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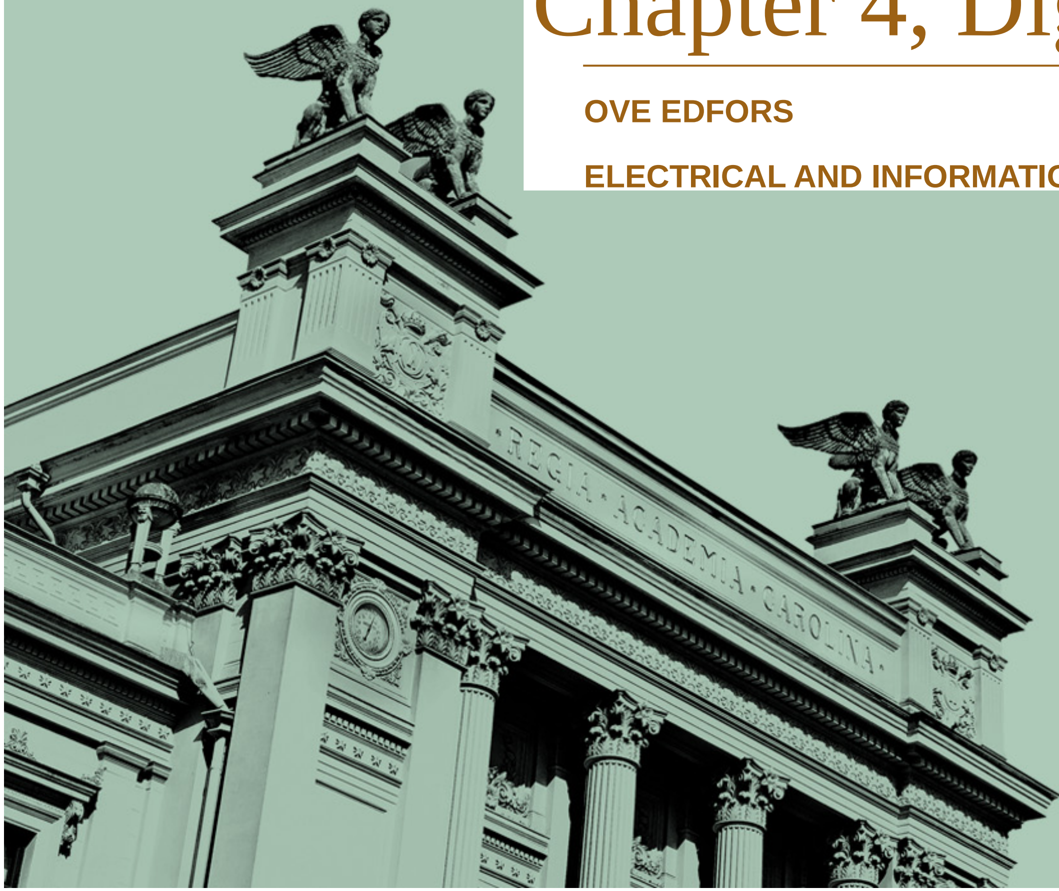
Information Transmission Chapter 4, Digitial modulation

OVE EDFORS

ELECTRICAL AND INFORMATION TECHNOLOGY



Learning outcomes

- After these lectures (slides span two lectures), the student should
  - understand the basic principles of how digital information is carried on analog signals (digital modulation), including amplitude, phase and frequency modulation/keying,
  - understand how the modulation pulse shape determines bandwidth of the signal and what the narrowest possible transmission bandwidth is for a certain data rate,
  - understand how one or more bits are mapped onto signal constellation points,
  - be able to perform basic calculations using relations between data rates, signal constellations, pulse chapes and transmission spectrum/bandwidths,
  - understand the fundamental principles of how digital information is detected at the receiver, including optimal receivers,
  - understand the relationships between receives signal quality and resulting bit-error rates,
  - be able to perform basic calculations on resulting receiver performance (bit-error rates) when the modulation type and the received signal quality are given.

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Some basics

- Each bit, or groups of bits, is represented by an analog waveform *v(t)*
- Symbol rate $R_t = 1/T$
- Symbol energy E_s
- The power of the signal is given by $E_s/T = E_s R_t$
- Data a_n,

$$s(t) = \sqrt{E_s} \sum_n a_n v(t - nT)$$

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Different modulation formats

- Amplitude modulation, ASK
- Phase modulation, PSK
- Frequency modulation, FSK

$$s(t) = A(t)\cos(2\pi f_c t + \varphi(t))$$

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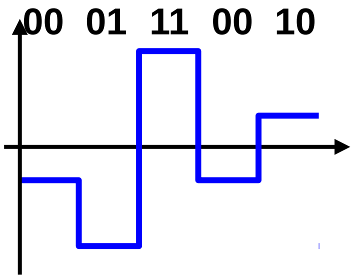
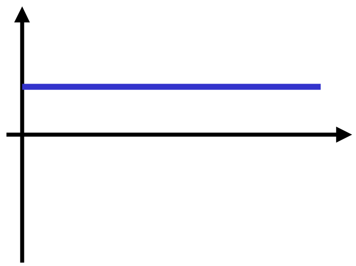
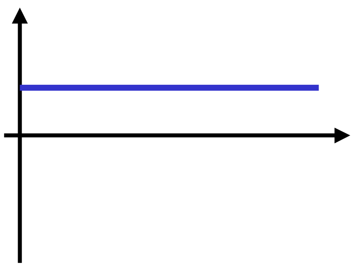
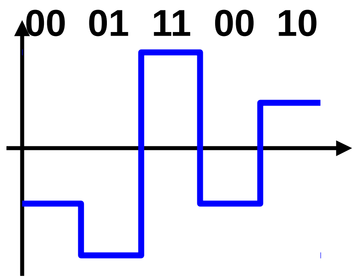
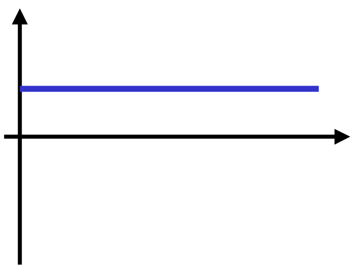
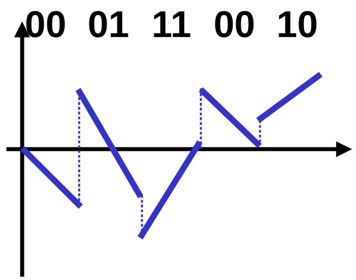
Amplitude, phase and frequency modulation

$$s(t) = A(t)\cos(2\pi f_c t + \varphi(t))$$

	$A(t)$	$\varphi(t)$	**Comment:**
4ASK			**- Amplitude carries information** **- Phase constant (arbitrary)**
4PSK			**- Amplitude constant (arbitrary)** **- Phase carries information**
4FSK			**- Amplitude constant (arbitrary)** **- Phase slope (frequency) carries information**

