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Information Transmission
Appendix B, Circuit theory

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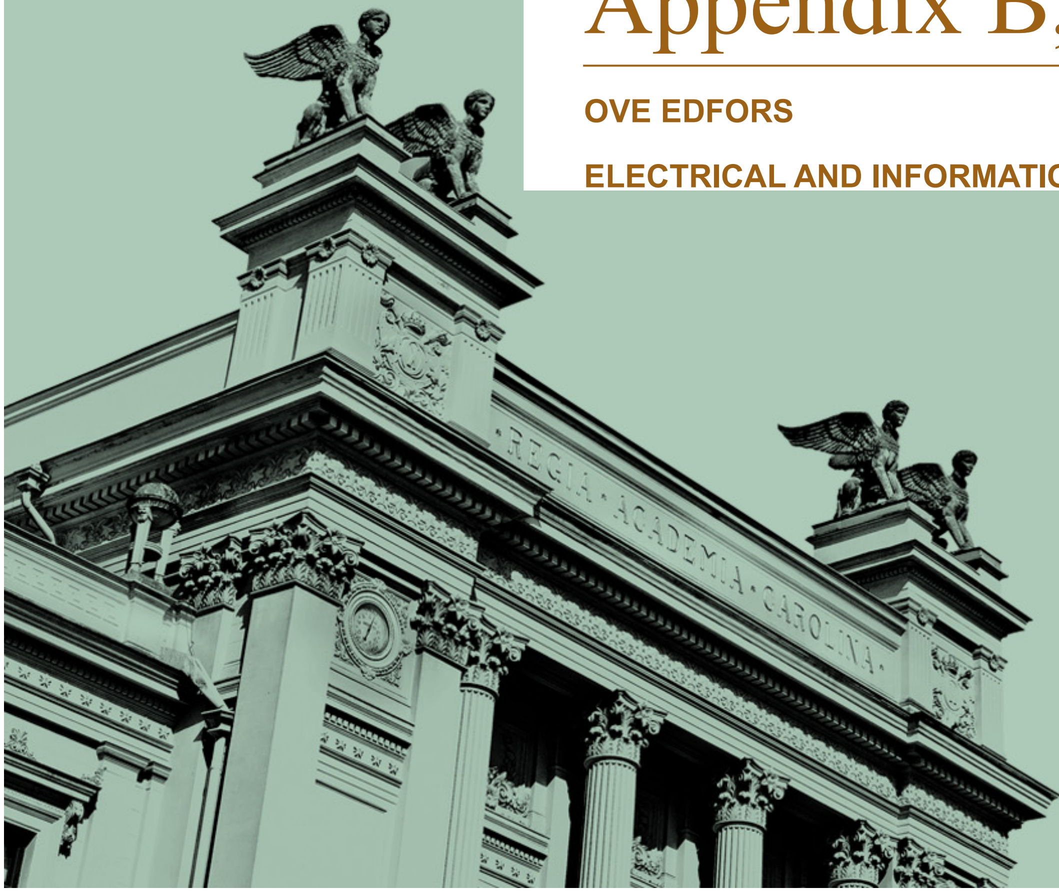
ELECTRICAL AND INFORMATION TECHNOLOGY



Learning outcomes

After this lecture the student should

- Know the properties of, and be able to perform basic calculations with, resistors, inductors and capacitors
- Know how resistors, inductors, and capacitors, behave when sinusoidal signals are applied.
- Know Kirchhoff's voltage and current laws and understand how they are applied to perform basic calculations on electronic circuits.
- Understand the impedance concept and how to caclculate the total impedance of impedances connected in serial or parallel

