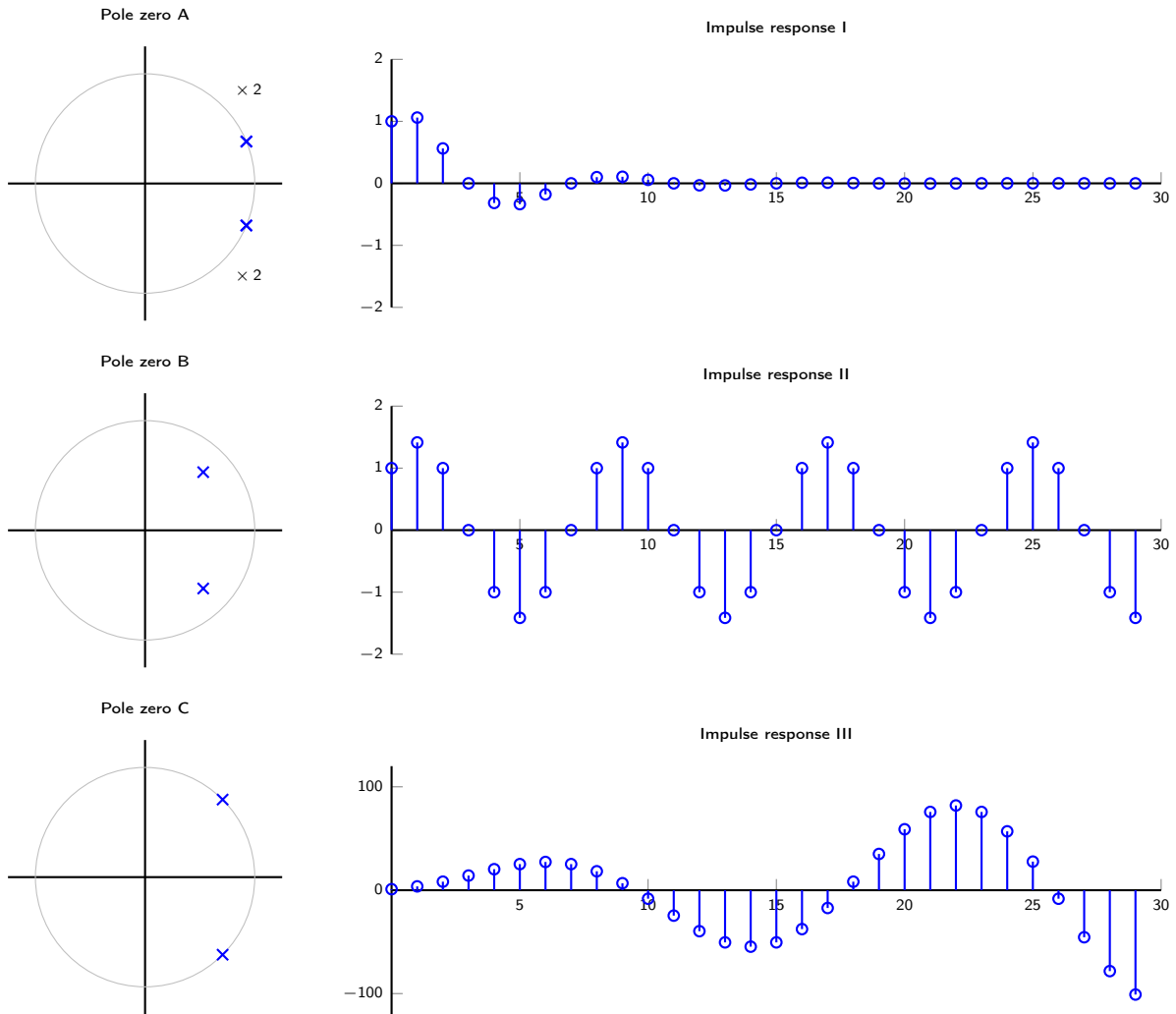
Problem E3.2

A second-order, linear, and time-invariant system is defined by:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{7}$$

a) Compute the output signal $y(n)$ when the input signal is $x(n) = 3\sin\left(\frac{\pi}{2}n\right)u(n)$ where:

$$y(-1) = 1/3 \tag{8}$$

and

$$b_0 = 0 \quad b_1 = 1/5 \quad b_2 = 0 \tag{9}$$

$$a_1 = 1/2 \quad a_2 = 0 \tag{10}$$

b) Let the input signal be $x(n) = \sin(2\pi f_0 n)$ för $-\infty < n < \infty$. The coefficients are chosen so that the output becomes $y(n) = 0$ för $-\infty < n < \infty$. This occurs when

$$b_0 = b_1 = b_2 = 2 \tag{11}$$

and

$$a_1 = -\sqrt{2} \quad a_2 = 1/4 \tag{12}$$

What is the frequency f_0 in the input signal?

c) Again, the input signal is $x(n) = \sin(2\pi f_0 n)$, $-\infty < n < \infty$. When the coefficients are

$$b_0 = 1 \quad b_1 = 1 \quad b_2 = 1/4 \tag{13}$$

and

$$a_1 = -1 \quad a_2 = 1 \tag{14}$$

the output signal becomes $y(n) = A_0 \sin(2\pi f_0 n + \theta_0) + A_1 \sin(2\pi f_1 n + \theta_1)$ för $-\infty < n < \infty$. What is the frequency f_1?

