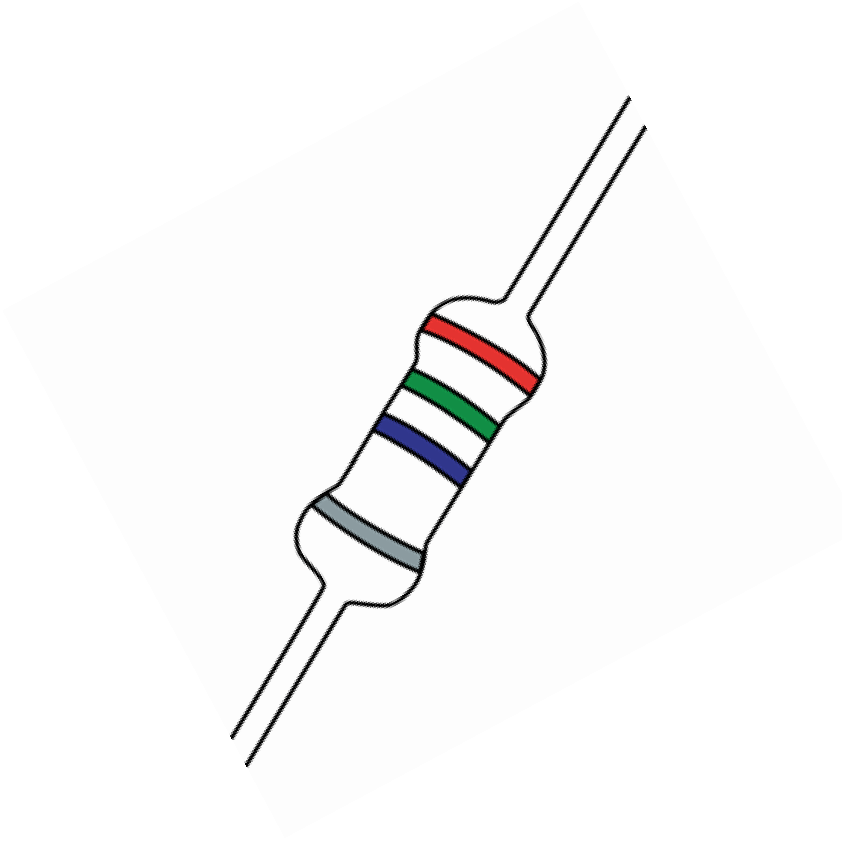
Föreläsning 6

Tidsberoende Signaler – växelström (AC)

Komplexa Tal

(SEM: Nodanalys – om tid!)

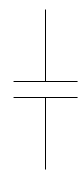


Senaste föreläsning

- Resistanser

$$v(t) = i(t)R$$

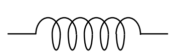
- Kondensatorer – Kapacitans



$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int i(t')dt'$$

- Spolar – Induktans



$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int v(t')dt'$$

Tidsberoende Signaler – sin(ωt)