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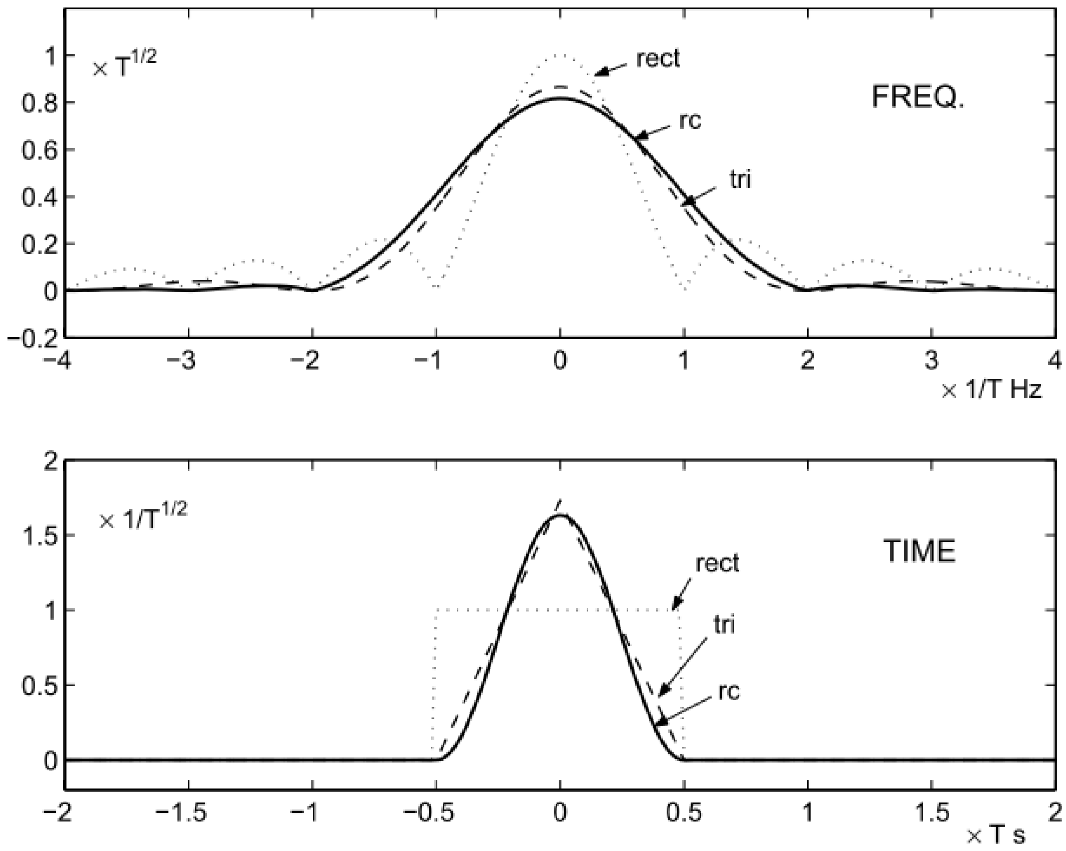
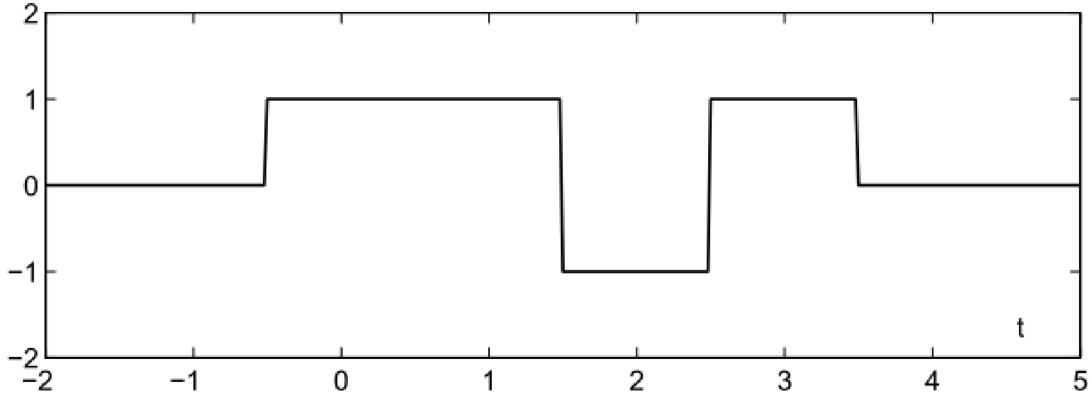
The pulse shape determines the bandwidth occupied