Problem E3.3

An LTI system is described by the difference equation

$$y(n) - y(n-1) + \frac{3}{16} y(n-2) = x(n) \tag{15}$$

Compute the output signal $y(n)$ when

$$x(n) = \left(\frac{1}{2}\right)^n \cdot u(n) + \sin\left(2\pi \frac{1}{4} n\right) \quad -\infty < n < \infty \tag{16}$$

Problem E3.4

A system is described by the difference equation $y(n) - y(n-1) + 0.5y(n-2) = x(n)$. The initial values are $y(-1) = 0$ and $y(-2) = 2$, and the input signal is $x(n) = u(n)$. When $n \geq 0$ compute

a) the zero input solution, $y_{zi}(n)$.

b) the zero state solution, $y_{zs}(n)$.

c) the transient solution, $y_{tr}(n)$.

d) the stationary solution, $y_{ss}(n)$.

Problem E3.5

A system is described by the difference equation

$$y(n) - \frac{1}{4} y(n-1) = x(n) \tag{17}$$

Compute the output signal $y(n)$ when

$$x(n) = \begin{cases} \sin\left(2\pi \frac{1}{4} n\right) & n < 0 \\ 0 & n \geq 0 \end{cases} \tag{18}$$

The Fourier Transform, LTI Systems, and Sampling, Chapter 4, 5, and 6

Problem N4.1

The transfer function of a system is given as

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})}$$

Find the output signal $y(n)$ if the input signal is given by

$$x(n) = u(n-1) + sin(2\pi \frac{1}{4} n + \frac{\pi}{4})$$

Problem N4.2

The transfer function of a system is given as

$$H(z) = \frac{1 - 0.5z^{-1}}{1 - z^{-1}} \tag{19}$$

Find the output signal if the input signal is given by

$$x(n) = 0.5^{n-1} u(n-1) + sin(2\pi \frac{1}{3} n) \tag{20}$$

Problem 4.8

Two discrete signals $s_k(n)$ and $s_l(n)$ are orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} s_k(n)s_l^*(n) = \begin{cases} A_k & k = l \\ 0 & k \neq l \end{cases} \tag{21}$$

If $A_k = 1$ the signal are orthonormal.

a) Show the relation

$$\sum_{n=0}^{N-1} e^{j2\pi \frac{kn}{N}} = \begin{cases} N & k = 0 \quad k = \pm N \quad k = \pm 2N \quad \dots \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

b) Illustrate a) for $N = 6$ by drawing $s_k(n) = e^{j2\pi \frac{kn}{N}}$ for $k = 1, 2, 3, 4, 5, 6$ and $n = 0, 1, 2, 3, 4, 5$.

c) Show that harmonic signals $s(n) = e^{j2\pi \frac{kn}{N}}$ arer orthogonal over any interval with length N.

Problem 4.9

Compute the Fourier transforms of the following signals

a) $x(n) = u(n) - u(n-6)$.

b) $x(n) = 2^n u(-n)$.

c) $x(n) = (0.25)^n u(n+4)$.

d) $x(n) = (\alpha^n \sin(\omega_0 n)) u(n)$ för $|\alpha| < 1$.

g) $x(n) = \left\{ \begin{matrix} -2 & -1 & \underline{0} & 1 & 2 \end{matrix} \right\}$.

Problem 4.10

Compute the time-domain signal of the Fourier transform

a) $X(\omega) = \begin{cases} 0 & 0 \leq \omega \leq \omega_0 \\ 1 & \omega_0 < \omega \leq \pi \end{cases}$

b) $X(\omega) = \cos^2 \omega$

The Fourier transforms are symmetric for negative frequencies so that the signals have real values.

Problem 4.12

Compute the time-domain signal of the Fourier transform

c) $X(\omega) = \begin{cases} 2 & \omega_c - W/2 \leq |\omega| \leq \omega_c + W/2 \\ 0 & \text{otherwise} \end{cases}$

Problem 4.14

The sequence $x(n)$ is given by $x(n) = \left\{ \begin{matrix} -1 & 2 & \underline{-3} & 2 & -1 \end{matrix} \right\}$ and its Fourier transform is $X(\omega)$. Compute the following, without calculating $X(\omega)$ explicitly

a) $X(0)$

b) $\arg X(\omega)$

c) $\int_{-\pi}^{\pi} X(\omega)d\omega$

d) $X(\pi)$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

MATLAB: Compute $X(\omega)$ and plot $|X(\omega)|$ and $\arg X(\omega)$.

Problem 5.2

a) Compute and draw the Fourier transform $W_R(\omega)$ for a rectangular window

$$w_R(n) = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

b) Compute and draw the Fourier transform for a triangular window $w_T(n)$

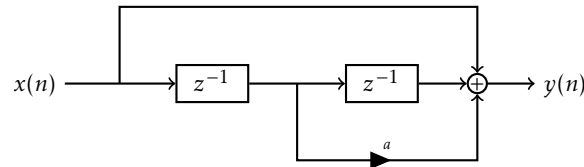$$w_T(n) = \begin{cases} n & 0 \le n \le M/2 \\ M - n & M/2 < n \le M \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

by using the convolution of two rectangular windows as in a.

MATLAB: Plot $W_R(\omega)$ for $M = 2$, $M = 4$, and $M = 5$.

Problem 5.17

A time-discrete circuit is given by the figure with $a = -2\cos(\omega_0)$

a) Compute the impulse response $h(n)$.

b) Draw $|H(\omega)|$ and $\arg H(\omega)$.

c) Compute $y(n)$ when $x(n) = 3\cos(\pi/3\, n + \pi/6)$ for $-\infty < n < \infty$ and $\omega_0 = \pi/2$.