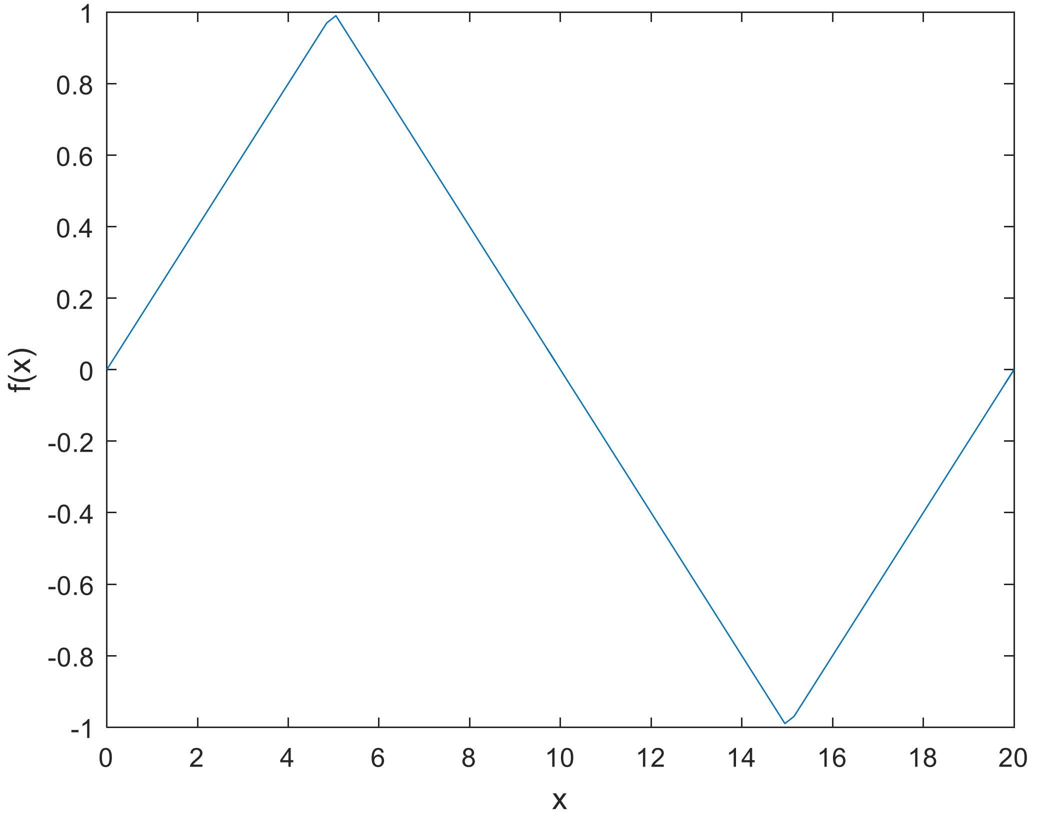
$$v(t) = V_0 \sin(2\pi f t + \phi)$$

Växelström/spänning - eluttag
f=50 Hz
V_0=325V (toppvärde)

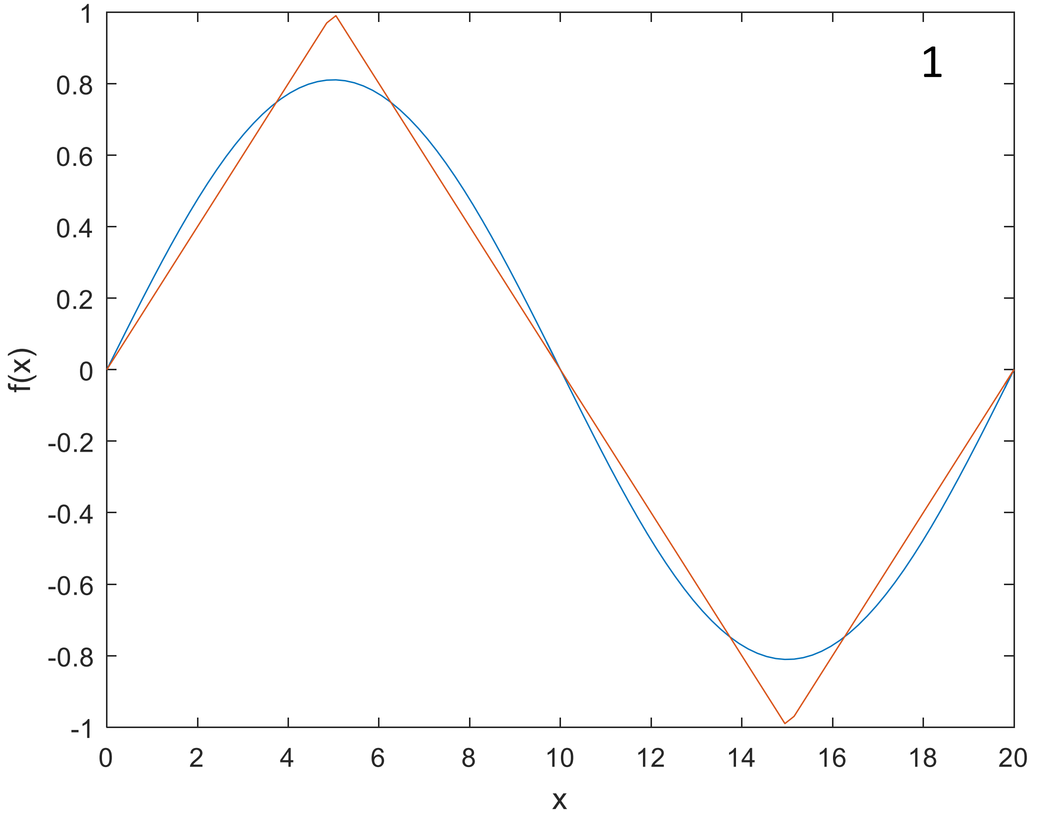
Fourieranalys – alla fysiskt realiserbara signaler kan skrivas som en summa av sinus och cosinus-termer (!!!)

$$v(t) = \sum_{n=0} A_n \sin(n \cdot \omega_0 t) + B_n \cos(n \cdot \omega_0 t)$$