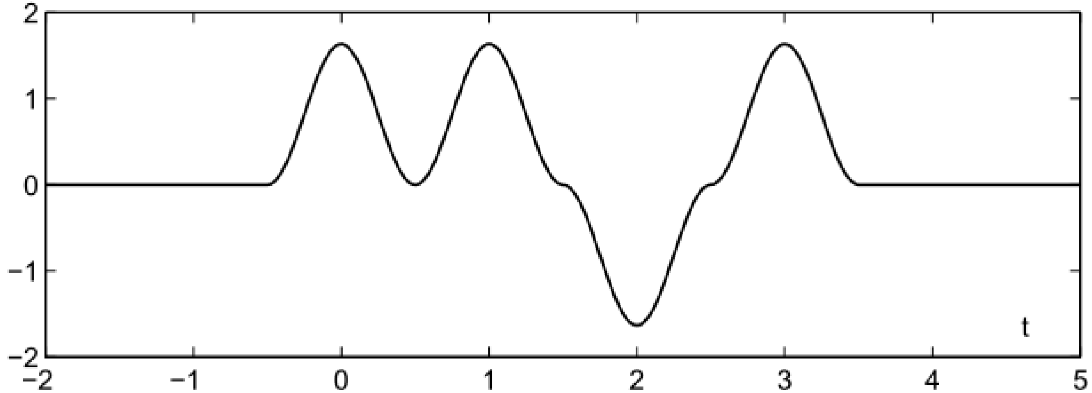


Train of pulses representing 0 0 1 0

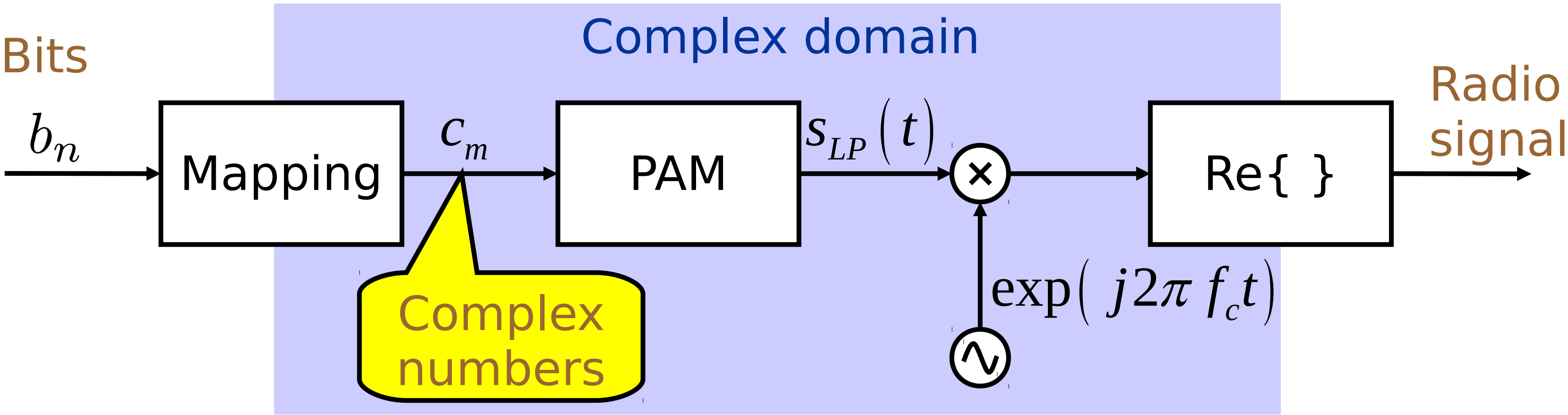
- Square pulses



- Raised cosine



The modulation process



PAM: $$s_{\mathrm{LP}}(t)=\sum_{m=-\infty}^{\infty} c_m v(t-mT_s)$$

Symbol time

"Standard" basis pulse criteria

$$\int_{-\infty}^{\infty} |v(t)|^2 dt = 1 \text{ or } = T_s$$ (energy norm.)

$$\int_{-\infty}^{\infty} v(t)v^*(t-mT_s)\,dt = 0 \text{ for } m \neq 0$$ (orthogonality)

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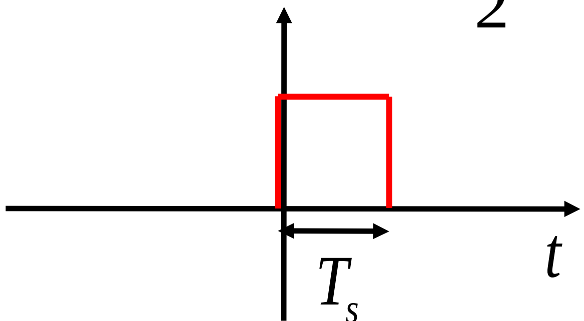
Basis pulses and spectrum

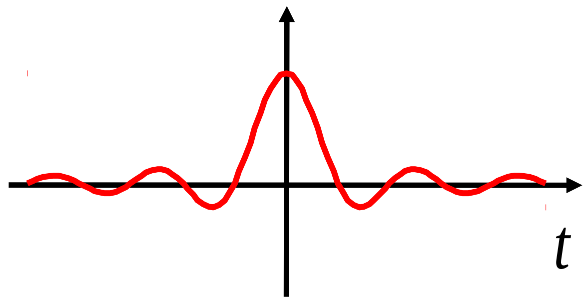
Assuming that the complex numbers c_m representing the data are independent, then the power spectral density of the base band PAM signal becomes:

$$S_{\mathrm{LP}}(f) \sim \left| \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} dt \right|^2$$

which translates into a radio signal (band pass) with

$$S_{\mathrm{BP}}(f) = \frac{1}{2}\left(S_{\mathrm{LP}}(f - f_c) + S_{\mathrm{LP}}(-f - f_c) \right)$$