Problem 5.25

Draw $|X(f)|$ corresponding to the following pole-zero configurations.
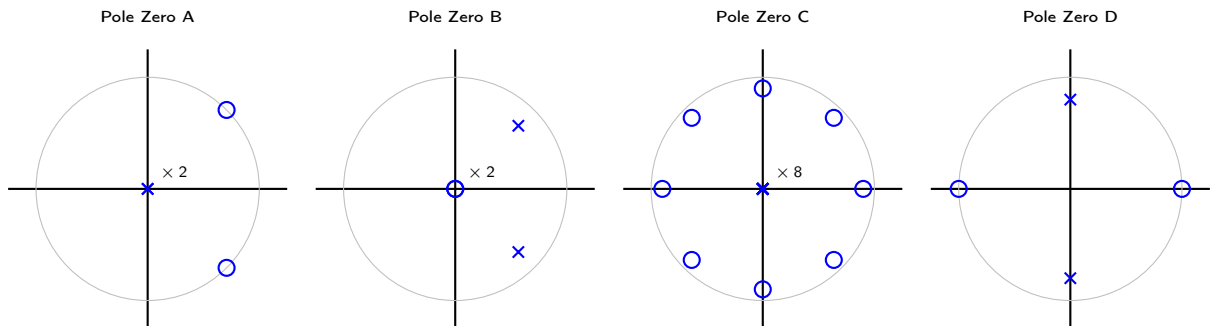
Problem 5.26

Design a filter that blocks the frequency $\omega_0 = \pi/4$ and compute the output signal when the input signal is $x(n) = \sin(\pi/4\, n)u(n)$ for $n = 0, 1, 2, 3, 4$.

Problem 5.35

A second-order system has a double pole at $p_{1,2} = 0.5$ and two zeros at $z_{1,2} = e^{\pm j3\pi/4}$. Compute the gain so that $|H(0)| = 1$.

Problem 5.39

Compute the 3 dB bandwidth for the filters where $0 < a < 1$.
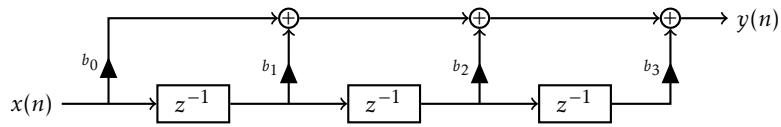
$$H_1(z) = \frac{1-a}{1-az^{-1}} \tag{25}$$

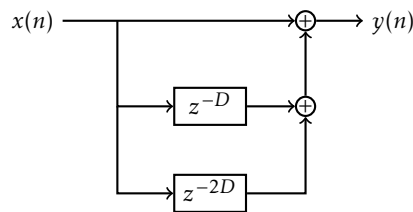$$H_2(z) = \frac{1-a}{2} \cdot \frac{1+z^{-1}}{1-az^{-1}} \tag{26}$$

$$\tag{27}$$

Problem E4.1

Design the circuit below so that the constant amplification is 1 and the frequencies $\omega = \pi/2$ and $\omega = \pi$ are blocked. Compute the amplitude and phase functions.
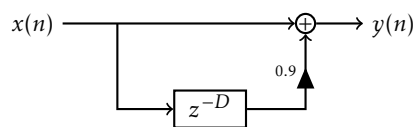

Problem E4.2

To create an echo effect, we use the circuit below.

a) Compute the poles and zeros for $D = 500$.

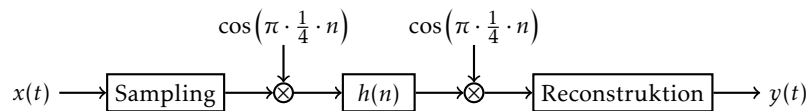b) Compute and draw the amplitude function $|H(f)|$.

Problem E4.3

To create an echo effect, we use the circuit below.

a) Compute the impulse response.

b) Compute the amplitude function $|H(f)|$.

c) Compute the poles and zeros for $D = 500$.

Problem E4.4

The input signal to the system below is $x(t) = \cos(2\pi 1000t) + \cos(2\pi 6000t)$. The impulse response is $h(n) = u(n) - u(n-8)$. The sampling rate is 8 kHz.

Compute the output signal $y(t)$ for ideal reconstruction.

Problem N4.3

During a vocal recording, a poorly insulated wire picked up interference from the 50 Hz power line, resulting in an unwanted buzzing tone in the final recording. The recording used a 44100 Hz sample rate.

a) Design a notch filter with a single zero- and pole-pair that would reduce the buzzing tone as much as possible. (Assume that a pole radius of 0.9 gives adequate performance. A polar form-description of the filter is sufficient.)

b) A sharp "S"-sound with a frequency of 3000 Hz was also noticed in the final recording. Add a single zero- and pole-pair to the filter from **a)** to reduce it as much as possible. (Assume that a pole radius of 0.9 gives adequate performance. A polar form-description of the filter is sufficient.)
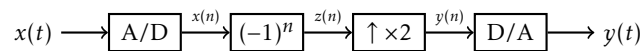
Sampling

Problem E4.5

Assume a continuous signal $x_a(t) = e^{-10 \cdot t} \cdot u(t)$, t in seconds.

a) Compute the Fourier transform $X_a(F)$ and $|X_a(F)|^2$.

b) We sample the signal with $F_s = 100\,\text{Hz}$. Before the sampling, we filter it with an anti-aliasing filter. Assume that this is an ideal low pass filter with a cutoff frequency of $F_s/2 = 50\,\text{Hz}$. How much of the energy in the signal $x_a(t)$ is blocked by the filter?

c) The filtered signal is sampled with F_s. Compute the absolute value of the Fourier transform of the sampled signal $y(n)$. Also compute the Fourier transform of the sampled signal without the anti-aliasing filter.