V(t) - triangelvåg

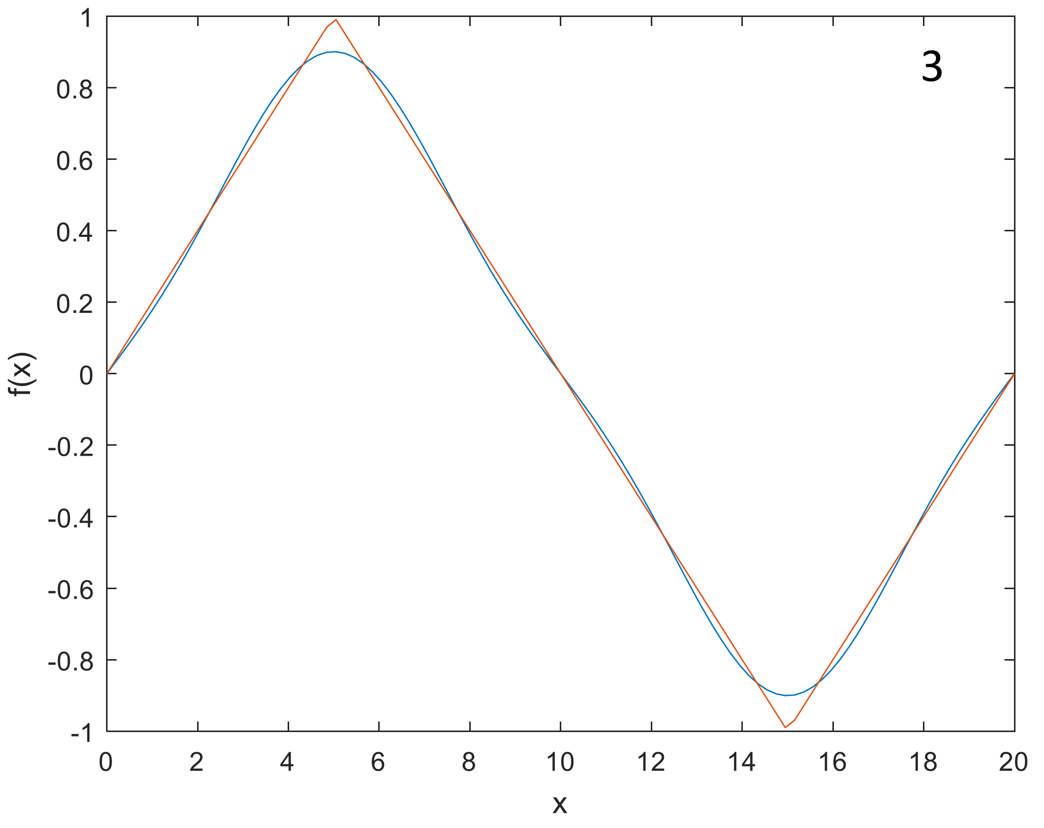


V(t) - triangelvåg



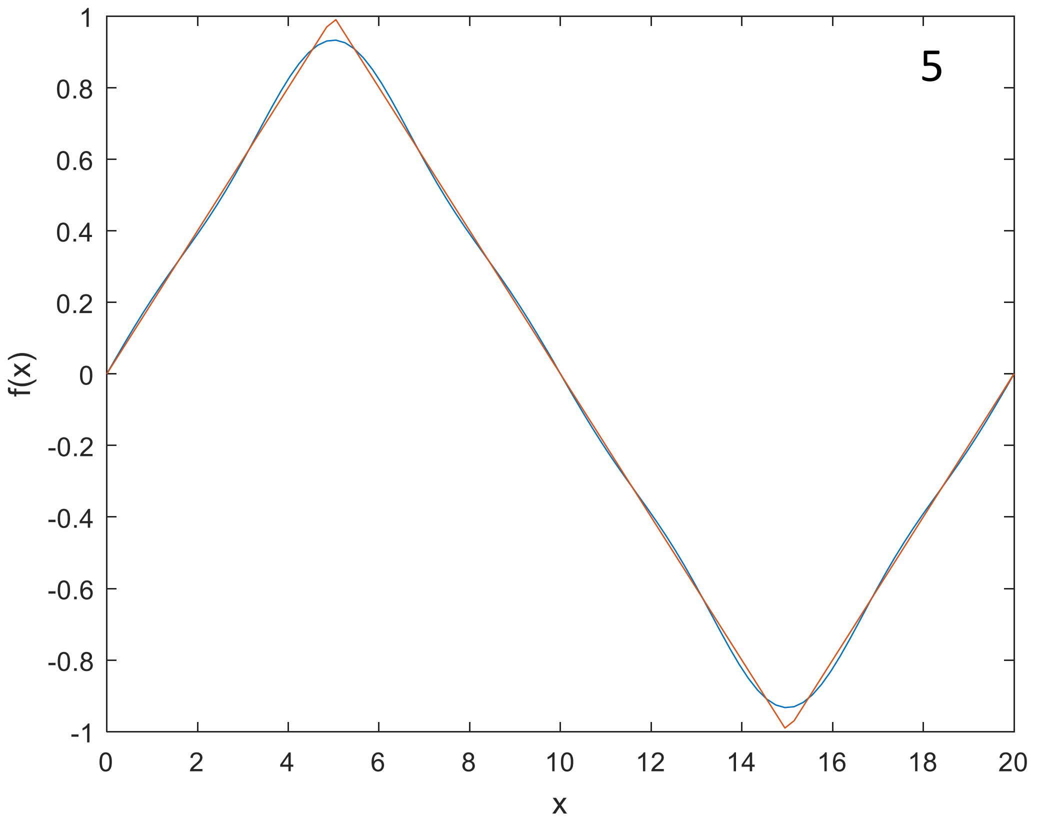
$$f(x) \approx \frac{8}{\pi^2} \sin\left(\frac{\pi x}{20}\right)$$