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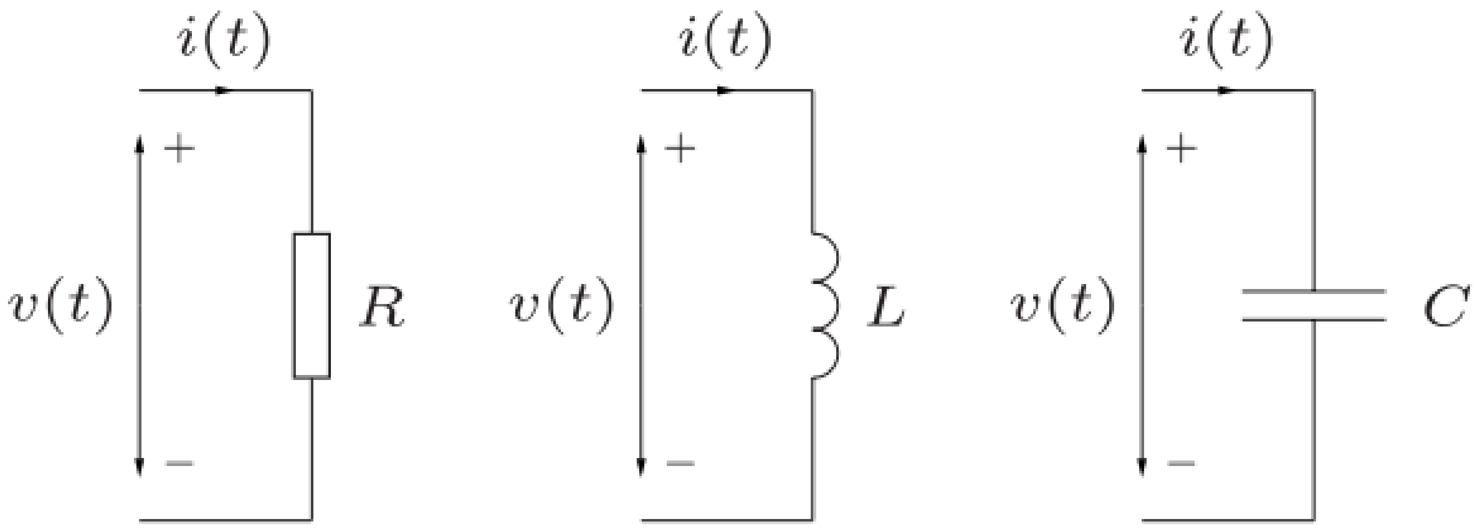
Resistors, Inductors, Capacitors



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Resistors

Ohm's law: the voltage $v(t)$ volt [V] across a resistor with resistance R ohm [Ω] is proportional to the current $i(t)$ ampére [A] through the resistance.

$$v(t) = Ri(t) \tag{B.1}$$

For $i(t) = e^{j\omega_0 t}$ we have

$$\boxed{v(t) = \underbrace{R}e^{j\omega_0 t}} \tag{B.2}$$

Does not depend on frequency.

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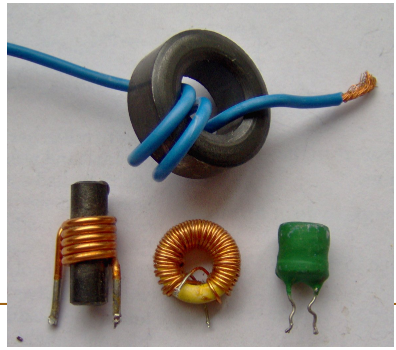

Inductors

The voltage across an inductor with inductance L henry [H] is proportional to the derivative of the current $i(t)$ through the inductor.

$$v(t) = L\frac{di(t)}{dt} \qquad \text{(B.3)}$$

For $i(t) = e^{j\omega_0 t}$ we have

$$\boxed{v(t) = j\omega_0 L e^{j\omega_0 t}} \qquad \text{(B.4)}$$

Depends (increases) with frequency.

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Capacitors

The charge $q(t)$ coulombs [C] of a capacitor with capacitans C farad [F] is proportional to the voltage across the capacitor:

$$q(t) = Cv(t)$$

Since $q(t) = \int_{-\infty}^{t} i(\tau)d\tau$, we have

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

For $i(t) = e^{j\omega_0 t}$ we have

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} e^{j\omega_0\tau}d\tau = \frac{1}{j\omega_0 C}e^{j\omega_0 t}$$

Depends on (decreases with) frequency.