Problem E4.6

Compute the output signal from the circuit below when the input signal is $x_a(t) = 2 \cdot \cos(2\pi \cdot F_0 t)$ då $F_0 = 600\,\text{Hz}$ and $F_s = 1000\,\text{Hz}$.

$$x(t) \longrightarrow \boxed{\text{A/D}} \xrightarrow{x(n)} \boxed{(-1)^n} \xrightarrow{z(n)} \boxed{\uparrow \times 2} \xrightarrow{y(n)} \boxed{\text{D/A}} \longrightarrow y(t)$$

The sign change performs the operation $x_1(n) = (-1)^n x(n)$ and the collector

$$y(n) = \begin{cases} z\left(\frac{n}{2}\right) & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \tag{28}$$

The reconstruction is ideal with the sampling rate $2F_s$.

The Discrete Fourier Transform DFT, Chapter 7

Problem 7.1

The first five values in an eight-point DFT of a real sequence are given by

$$\left\{ \begin{matrix} 0.25 & 0.125 - j0.3018 & 0 & 0.125 - j0.0518 & 0 \end{matrix} \right\} \tag{29}$$

Compute the remaining three points.

MATLAB: Check the result in MATLAB.

Problem 7.2

Compute the circular convolution between the following signals $(N = 8)$

a) $x_1 = \left\{ \begin{matrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{matrix} \right\}$ and $x_2 = \sin(\frac{3\pi}{8} n)$ for $0 \le n \le N - 1$.

Problem 7.3

Given an N-point DFT $X(k)$, $0 \le k \le N - 1$ of $x(n)$, $\le n \le N - 1$. Define

$$X_0(k) = \begin{cases} X(k) & 0 \le k \le k_c \quad N - k_c \le k \le N - 1 \\ 0 & k_c < k < N - k_c \end{cases} \tag{30}$$

Compare $x_0(n) = \text{IDFT}\{X_0(k)\}$ with $x(n)$. Vhat happens?

Problem 7.4

The sequences $x_1(n) = \cos\frac{2\pi}{N} n$ and $x_2(n) = \sin\frac{2\pi}{N} n$ are given for $0 \le n \le N - 1$.

a) Compute the circular convolution between $x_1(n)$ and $x_2(n)$.

b) Compute the circular correlation between $x_1(n)$ och $x_2(n)$.

c) Compute the circular auto-correlation for för $x_1(n)$.

d) Compute the circular auto-correlation for för $x_2(n)$.

Problem 7.7

The sequence $x(n)$, $0 \le n \le N-1$, has the DFT $X(k)$. Compute the DFTs of the following sequences, using terms of $X(k)$

1) $x_c(n) = x(n)\cos\left(2\pi \cdot \frac{k_0}{N} \cdot n\right)$.

2) $x_s(n) = x(n)\sin\left(2\pi \cdot \frac{k_0}{N} \cdot n\right)$.

Problem 7.8

Compute the circular convolution between $x_1(n) = \left\{\begin{array}{cccc} \underline{1} & 2 & 3 & 1 \end{array}\right\}$ and $x_2(n) = \left\{\begin{array}{cccc} \underline{4} & 3 & 2 & 2 \end{array}\right\}$. Compare with the linear convolution.

Problem 7.9

Compute the circular convolution between $x_1(n) = \left\{\begin{array}{cccc} \underline{1} & 2 & 3 & 1 \end{array}\right\}$ and $x_2(n) = \left\{\begin{array}{cccc} \underline{4} & 3 & 2 & 2 \end{array}\right\}$ by using DFT and IDFT.

Problem 7.10

Compute the energy for the sequence $x(n) = \cos\left(2\pi \cdot \frac{k}{N} \cdot n\right)$ for $0 \le n \le N-1$, where k is an integer.

MATLAB: Compute the energy in MATLAB by using `fft`.

Problem 7.11

The sequence $x(n) = \left\{\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}\right\}$ has the DFT $X(k)$. Compute the DFTs of the following sequences:

a) $\left\{\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right\}$.

b) $\left\{\begin{array}{cccccccc} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right\}$.

Problem 7.18

The input signal to an LTI system $H(\omega)$ is $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-kN)$. We compute an N-point DFT for for the output signal for the points $0 \le n \le N-1$. Compute the connection between $Y(k)$ and $H(\omega)$.

Problem 7.23

Compute the N-point DFT for

a) $x(n) = \delta(n)$

b) $x(n) = \delta(n-n_0)$, $0 < n_0 < N$

c) $x(n) = a^n$, $0 \le n \le N-1$

d) $x(n) = \begin{cases} 1 & 0 \le n \le N/2-1 \quad N \text{ even} \\ 0 & N/2 \le n \le N-1 \end{cases}$

e) $x(n) = e^{j2\pi/N \, k_0 n}$, $0 \le n \le N-1$

f) $x(n) = \cos(2\pi/N \, k_0 \, n)$, $0 \le n \le N-1$

g) $x(n) = \sin(2\pi/N \, k_0 \, n)$, $0 \le n \le N-1$

h) $x(n) = \begin{cases} 1 & n \quad \text{even} \\ 0 & n \quad \text{odd} \quad 0 \le n \le N-1 \end{cases}$

Problem 7.24

Given $x(n) = \left\{\begin{array}{cccc} 1 & 2 & 3 & 1 \end{array}\right\}$. Compute the 4-point DFT of $x(n)$ by solving a 4th-order equation system for the inverse DFT.