V(t) - triangelvåg



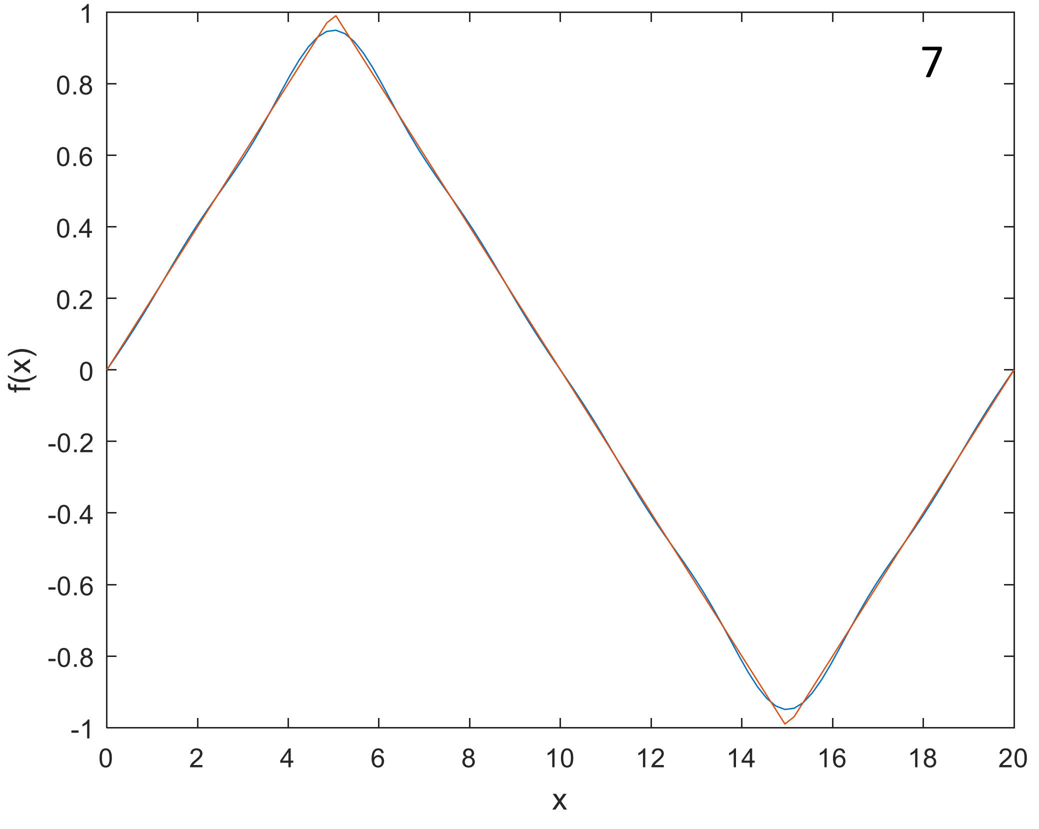
$$f(x) \approx \frac{8}{\pi^2} \sin\left(\frac{\pi}{20} x\right) - \frac{1}{9} \frac{8}{\pi^2} \sin\left(3 \frac{\pi}{20} x\right)$$

V(t) - triangelvåg



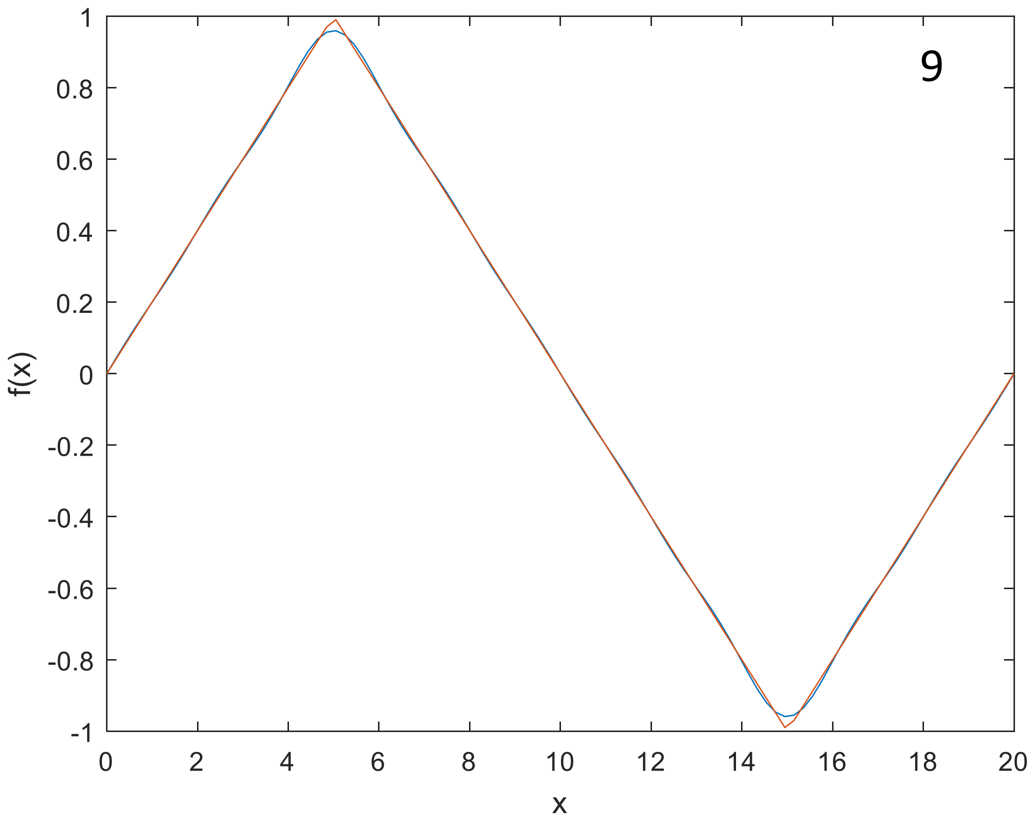
$$f(x) \approx \frac{8}{\pi^2}\sin\left(\frac{\pi}{20}x\right) - \frac{1}{9}\frac{8}{\pi^2}\sin\left(3\frac{\pi}{20}x\right) + \frac{1}{25}\frac{8}{\pi^2}\sin\left(5\frac{\pi}{20}x\right)$$