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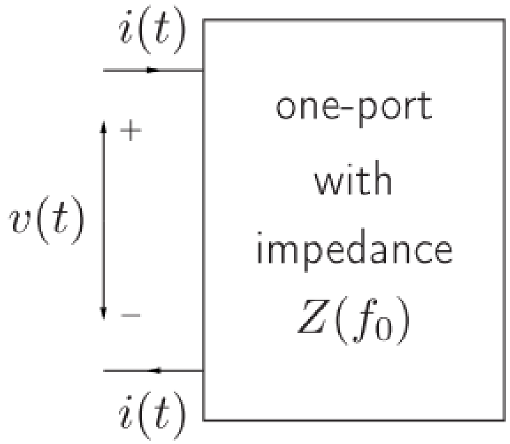
Impedance

The one-port shown in Fig. B.2 is a network consisting of resistors, inductors, and capacitors. The voltage across the one-port is $v(t) = e^{j\omega_0 t}$.

The current $i(t)$ will also be a complex exponential signal;
but, in general, with different amplitude and phase.



We have

$$\boxed{v(t) = Z(f_0)i(t)}$$

where $Z(f_0)$ is called the impedance.

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Impedance

The impedance is in general complex and dependent on the frequency f_0, but if the one-port consists only of resistors, then its impedance will always be real and independent of the frequency f_0; that is, it is a resistance.

Ohm's law for alternating current holds only for stationary sinusoidal voltages and currents (including the special case when $f_0 = 0$).

Kirchhoff's current law

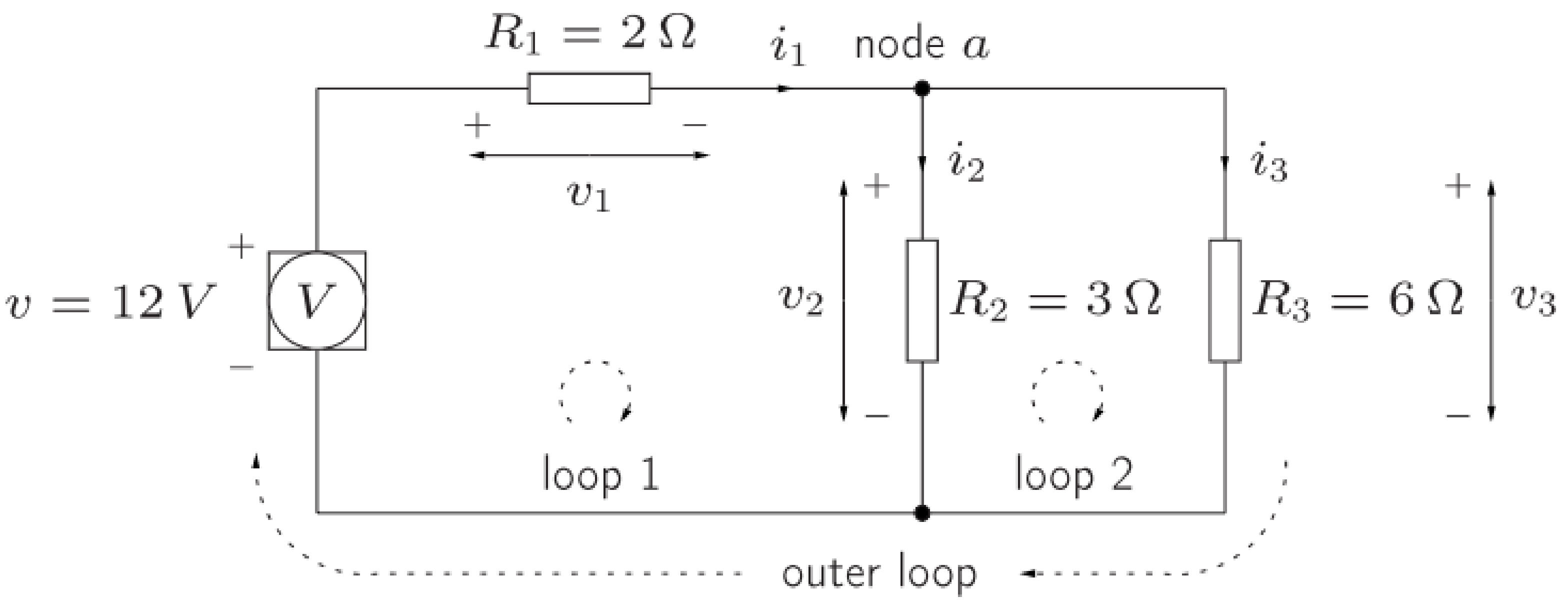
Kirchhoff's current law (KCL): *The algebraic sum of the currents entering any node is identically zero for all instants of time.*