Problem 7.25

a) Compute the Fourier transform of $x(n) = \{\ 1\quad 2\quad \underline{3}\quad 2\quad 1\quad 0\ \}$.

b) Compute the six-point DFT of $v(n) = \{\ 3\quad 2\quad 1\quad 0\quad 1\quad 2\ \}$.

c) Compare $X(k)$ and $V(k)$ i a) och b).

Problem N5.0

Solve the following convolution using the overlap-and-add convolution method.

$x(n) = [3,1,-1,4,5,7,6,2,1,0,3,4]$
$h(n) = [1,2,0]$

Problem N5.1

Solve the following convolution using the overlap-and-add convolution method.

$x(n) = [1,1,-1,-5,3,0,1,3,-1,0]$
$h(n) = [0,-1,2,4]$

Problem E5.1

Compute and draw $|X(k)|$ where $X(k)$ is the eight-point DFT of $x(n)$ with $x(n) = \{\ 1\quad 1\quad 1\quad 0\quad 0\quad 0\quad 0\quad 1\ \}$

Problem E5.2

Let $X(k)$ be the eight-point DFT of $x(n) = \{\ 0\quad 1\quad 1\quad 3\quad 8\quad 7\quad 2\quad 2\ \}$.

a) Compute $y(n) = \text{IDFT}\{X^*(k)\}$.

b) Compute $y(n) = \text{IDFT}\{(-1)^k \cdot X(k)\}$.

MATLAB: Check the calculations in MATLAB with `fft` and `ifft`. Let $x(n)$ be a sound signal and listen to $y(n)$ in a) and b).

Problem E5.3

The input signal $x(n)$ to the system $y(n) = ay(n-1) + x(n)$ where $0 < a < 1$ is periodical with the period N. Compute the impulse response for an FIR filter, which gives the same stationary solution as the system above with the input signal $x(n)$.

Problem E5.4

The student Hans is deeply interested in signal processing and finds a tuning fork. Sadly, there is no information about what pitch the tuning fork has. Hans decides to use a spectral analyzer. This machine is digital and performs a DFT on the input signal. Hans sets the spectral analyzer from 0 Hz to 200 Hz (sampling rate 400 Hz) and sees a top at the frequency 138 Hz. Which values can the pitch of the tuning fork have? Give a motivation for your answer.

Problem E5.5

Below are four sequences $x_i(n)$ (1–4) and eight sequences $X_j(k)$ (a–h). Choose the correct pairs $x(n) \longleftrightarrow X(k)$.

$1\ \{\ 1+j\quad 0\quad 0\quad 0\ \}$

$2\ \{\ 0.5\quad 0.5\quad 0.5\quad -0.5\ \}$

$3\ \{\ 1+j\quad j\quad 0\quad 1\ \}$

$4\ \{\ 0.75\quad 0.25\quad -0.25\quad 0.25\ \}$

$a\ \{\ 1\quad 1\quad 0\quad 0\ \}$

$b\ \{\ j\quad 0\quad j\quad 0\ \}$

$c\ \{\ 1+j\quad 1+j\quad 1+j\quad 1+j\ \}$

$d\ \{\ j\quad 0\quad -j\quad 0\ \}$

$e\ \{\ 1\quad -j\quad j\quad 1\ \}$

$f\ \{\ 1\quad -j\quad 1\quad j\ \}$

$g\ \{\ 2+2j\quad 2+2j\quad 0\quad 0\ \}$

$h\ \{\ 1\quad 1\quad 0\quad 1\ \}$

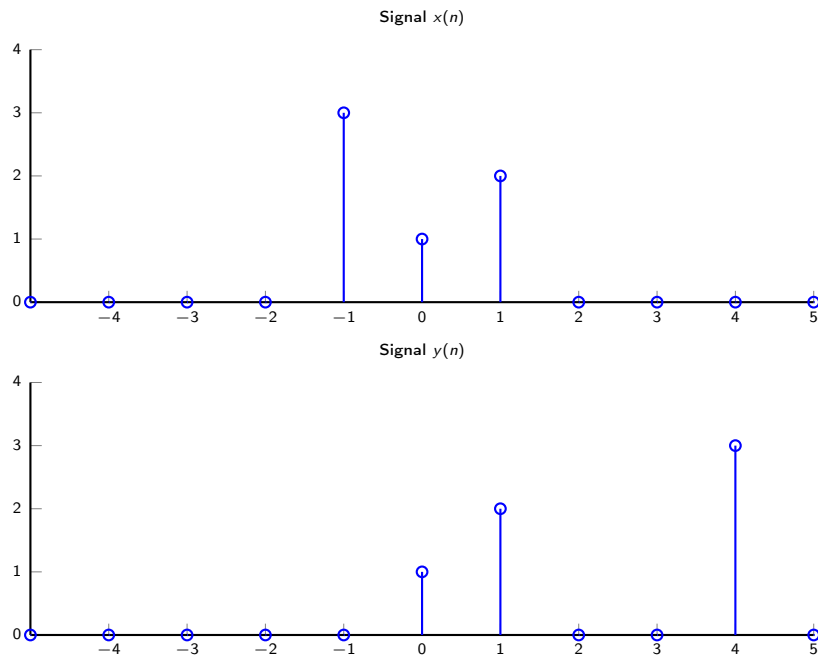**MATLAB:** Use MATLAB to check the result.

Problem E5.6

Given the impulse response

$$h(n) = \frac{1}{4}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{4}\delta(n-2) + \frac{1}{4}\delta(n-3) \tag{31}$$

a) Compute the Fourier transform. Compute the DFT for $N \geq 4$. Connection?

b) What is $H(k)$ for $k = 0\dots3$ when $N = 4$? What is $h_p(n) = \text{IDFT}\{H(k)\}$?