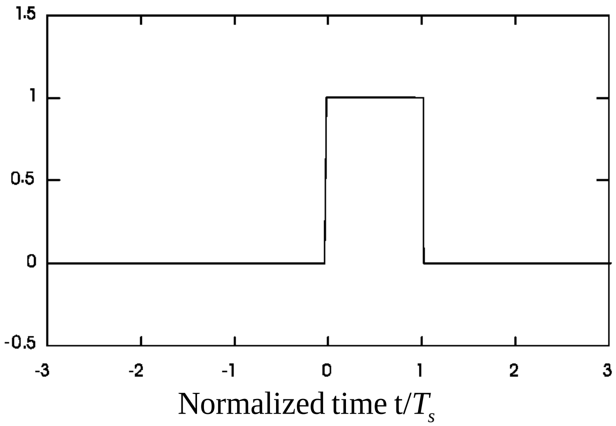


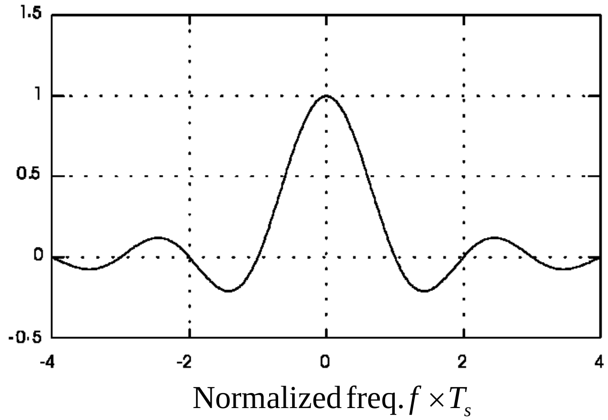
Many possible pulses

Basis pulses

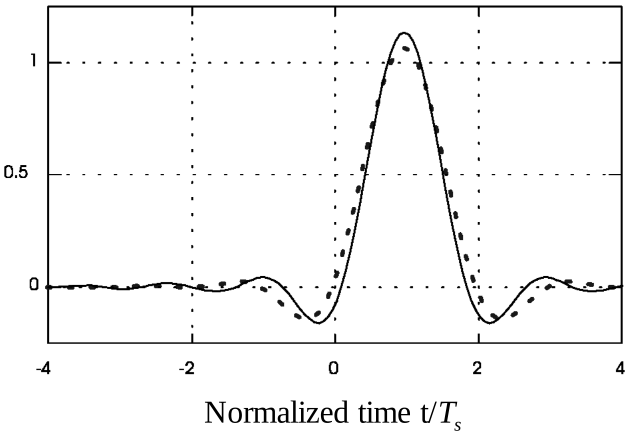
TIME DOMAIN FREQ. DOMAIN

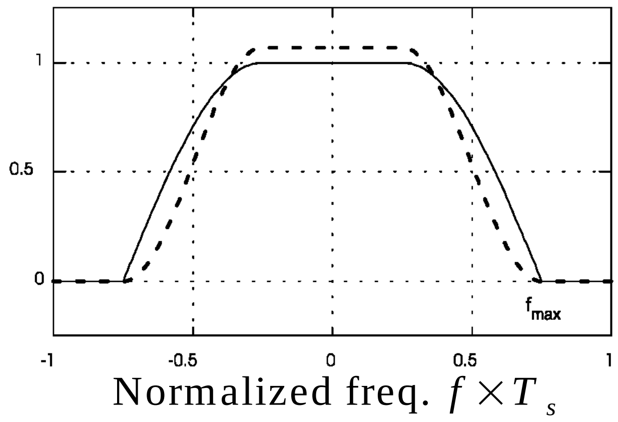
