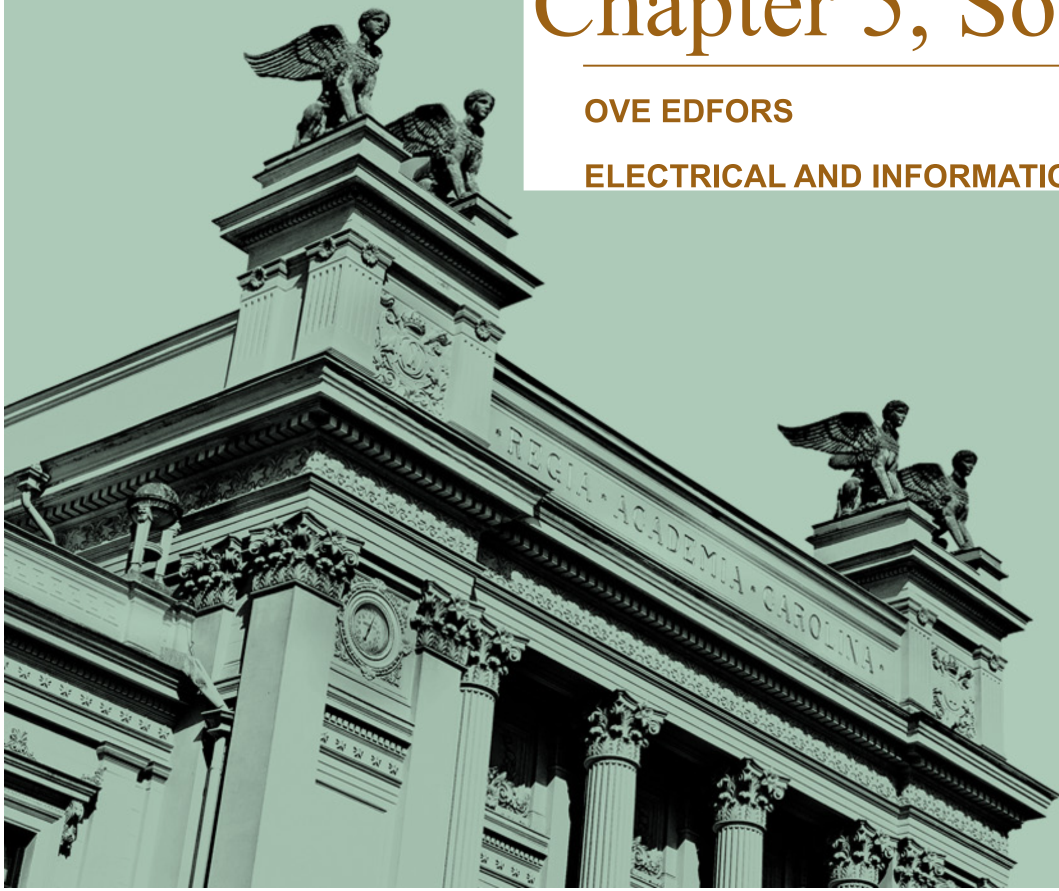
Lund University

Information Transmission
Chapter 5, Source coding

OVE EDFORS

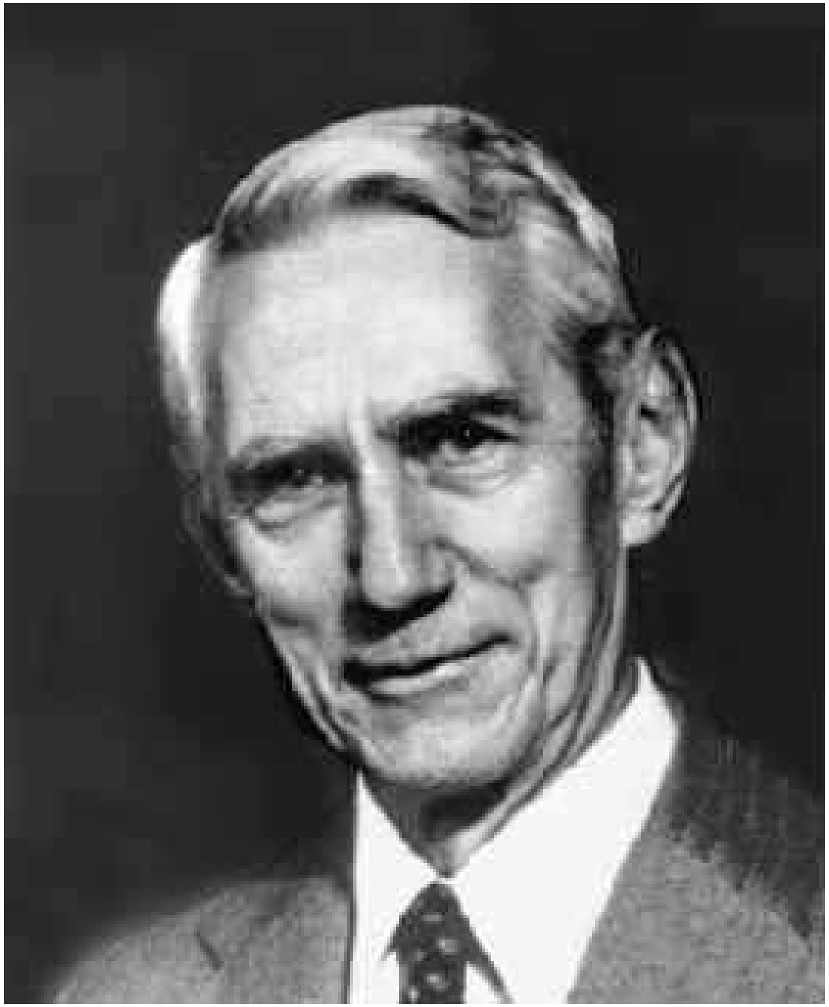
ELECTRICAL AND INFORMATION TECHNOLOGY



Learning outcomes

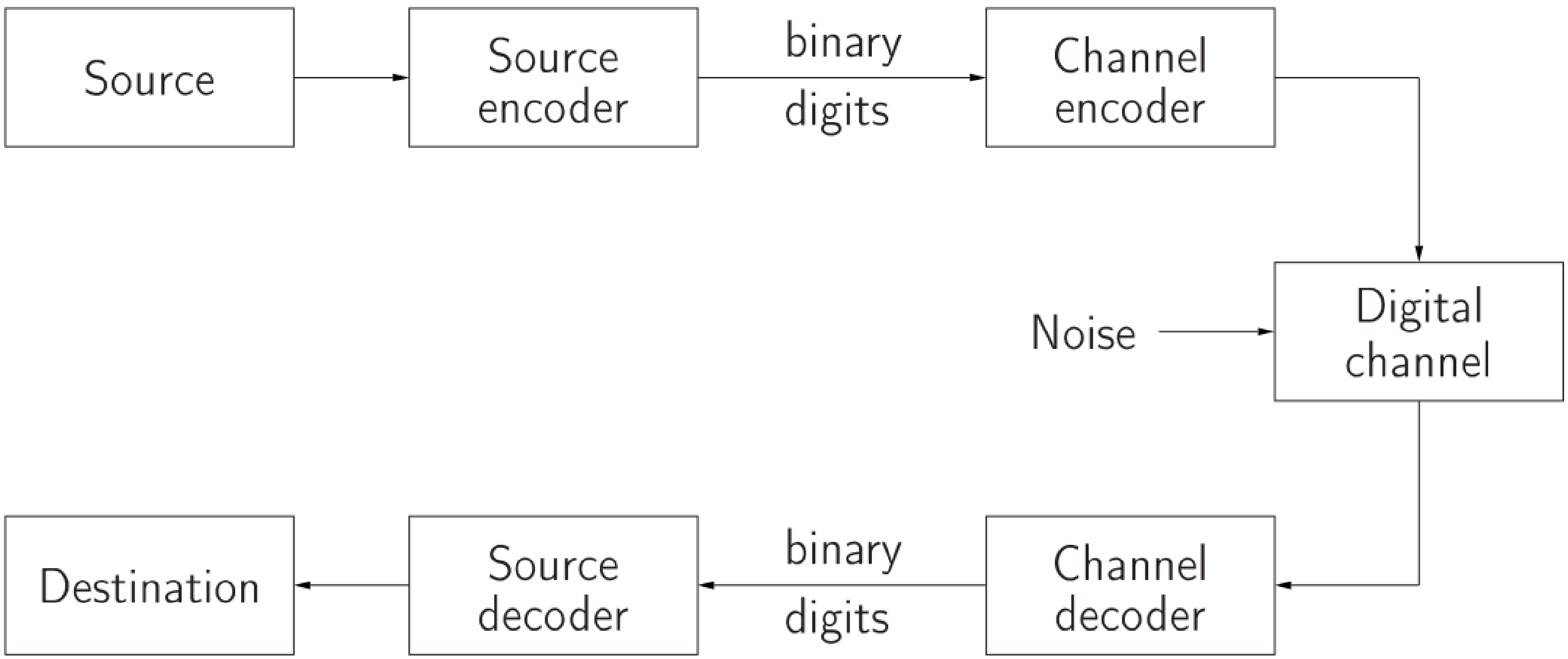
- After this lecture the student should
  - understand the basics of source coding,
  - know what a prefix free source code is,
  - know how to calculate average codword length,
  - understand the limits on source coding,
  - understand the concept of universal source coding, and
  - be able to perform encoding and decoding according to the Lempel-Ziv-Welch algorithm

What did Shannon promise?