V(t) - triangelvåg



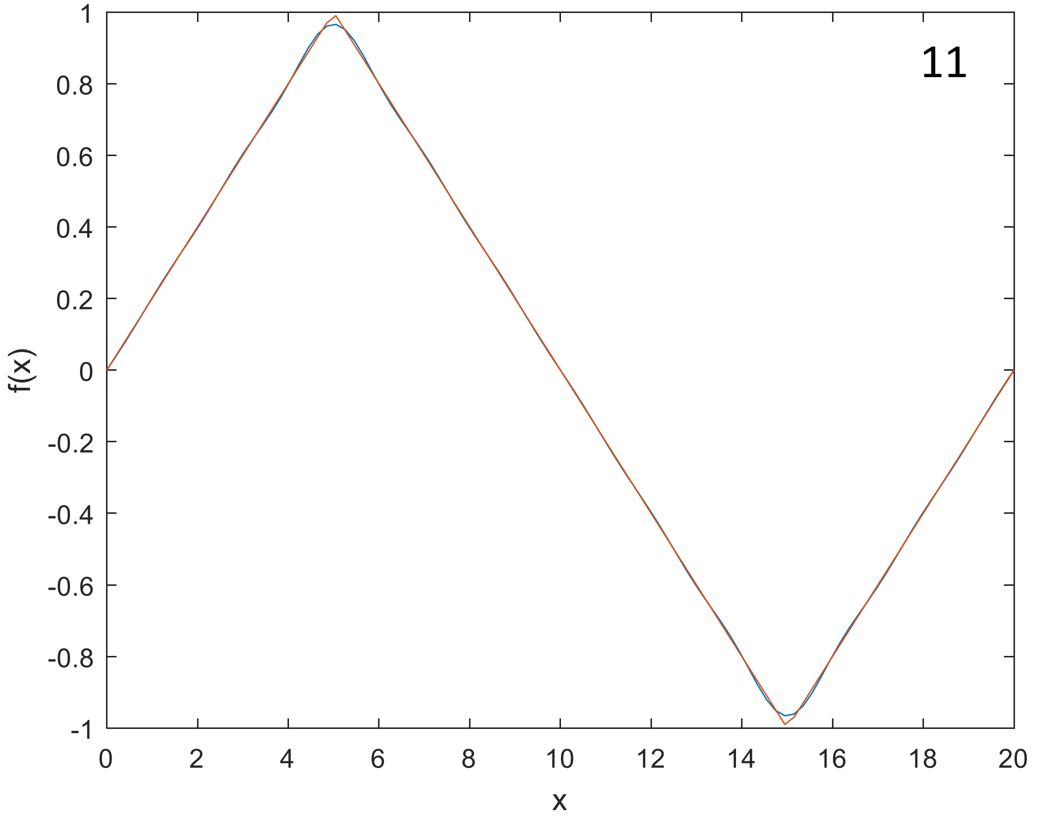
$$f(x) \approx \frac{8}{\pi^2}\sin\left(\frac{\pi}{20}x\right) - \frac{1}{9}\frac{8}{\pi^2}\sin\left(3\frac{\pi}{20}x\right) + \frac{1}{25}\frac{8}{\pi^2}\sin\left(5\frac{\pi}{20}x\right) - \frac{1}{49}\frac{8}{\pi^2}\sin\left(7\frac{\pi}{20}x\right)$$