Rectangular [in time]





(Root-) Raised-cosine [in freq.]





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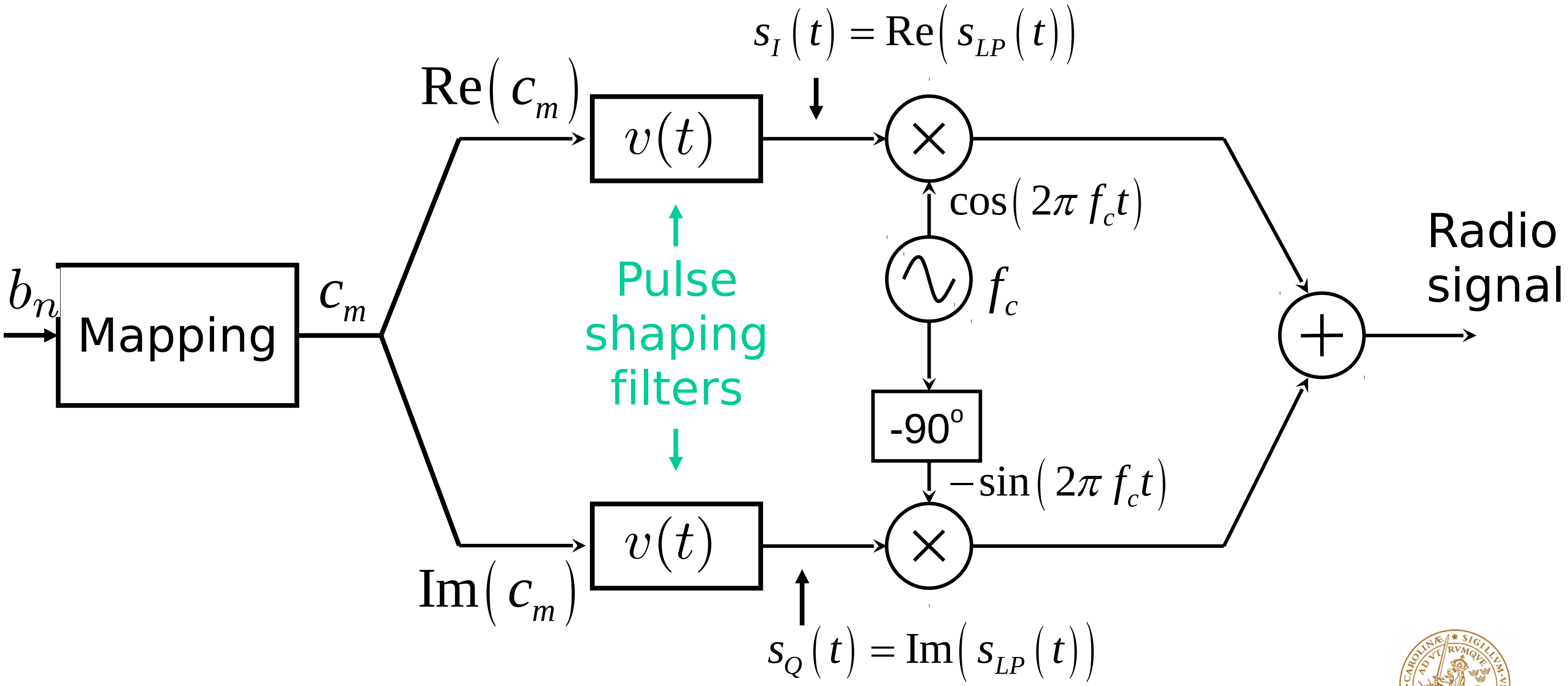
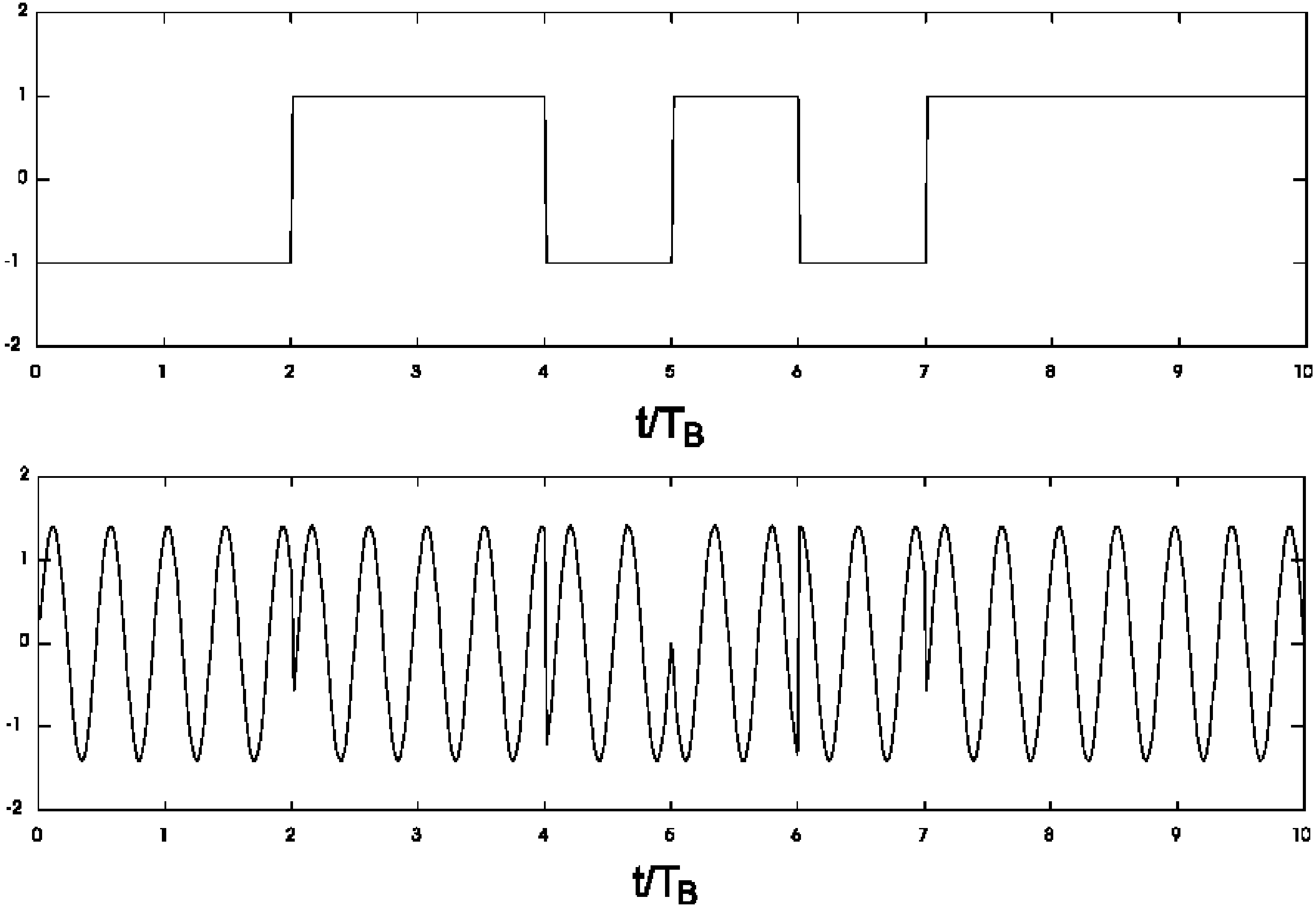
Interpretation as IQ-modulator