Problem E5.7

The following two signals are given

Signal $x(n)$

Signal $y(n)$

The Fourier transforms of the signals are sampled in the points $f = k/N$ för $k = 0\dots N-1$ where $N = 5$. How is $Y(k/N)$ related to $X(k/N)$?

Problem E5.8

The impulse response to an LTI system is

$$h(n) = \delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2) \tag{32}$$

and the input signal is

$$x(n) = \delta(n) + \delta(n-1) \tag{33}$$

a) Compute the output signal with convolution.

b) Compute the output signal with the Fourier transform.

c) Let $f = k/N$ för $k = 0\dots N-1$ in the expression for $Y(f)$. If N is less than a certain number M there is aliasing in the time domain. Conclusion? How large is M? How large is M in the general case, when the impulse response has length P and the input signal has length Q?
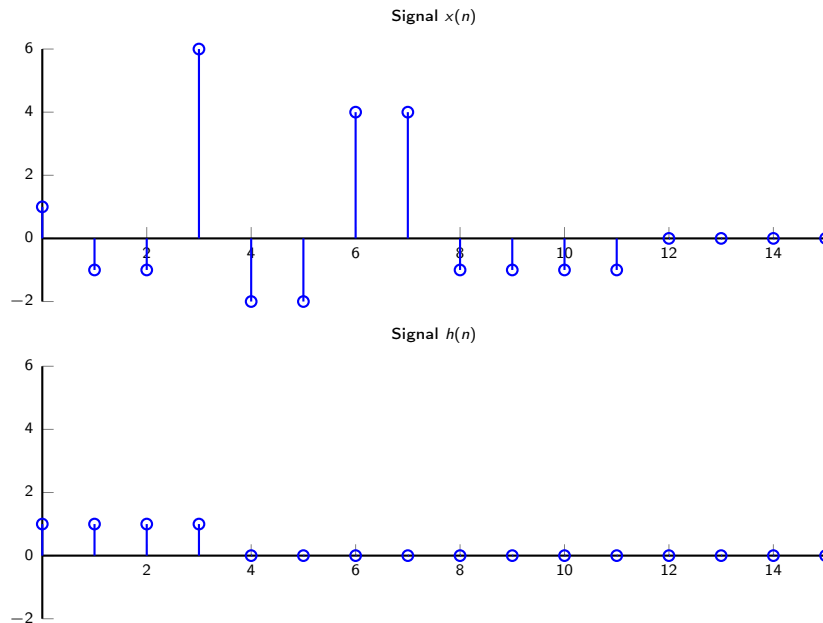
Problem E5.9

The sequence $h(n)$ is the impulse response to a transversal filter of order L with the coefficients b_k, $k = 0\dots L$. The signal $x(n)$ is a truncated step function with length Q, that is,

$$x(n) = \begin{cases} 1 & n = 0\dots Q-1 \quad Q > L+1 \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

We want to compute the output signal from the filter, but mistakenly use the DFT length $N = Q$ instead of $N \geq Q + P - 1 = Q + L$, where P is the length of $h(n)$). Given this choice of N compute the erroneous output signal $y_p(n)$, $n = 0\dots N-1$.

Problem E5.10

The signals $x(n)$ and $h(n)$ in the figures are the input signals and the impulse responses to an LTI system. We want to compute the output signal $y(n) = h(n) * x(n)$ with an eight-point DFT. This can be done by splitting the input signals in segments of length four. Do this and compute each output signal segment with a convolution.

Signal $x(n)$

Signal $h(n)$

Problem E5.11

We are given an infinite sequence $x(n) = (\frac{1}{2})^n u(n)$ with the Fourier transform $X(f)$. From this, we form a finite sequence $y(n)$ so that $y(n) = 0$ where $k < 0$ and $k > 9$ by computing

$$y(n) = \text{IDFT}\{Y(k)\} \quad n = 0 \ldots 9 \tag{35}$$

where

$$Y(k) = X(f)|_{f=\frac{k}{10}} \quad k = 0 \ldots 9 \tag{36}$$

Compute $y(n)$.

Problem E5.12

It is possible to compute filters with the overlap-and-save-method. The input signal $x(n)$ is split in segments with the length N

$$x_i(n) = x(n + i(N - M + 1)) \quad 0 \le n \le N - 1 \tag{37}$$

where M is the length of the filter's impulse response. For each segment we calculate

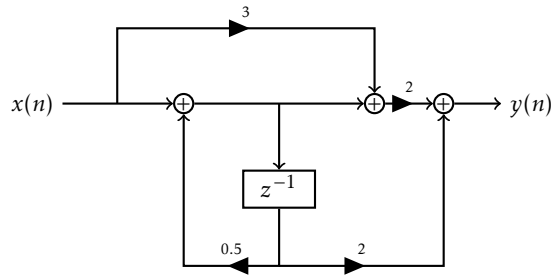$$X_i(k) = \text{DFT}\{x_i(n)\} \tag{38}$$

and

$$y_i(n) = \text{IDFT}\{X_i(k) H(k)\} \tag{39}$$

where $H(k)$ is the DFT $\{h(n)\}$. Decide how $y_i(n)$, $i = 0, 1, 2\ldots$ shoudl be combined to give $y(n) = x(n) * h(n)$. Show this in a figure.

Realizations, Chapter 9

Problem 9.3

Compute the system function and impulse response for the following circuit:

Problem 9.9

Compute realizations in Direct Form I, Direct Form II, cascade form, and parallel form for the systems below.

a) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$.

f) $y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$.

Which of the systems are stable?