V(t) - triangelvåg

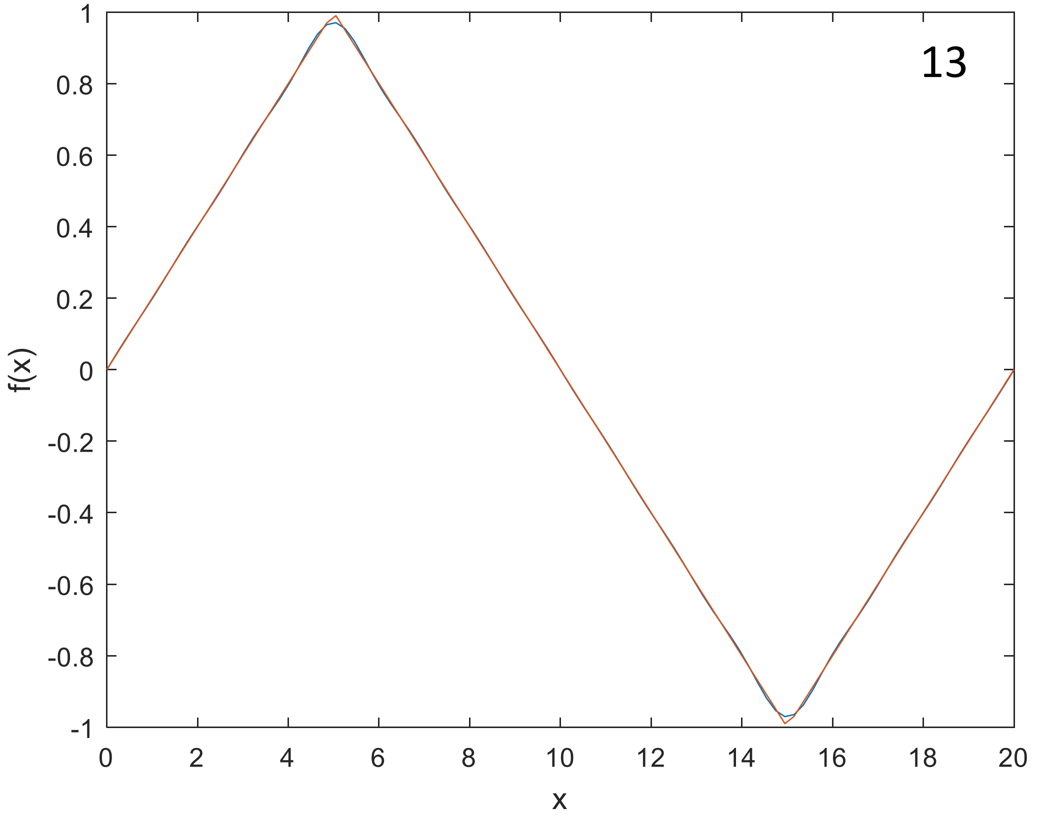


$$f(x) \approx \frac{8}{\pi^2}\sin\left(\frac{\pi}{20}x\right) - \frac{1}{9}\frac{8}{\pi^2}\sin\left(3\frac{\pi}{20}x\right) + \frac{1}{25}\frac{8}{\pi^2}\sin\left(5\frac{\pi}{20}x\right) - \frac{1}{49}\frac{8}{\pi^2}\sin\left(7\frac{\pi}{20}x\right) + \cdots$$