For real valued basis functions *v*(*t*) we can view PAM as:



(Both the rectangular and the (root-) raised-cosine pulses are real valued.)

Binary phase-shift keying (BPSK)
Rectangular pulses

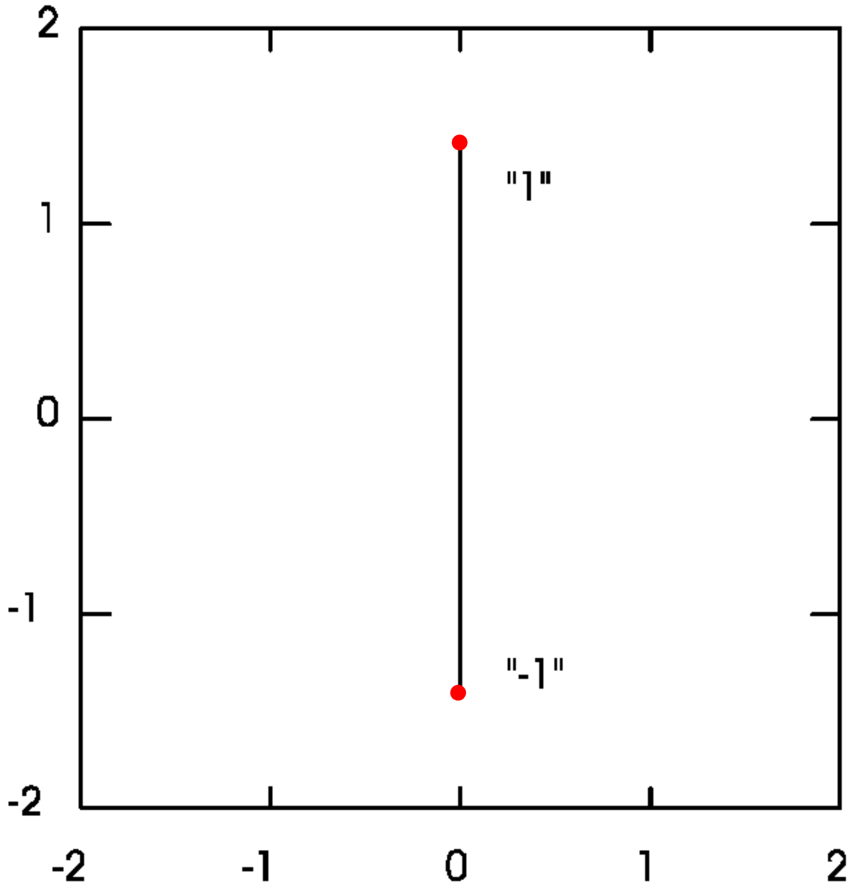


Base-band

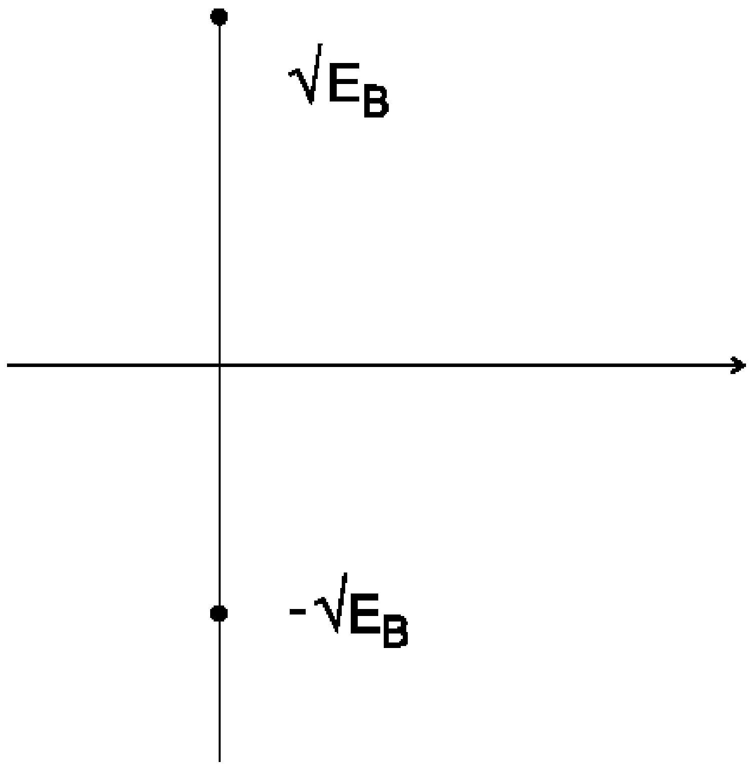
Radio signal

Binary phase-shift keying (BPSK)
Rectangular pulses

Complex representation

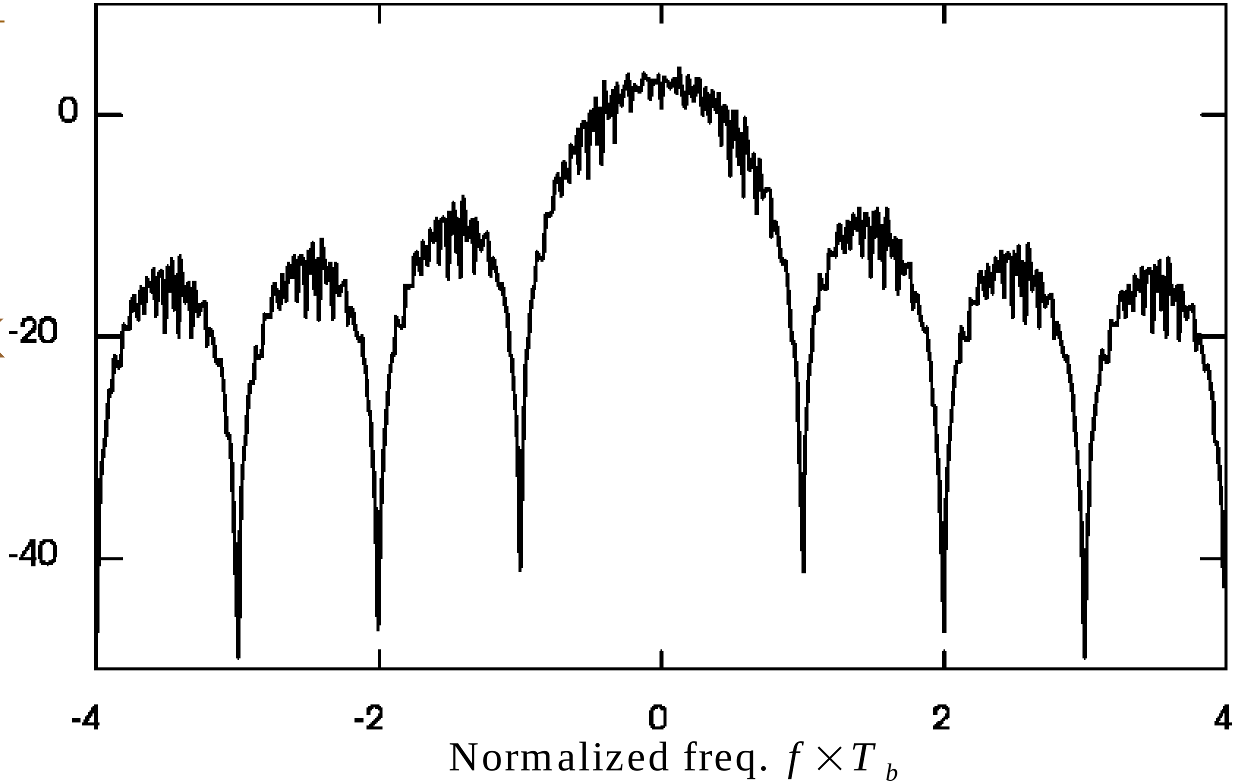


Signal constellation diagram



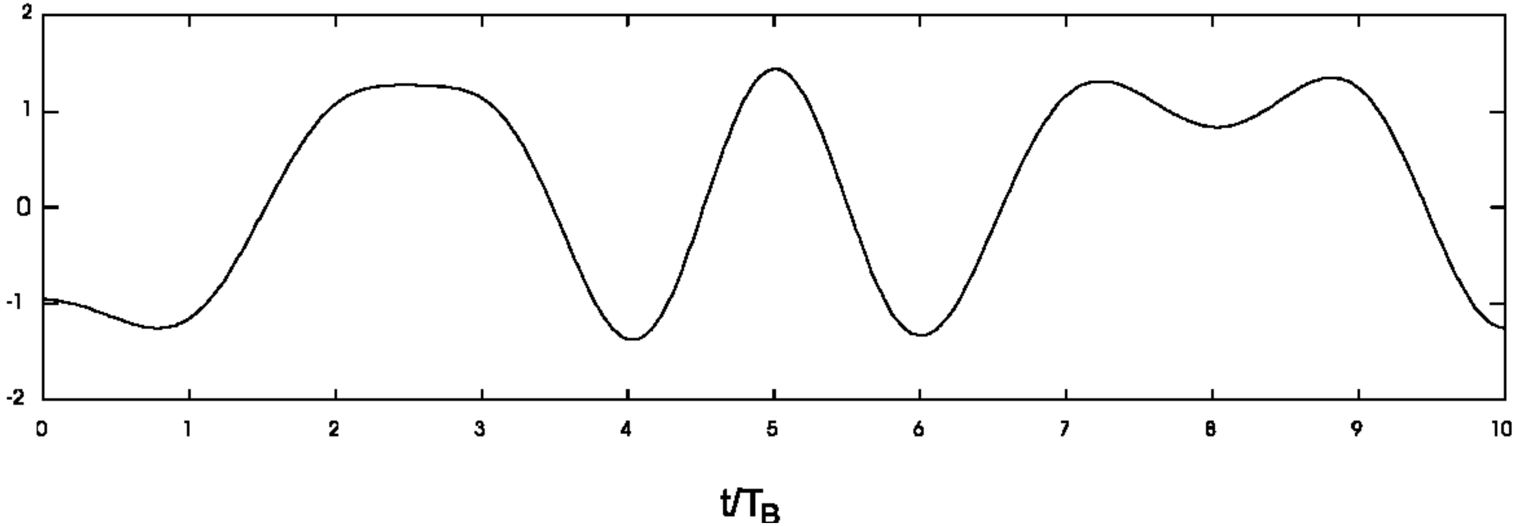
Binary phase-shift keying (BPSK)
Rectangular pulses