KCL: Sum of currents flowing into a node = sum of currents leaving the node

Kirchhoff's voltage law

Kirchhoff's voltage law (KVL): *The algebraic sum of the voltages around any closed path, or loop, in a circuit is identically zero for all instants of time.*

Example 1

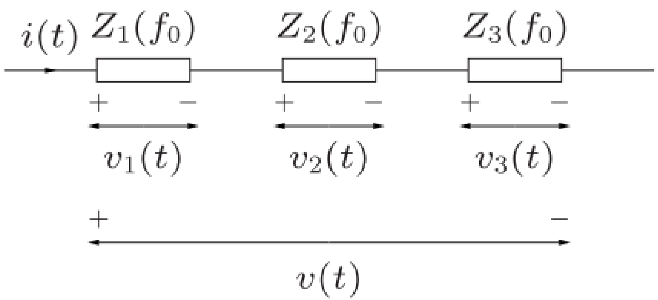


What is i_1, i_2, i_3, v_1, v_2, v_3?

Serial impedance

Consider three impedances connected in a serial manner.
Assuming that the current $i(t)$ is sinusoidal with frequency f_0, let $Z_s(f_0)$ denote the impedance of this serial circuit.

$$v(t) = Z_s(f_0)i(t)$$



The impedance for the serial circuit is

$$Z(f_0) = Z_1(f_0) + Z_2(f_0) + Z_3(f_0)$$

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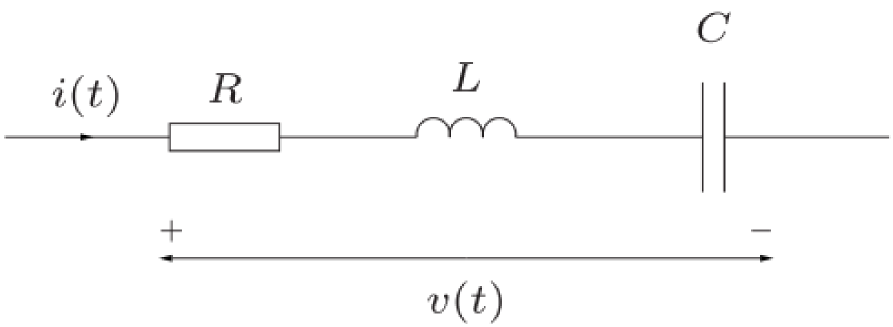
Example 2

Consider the circuit

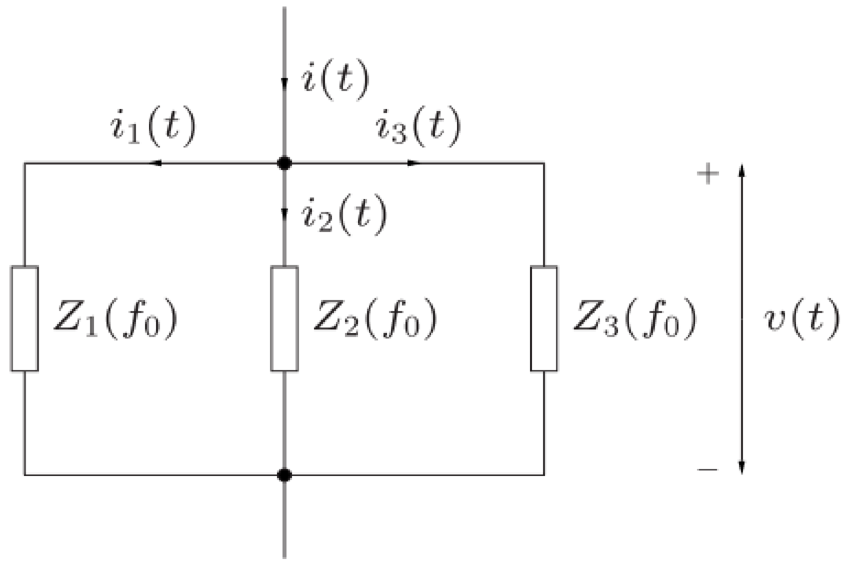


The current $i(t)$ is a sinusoid of frequency f_0 Hz.