- We can represent a source sequence X of length *n* uniquely by, on the average, $n*H(X)$ bits

A schematic communication system



Prefix free source code

We say that a sequence of length *l* is a *prefix* of a sequence if the first *l* symbols of the latter sequence is identical to the first sequence; in particular, a sequence is a prefix of itself.

Then we require that *no codeword is the prefix of another codeword* and call such a code a *prefix-free source code*.

The sequence 10011 has the prefixes: 1, 10, 100, 1001, and 10011.

The source code with codewords {00,01,1} is prefix-free, but {00,10,1} is not, since 1 is prefix of 10.

LUND UNIVERSITY

Example

Consider the source code

u	$P_U(u)$	$\boldsymbol{x}$
u_1	0.45	0
u_2	0.30	10
u_3	0.15	110
u_4	0.10	111

What is the average codeword length?

LUND UNIVERSITY

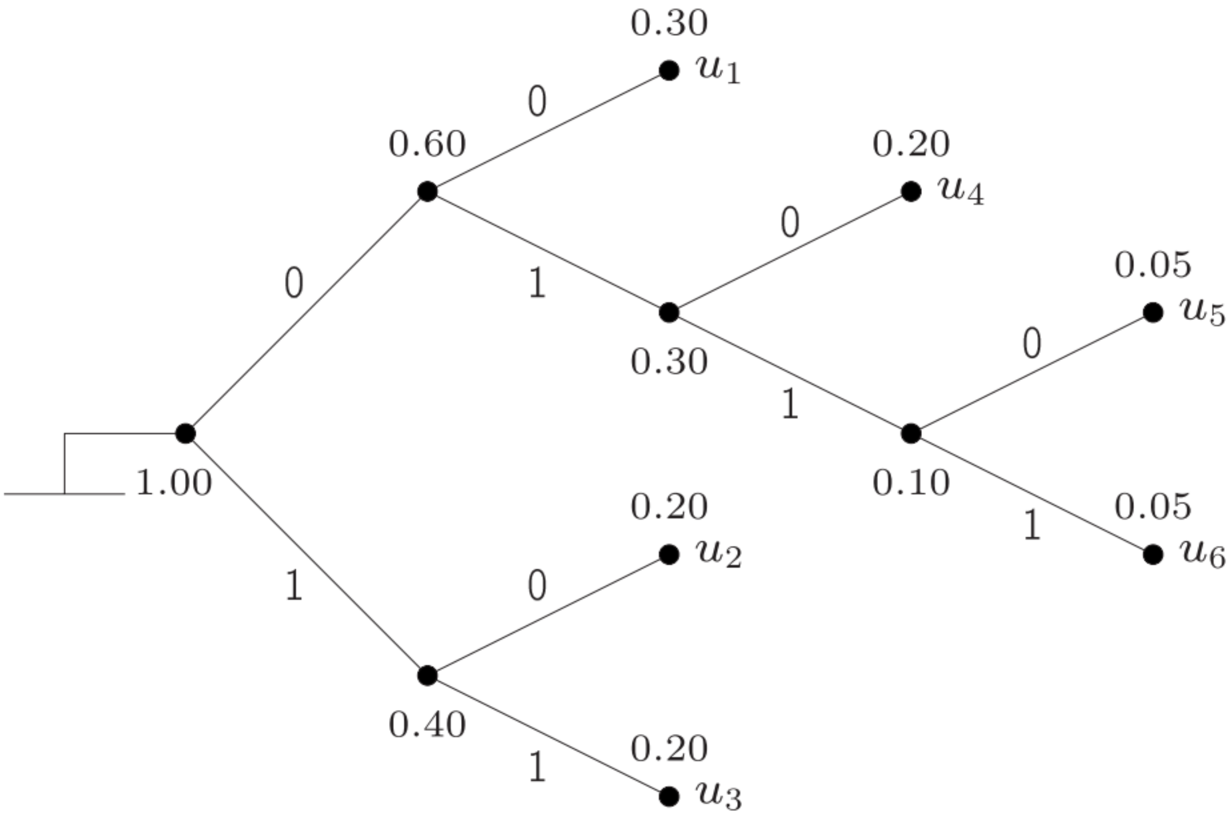
Path length lemma

In a rooted tree with probabilities, the average depth of the leaves is equal to the sum of the probabilities of the nodes (including the root).