Power spectral density for BPSK

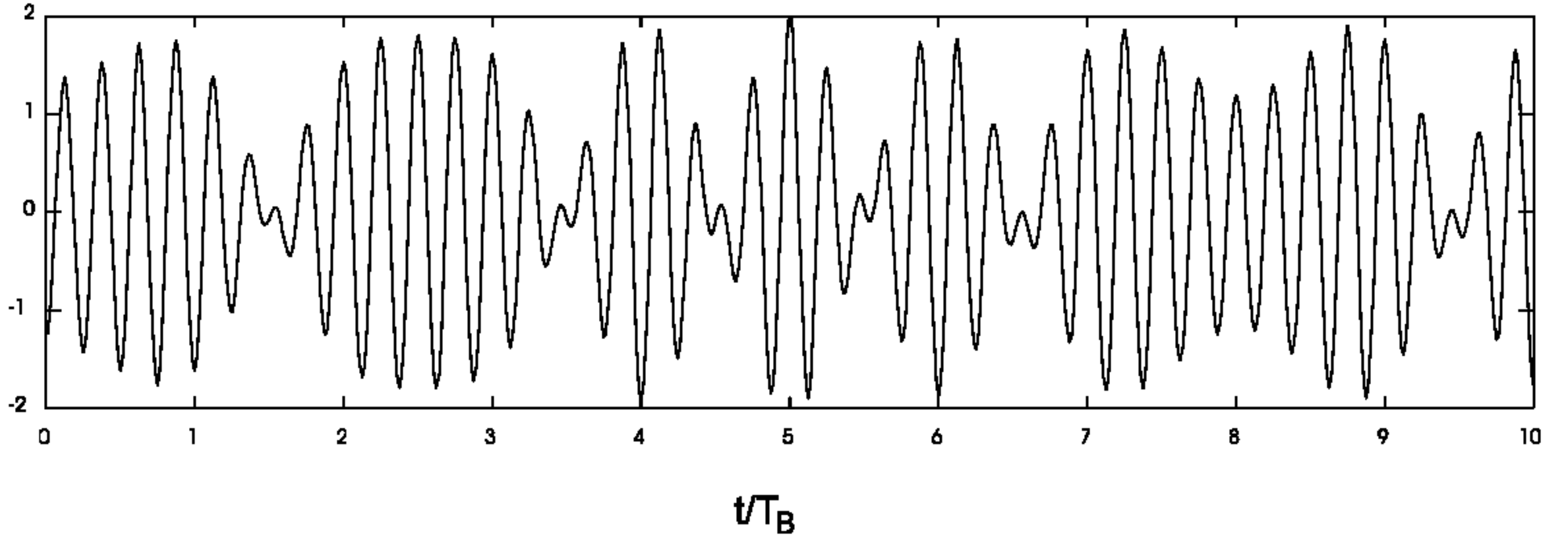


Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

Base-band

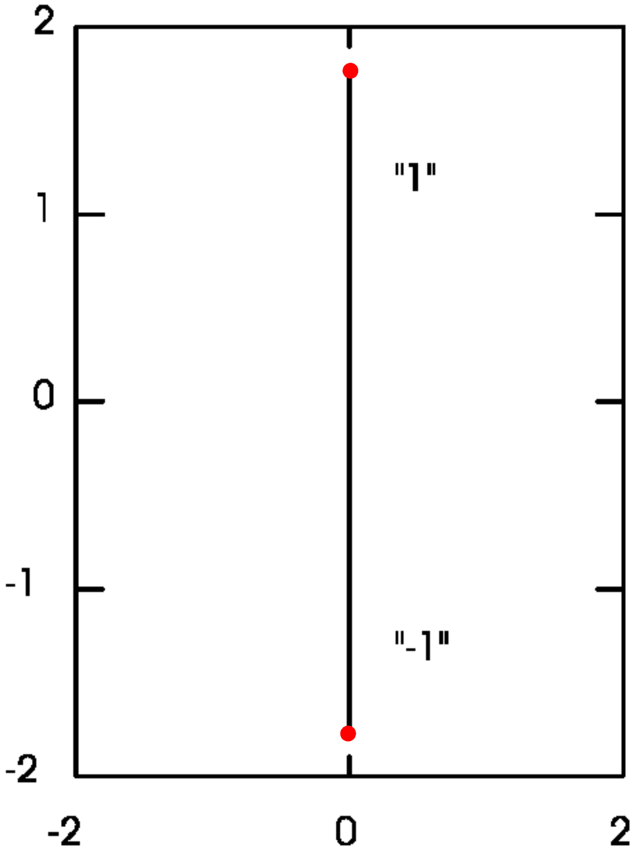


Radio signal

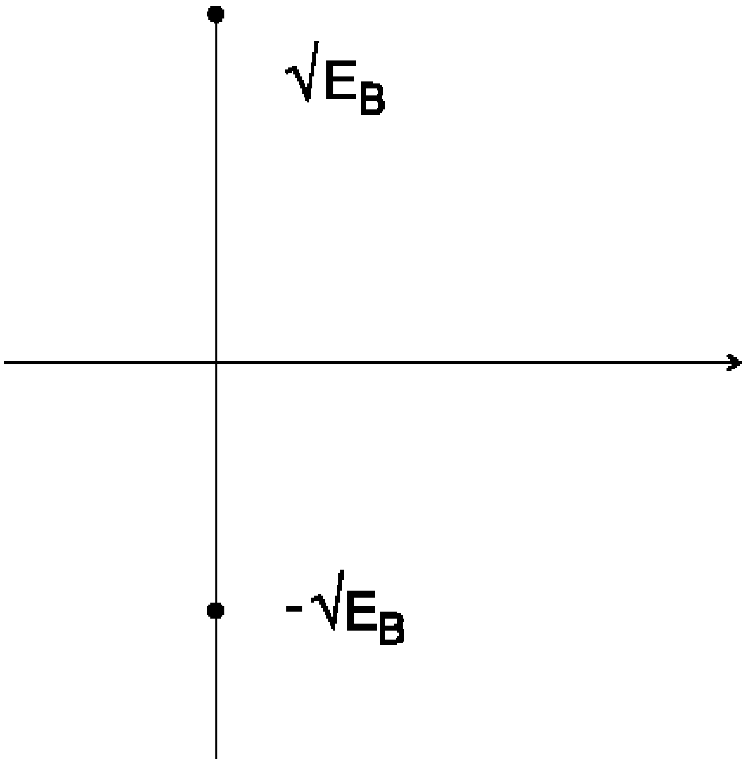


Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

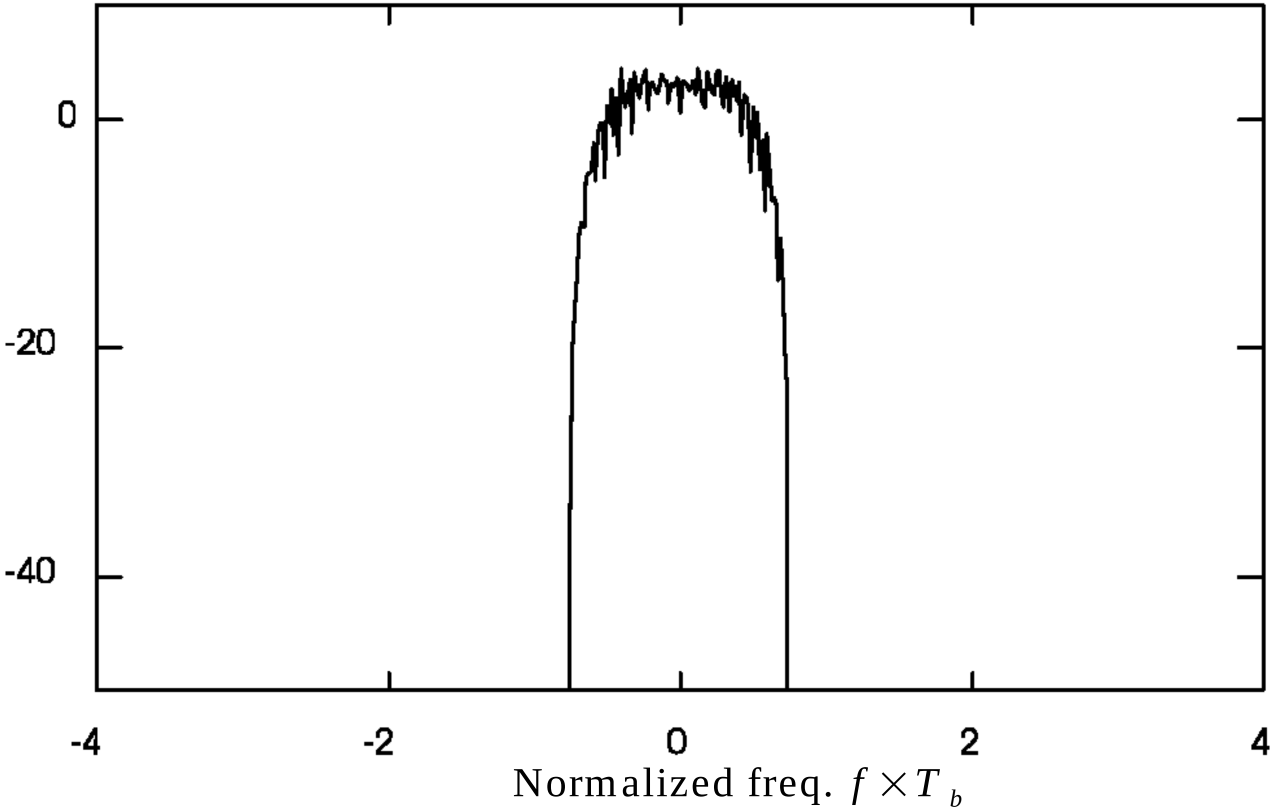
Complex representation



Signal constellation diagram