V(t) - triangelvåg



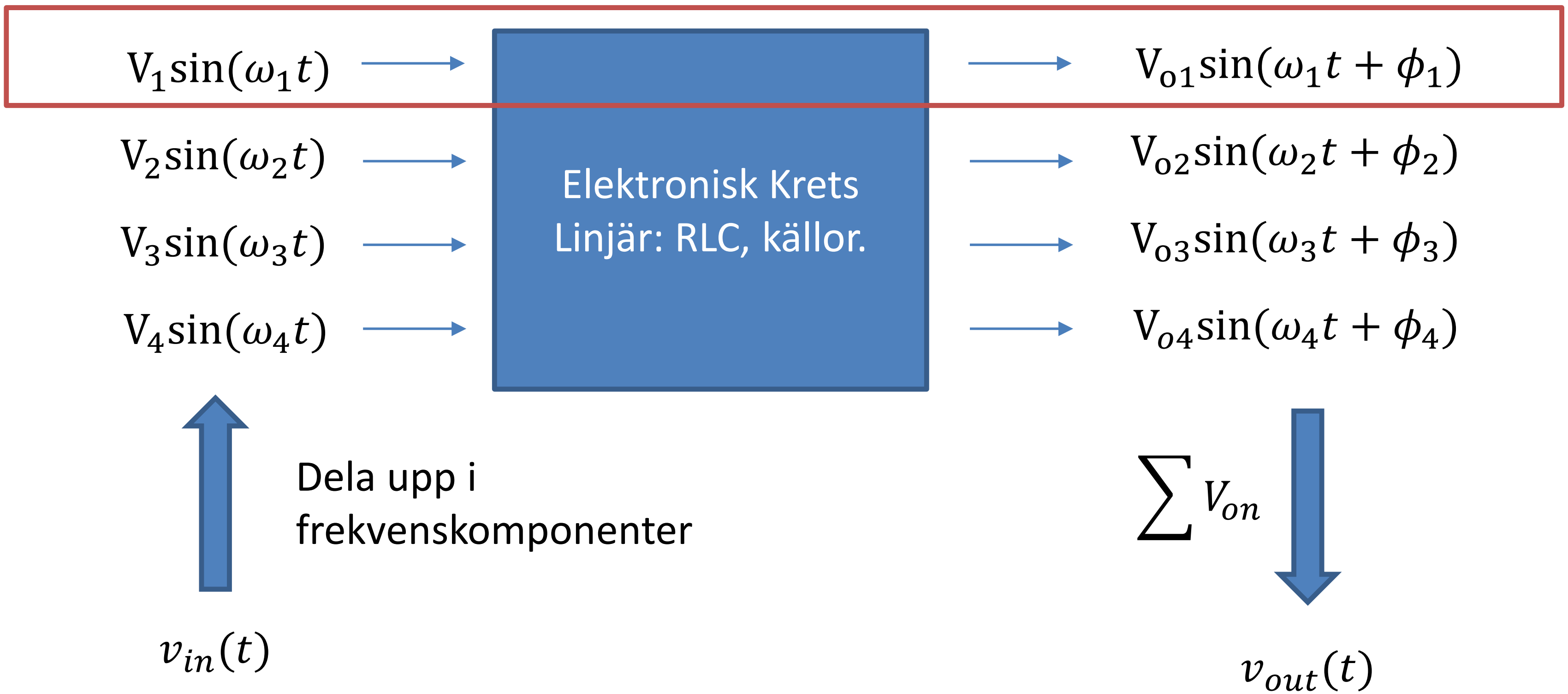
V(t) - triangelvåg



Alla signaler kan uttryckas som en ***summa*** av cosinustermer!
Superposition – vi kan hantera termerna var för sig!

Tidsbeorende signaler - superposition

Linjär krets: Amplitud och fas ändras
Frekvensen (ω) är konstant!



Frekvensområden

$V_1 \sin(\omega_1 t)$

$V_2 \sin(\omega_2 t)$

$V_3 \sin(\omega_3 t)$

$V_4 \sin(\omega_4 t)$

...

Audio: 20 Hz – 20 kHz

$$20 < \frac{\omega}{2\pi} < 20\ kHz$$

$V_1 \sin(\omega_1 t)$

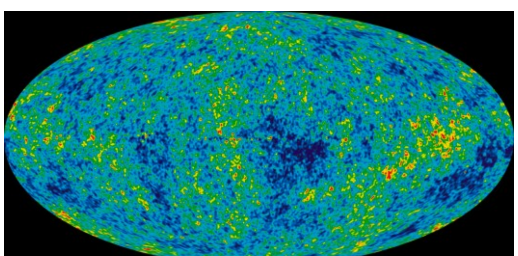
$V_2 \sin(\omega_2 t)$

$V_3 \sin(\omega_3 t)$

$V_4 \sin(\omega_4 t)$

...

Kommunikation:
900 MHz – 60 GHz



$V_1 \sin(\omega_1 t)$

$V_2 \sin(\omega_2 t)$

$V_3 \sin(\omega_3 t)$

$V_4 \sin(\omega_4 t)$

...

Mikrovågsbakgrund från Big Bang:
70-800 GHz

Komplexa Tal

$$z = a + jb$$

$$z = |z|(\cos(\phi) + \mathrm{j} \cdot \sin(\phi))$$

$$z = |z|e^{j\phi}$$

$$|z| = a^2 + b^2$$

$$\phi = \arctan\left(\frac{b}{a}\right)$$

$$Re(z) = |z|\cos(\phi)$$

$$Im(z) = |z|sin(\phi)$$

$$e^a e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

$$v(t) = V_0 \cos(\omega t + \theta) = Re\left(V_0 e^{j\theta} e^{j\omega t}\right)$$

3-fas

$$\frac{325}{\sqrt{2}} = 230V$$

Spänning (Effektivvärde) mellan de olika faserna och nolla. *Vanliga eluttag.*

$$\frac{563}{\sqrt{2}} = 400V$$

Spänning (Effektivvärde) två faser.
Applikationer som kräver högre effekt. *Spisar.*

Komplexa Tal

Addition är enkelt i rektangulär form:

$$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Multiplikation är enkelt i rektangulär form:

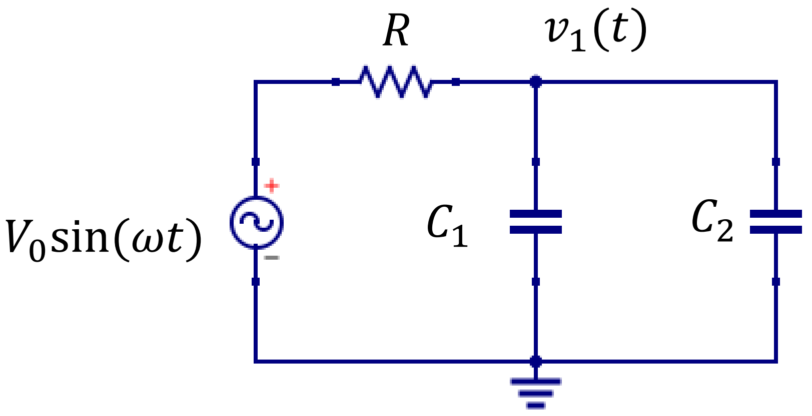
$$z_1 z_2 = |z_1||z_2|e^{j(\theta_1 + \theta_2)}$$