Problem 9.15

Compute the parameters K_m for a lattice FIR filter with the equation

$$H(z) = A_2(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2} \tag{40}$$

MATLAB: Compute the parameters with MATLAB.

Problem 9.19

a) Compute and draw the zeros for the lattice FIR filter with the parameters $K_1 = \frac{1}{2}$, $K_2 = -\frac{1}{3}$, and $K_3 = 1$.

b) Same as in a) with $K_3 = -1$.

c) In a) and b) all zeros are on the unit circle. Can this result be generalized? How?

d) Compute the phase function for the filters in a) and b). Conclusion?

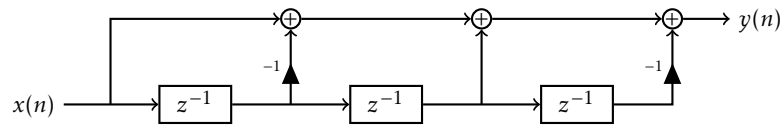## Examples of Filter Design

Problem E8.1

Given the FIR filter below.
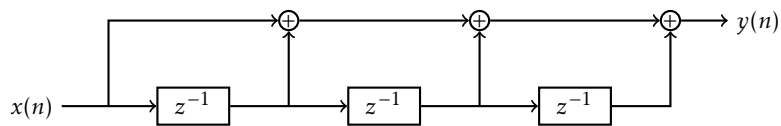
a) Compute poles and zeros.

b) Compute and draw the amplitude function $|H(f)|$.

c) Compute and draw the phase function.

d) At what frequencies are $|H(f)| = 0$?

e) What type of filter (LP, BP, HP) is it?

Problem E8.2

Given the FIR filter below.

a) Compute poles and zeros.

b) Compute and draw the amplitude function $|H(f)|$.

c) Compute and draw the phase function.

d) At what frequencies are $|H(f)| = 0$?

e) What type of filter (LP, BP, HP) is it?

Problem E8.3

We want to filter a signal by creating an output signal as the moving average of the five last values, like this:

$$y(n) = \frac{1}{5} \sum x(k) = 0.2\left[x(n-4) + x(n-3) + x(n-2) + x(n-1) + x(n)\right] \tag{41}$$

a) Compute the impulse response $h(n)$.

b) Compute the amplitude function $|H(f)|$.

c) Draw $|H(f)|$ for $0 \leq f \leq 1$.

Problem E8.4

Using the signal $x(n) = \sin(\pi/2n)$ you want to create the signal $y(n) = \sin(\pi/2n + \pi/3)$. Design a circuit that accomplishes this. Try with a FIR filter.

Problem E8.5

Compute the impulse response $h(n)$ for a circuit with the following demands:

- $|H(0)| = |H(-1/5)| = |H(1/5)| = 1$

- $H(2/5) = H(-2/5) = 0$

- $h(n)$ is real

- $h(n)$ is causal

- the circuit has linear phase

MATLAB: Plot the spectrum in MATLAB and see if the demands are fulfilled.

Problem E8.6

A filter $H(z)$ is given by N poles in origo and N zeros in $z = -1$. The amplification for a stationary signal is 1 (0 dB). Compute the values of N for which the damping demands in the figure are fulfilled and compute the impulse response for the minimal value of N.

MATLAB: Plot the spectrum in MATLAB and see if the demands are fulfilled.
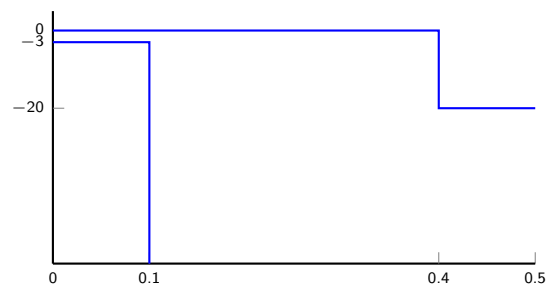
An Example with a FIR Filter and the Windowing Method

Design a FIR filter with the damping demand below using the windowing method and the equi-ripple method.
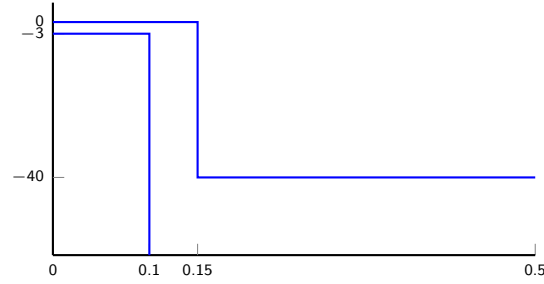
First select a window for the FIR filter. Hamming has the largest side lobe, $-55\,\mathrm{dB}$. Thus, we choose a Hamming window. An approximate value for M is found in Table 5.2:

$$\frac{1.7}{M} \approx 0.15 - 0.1 \quad \Rightarrow \quad M = 34 \tag{42}$$

Note that Table 5.2 defines the lower point at $-6\,\mathrm{dB}$, which means that the correct value is somewhat larger. A better value is seen in Figure 36. At the lower limit $f = 0.1$, $|H(f)| = -3\,\mathrm{dB}$, we get

$$x = -0.4 = (f - f_c) \cdot M = (0.1 - f_c) \cdot M \tag{43}$$

and at $f = 0.15$, $|H(f)| = -40\,\mathrm{dB}$ we get

$$x = 1.5 = (f - f_c) \cdot M = (0.15 - f_c) \cdot M \tag{44}$$

If we solve for f_c and M we get

$$f_c = 0.110 \tag{45}$$

$$M = 38 \tag{46}$$

Now let us use the equi-ripple method. The pass band ripple is

$$20\log \frac{1 + \delta_p}{1 - \delta_p} = 3 \quad \Rightarrow \quad \frac{1 + \delta_p}{1 - \delta_p} = 10^{0.15} = 1.41 \quad \Rightarrow \quad \delta_p = \frac{0.41}{2.41} = 0.17 \tag{47}$$

and

$$20\log \delta_s = -40 \quad \Rightarrow \quad \delta_s = 0.01 \tag{48}$$

$$\begin{cases} D_\infty = \dfrac{-20log\sqrt{0.17 \cdot 0.01} - 13}{14.6} = 1.0 \\ \Delta f = 0.15 - 0.1 = 0.05 \end{cases} \quad \Rightarrow \quad N = \frac{1.0}{0.05} + 1 = 21 \tag{49}$$