LUND UNIVERSITY

Example

- What is the average word length and the uncertainty of the source



u	$P_U(u)$	$\boldsymbol{x}$
u_1	0.30	00
u_2	0.20	10
u_3	0.20	11
u_4	0.20	010
u_5	0.05	0110
u_6	0.05	0111

LUND UNIVERSITY

Performance validation

We know from Shannon's source coding theorem that we cannot do better than the uncertainty of the source:

$$
\begin{aligned}
H(U) &= -\sum_{i=1}^{6} P_U(u_i) \log P_U(u_i) \\
&= -0.30 \log 0.30 - 0.20 \log 0.20 - 0.20 \log 0.20 \\
&\qquad - 0.20 \log 0.20 - 0.05 \log 0.05 - 0.05 \log 0.05 \\
&= 2.34
\end{aligned}
$$

The Huffman code is optimal so we cannot do better than W=2.40 when coding the source in the previous example.

We are 2.6 % above the ultimate limit H(U)=2.34, which cannot be reached if we encode the source symbol separately.

LUND UNIVERSITY

Reaching the limit

If we encode consecutive source symbols pairwise, that is, use the Huffman code for the source

$u_i u_j$	$P_{U_1 U_2}(u_i u_j)$
$u_1 u_1$	0.0900
$u_1 u_2$	0.0600
$u_1 u_3$	0.0600
$\vdots$	
$u_6 u_6$	0.0025

we will obtain an average codeword length per single source symbol that is closer to the uncertainty of the source, H(U)=2.34.

LUND UNIVERSITY

A universal source coding algorithm

The LZW algorithm is due to Ziv, Lempel, and Welch and belongs to the class of so-called *universal source-coding algorithms* which means that we do not need to know the source statistics.

The algorithm is easy to implement and for long sequences it approaches the uncertainty of the source; it is asymptotically optimum.

Basic procedure

1. Initialize the dictionary.
2. Find the longest string W in the dictionary that matches the current input.
3. Emit the dictionary index for W to output and remove W from the input.
4. Add W followed by the next symbol in the input to the dictionary.
5. Go to Step 2.

Suppose we want to compress the sentence:

DO_NOT_TROUBLE_TROUBLE_

UNTIL_TROUBLE_TROUBLES_YOU!

LUND UNIVERSITY

Step	Entry	# binary digits
0	*	—
1	D	8
2	O	$\lceil \log 1 \rceil + 8$
3	_	$\lceil \log 2 \rceil + 8$
4	N	$\lceil \log 3 \rceil + 8$
5	OT	$\lceil \log 4 \rceil$
6	T	$\lceil \log 5 \rceil + 8$
7	_T	$\lceil \log 6 \rceil$
8	TR	$\lceil \log 7 \rceil$
9	R	$\lceil \log 8 \rceil + 8$
10	OU	$\lceil \log 9 \rceil$
11	U	$\lceil \log 10 \rceil + 8$
12	B	$\lceil \log 11 \rceil + 8$
13	L	$\lceil \log 12 \rceil + 8$
14	E	$\lceil \log 13 \rceil + 8$
15	_TR	$\lceil \log 14 \rceil$
16	RO	$\lceil \log 15 \rceil$
17	OUB	$\lceil \log 16 \rceil$
18	BL	$\lceil \log 17 \rceil$
19	LE	$\lceil \log 18 \rceil$

Step	Entry	# binary digits
20	E_	$\lceil \log 19 \rceil$
21	_U	$\lceil \log 20 \rceil$
22	UN	$\lceil \log 21 \rceil$
23	NT	$\lceil \log 22 \rceil$
24	TI	$\lceil \log 23 \rceil$
25	I	$\lceil \log 24 \rceil + 8$
26	L_	$\lceil \log 25 \rceil$
27	_TRO	$\lceil \log 26 \rceil$
28	OUBL	$\lceil \log 27 \rceil$
29	LE_	$\lceil \log 28 \rceil$
30	_TROU	$\lceil \log 29 \rceil$
31	UB	$\lceil \log 30 \rceil$
32	BLE	$\lceil \log 31 \rceil$
33	ES	$\lceil \log 32 \rceil$
34	S	$\lceil \log 33 \rceil + 8$
35	_Y	$\lceil \log 34 \rceil$
36	Y	$\lceil \log 35 \rceil + 8$
37	OU!	$\lceil \log 36 \rceil$
38	!	$\lceil \log 37 \rceil + 8$

Evaluation

Without compression we need as many as 50*8=400 binary digits to represent the sentence as a string of 50 ASCII symbols. If we sum the number of binary digits needed for the 38 steps shown in the table we get only 271 binary digits.

A highly optimized version of the LZW algorithm we have described is used widely in practice to compress computer files under both the UNIX and Windows operating system, and in a CCITT standard for data-compression for modems.

LUND
UNIVERSITY