What is *v(t)*?

Parallel impedances

Consider three impedances connected in parallel:



What is the equivalent parallel impedance?

Parallel impedances

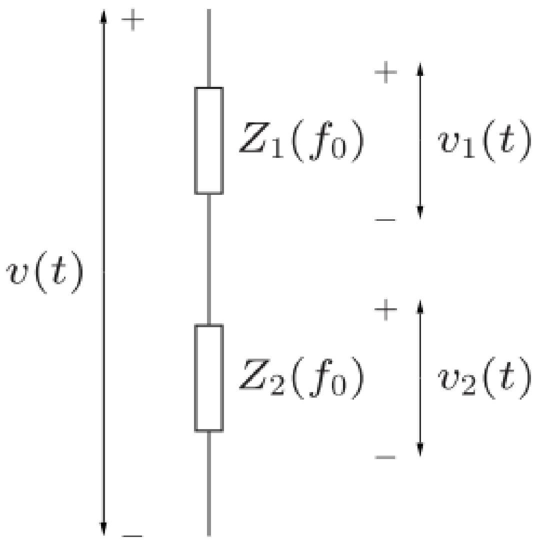
The impedance for the parallel circuit is written

$$\frac{1}{Z_p(f_0)} = \frac{1}{Z_1(f_0)} + \frac{1}{Z_2(f_0)} + \frac{1}{Z_3(f_0)}$$

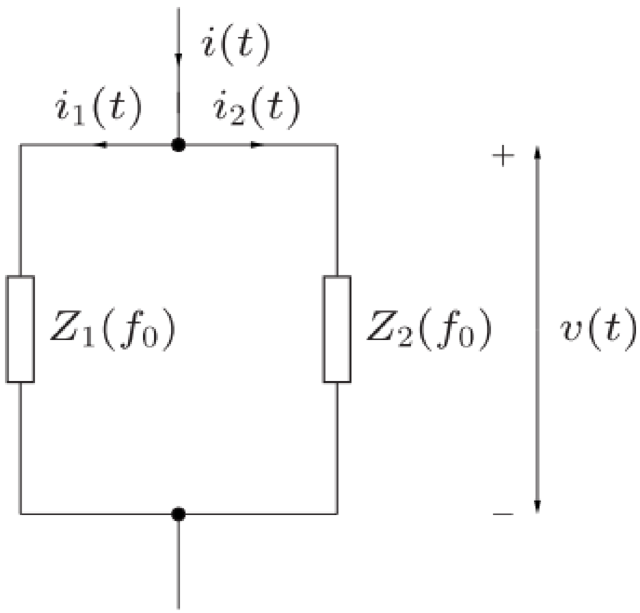
Often we have only two impedances in parallel

$$Z_p(f_0) = \frac{Z_1(f_0)Z_2(f_0)}{Z_1(f_0) + Z_2(f_0)}$$

Split of voltage and current



$$v_1(t) = \frac{Z_1(f_0)}{Z_1(f_0) + Z_2(f_0)} v(t)$$



$$i_1(t) = \frac{Z_2(f_0)}{Z_1(f_0) + Z_2(f_0)} i(t)$$

Summary

- Resistors, inductors & capacitors
  - We can apply Ohm's law, also for inductors and capacitors, by using $j\omega_0 L$ and $1/j\omega_0 C$ in place of ”resistance”.
  - Impedance is the relation between voltage and current (often from a combination of resistors, inductors and capacitors).
- Kirchoff's laws (general)
  - **Current law**: At all times, the sum of all currents into a node most equal the sum of currents leaving the node. (Charge can't accumulate in a node.)
  - **Voltage law**: At all times, the sum of voltages around any closed loop in a circuit must be zero.
- Typical circuits (special cases)
  - Serial and parallel impedance
  - Split of voltage and current

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