The equi-ripple filter gives the lower degree with the band-block damping $40\,\mathrm{dB}$ in the entire blocked band, while the FIR filter with a Hamming window has a larger side lobe, with a damping of around $55\,\mathrm{dB}$.
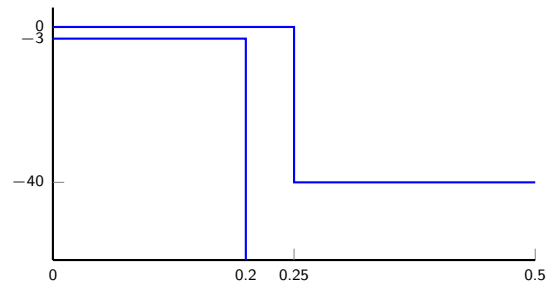
Problem E8.7

Compute the impulse response for a realizable filter fulfilling the demands below. The filter should have as low order as possible and exact linear phase.
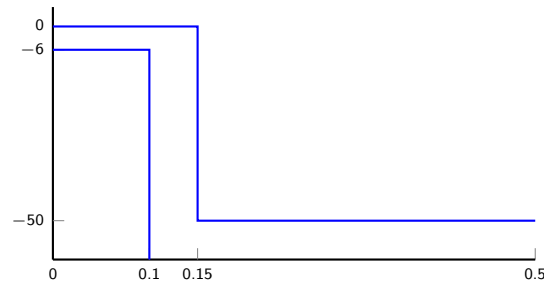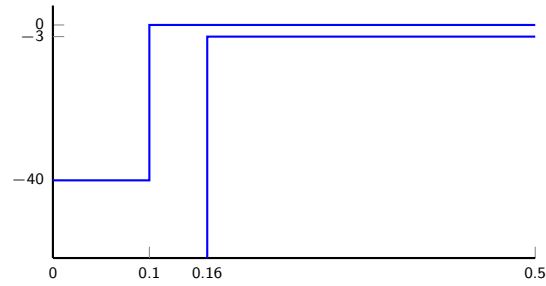
Problem E8.8

Design a FIR low pass filter fulfilling the demads given in the figure $H_d(f)$ with the cutoff frequency f_c. Use a Hamming window, which should give a band damping of at least 40 dB. Compute f_c to fulfill the demands with a minimal M (odd). Design a realization.

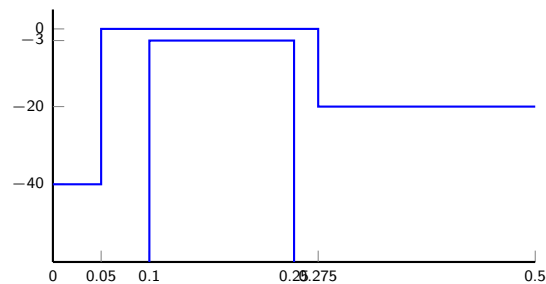**MATLAB:** Plot the spectrum in MATLAB and see if the demands are fulfilled.

Problem E8.9

Design a high pass filter according to the figure below. At the frequency 0.16 the damping **must** be 3dB. Design the impulse response for a FIR filter that fulfills the demands.

Problem E8.10

Design a band pass filter according to the figure below. At the frequencies 0.1 and 0.25 the damping **must** be 3 dB. Design the impulse response for a FIR filter that fulfills the demands.

MATLAB: Plot the spectrum in MATLAB and see if the demands are fulfilled.

Problem E8.11

The bandwidth for a band pass filter is defined as the difference in frequency between the two points, where the amplitude is $|H(\omega)| = 1/\sqrt{2}$. Compute the bandwidth for an FIR filter with the impulse response

$$h(n) = \left(0.4933 \cdot \frac{\sin(0.2466\pi(n-20))}{0.2466\pi(n-20)} \cdot \cos\left(\pi 0.4192(n-20)\right)\right) \tag{50}$$

$$\cdot \left(0.54 + 0.46 \cdot \cos\left(\frac{2\pi(n-20)}{40}\right)\right) \tag{51}$$

för $0 \leq n \leq 40$.

Problem E8.12

Two FIR filters are coupled in a cascade $H(z) = H_1(z) \cdot H_2(z)$ where

$$H_1(z) = 1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2} \tag{52}$$

Compute $H_2(z)$ so that the entire system has linear phase and the stationary amplification $|H(0)| = (1 - 2r\cos(\theta) + r^2)^2$. Give $\arg H(f)$ for $0 \le f \le 1/2$.

Problem E8.13

We wish to study a band pass signal with components in 2.5 MHz–3 MHz. The signal is sampled the the frequency $F_s = 10$ MHz and AD converted. However, the signal has generated overtones in 5 MHz–6 MHz and it has added disturbances in 9 MHz–10 MHz. We want to suppress the disturbances with a FIR filter with a damping of at least 40 dB. The filter should not affect the signal with more than 3 dB.