Tidsharmoniska Signaler -

$$v(t) = V_0 \cos(\omega t + \theta) = Re\left(V_0 e^{j\theta} e^{j\omega t}\right)$$

$V_0 e^{j\theta}$ Alla beräkningar kan göras med addition/multiplikation av komplexa tal!

Tidsberoende Signaler

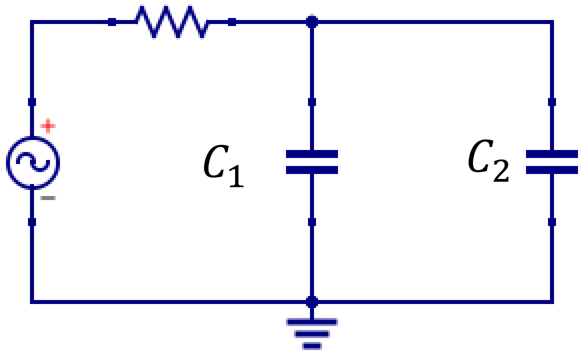


$$\frac{v_1(t) - V_0\sin(\omega t)}{R} + C_1\frac{dv_1(t)}{dt} + C_2\frac{dv_1(t)}{dt} = 0$$

Differentialekvationslösning. Relativt komplicerat.

Detta behöver göras om källan inte är rent tidsharmonisk.

Tidsberoende Signaler



Linjära system

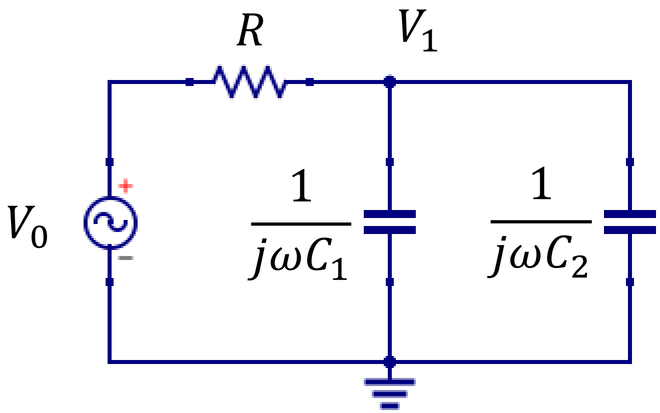
$$V_1 \sin(\omega_1 t) \longrightarrow V_{01} \sin(\omega_1 t + \phi_1)$$

$$\frac{V_1 sin(\omega t + \phi_1) \; - V_0 \sin(\omega t)}{R} + C_1 \frac{dv_1(t)}{dt} + C_2 \frac{dv_1(t)}{dt} = 0$$

$$\frac{V_1 sin(\omega t + \phi_1) - V_0 \sin(\omega t)}{R} + C_1 V_1 \omega \cos(\omega t + \phi_1) + C_2 V_1 \omega \cos(\omega t + \phi_1) = 0$$

+ en massa trigonimetriska identiter…

Tidsberoende Signaler – komplexa tal



V_1 – komplext tal. Både storlek $|V_1|$ och fas (ϕ)

$$\frac{V_1 - V_0}{R} + j\omega(C_1 + C_2)V_1 = 0$$

$$C = C_1 + C_2$$

$$V_1 = \frac{V_0}{1 + j\omega R(C_1 + C_2)}$$

$$v(t) = \frac{V_0}{\sqrt{1 + \omega^2 R^2 C^2}} \sin(\omega t - \arctan(\omega RC))$$

Komplexa Tal

Repetition

Komplex spänning <–> tidsharmonisk spänning

Mer detaljer och matematisk stringens : Analysen i HT2

Komplexa Tal - Sammanfattning

COMPLEX QUANTITIES AND THEIR USE IN ELECTRICAL ENGINEERING.

BY CHAS. PROTEUS STEINMETZ.

I.—INTRODUCTION.

In the following, I shall outline a method of calculating alternate current phenomena, which, I believe, differs from former methods essentially in so far, as it allows us to represent the alternate current, the sine-function of time, by a *constant* numerical quantity, and thereby eliminates the independent variable "time" altogether from the calculation of alternate current phenomena.

Herefrom results a considerable simplification of methods. Where before we had to deal with periodic functions of an independent variable, time, we have now to add, subtract, etc.,

Charles Proteus Steinmetz

"Complex Quantities and Their Use in Electrical Engineering", 1893