Binary phase-shift keying (BPSK) Raised-cosine pulses (roll-off 0.5)



Much higher spectral efficiency than BPSK (with rectangular pulses).

Power spectral density for BAM/BPSK

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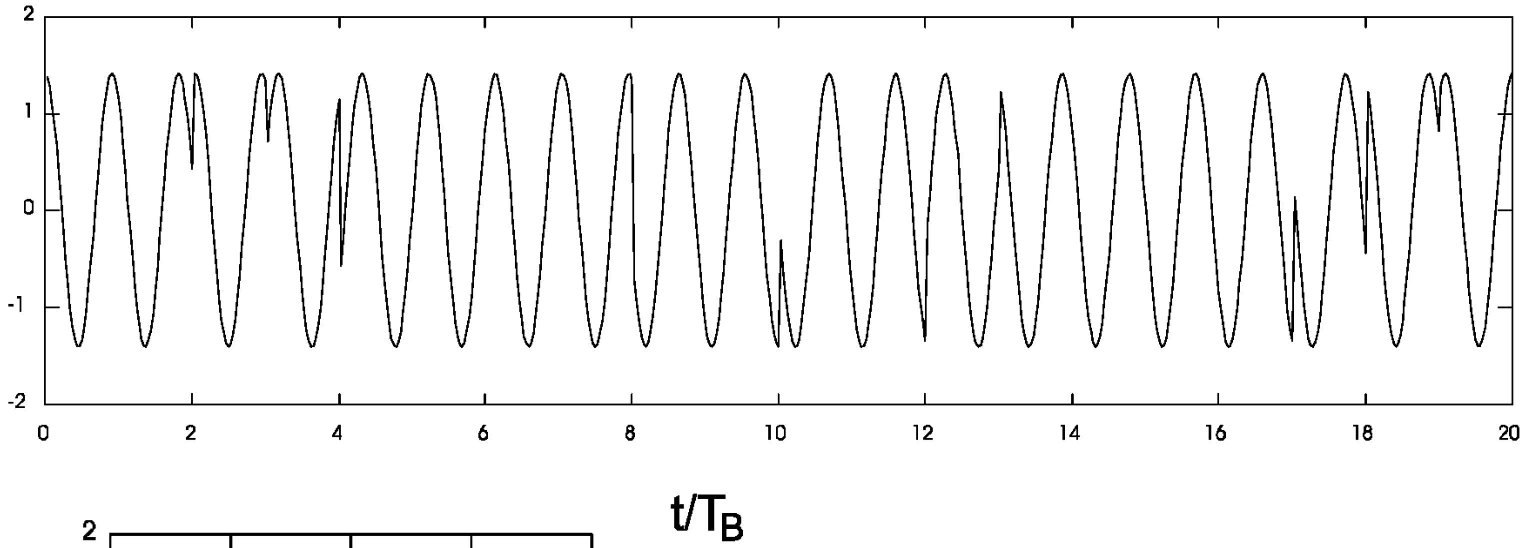
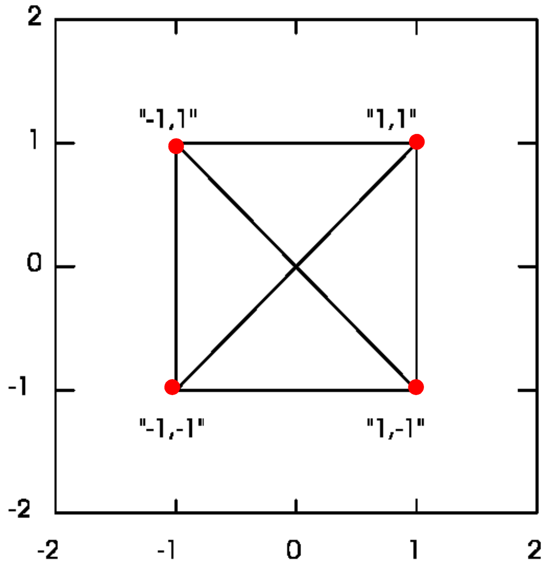
Quaternary PSK (QPSK or 4-PSK) Rectangular pulses



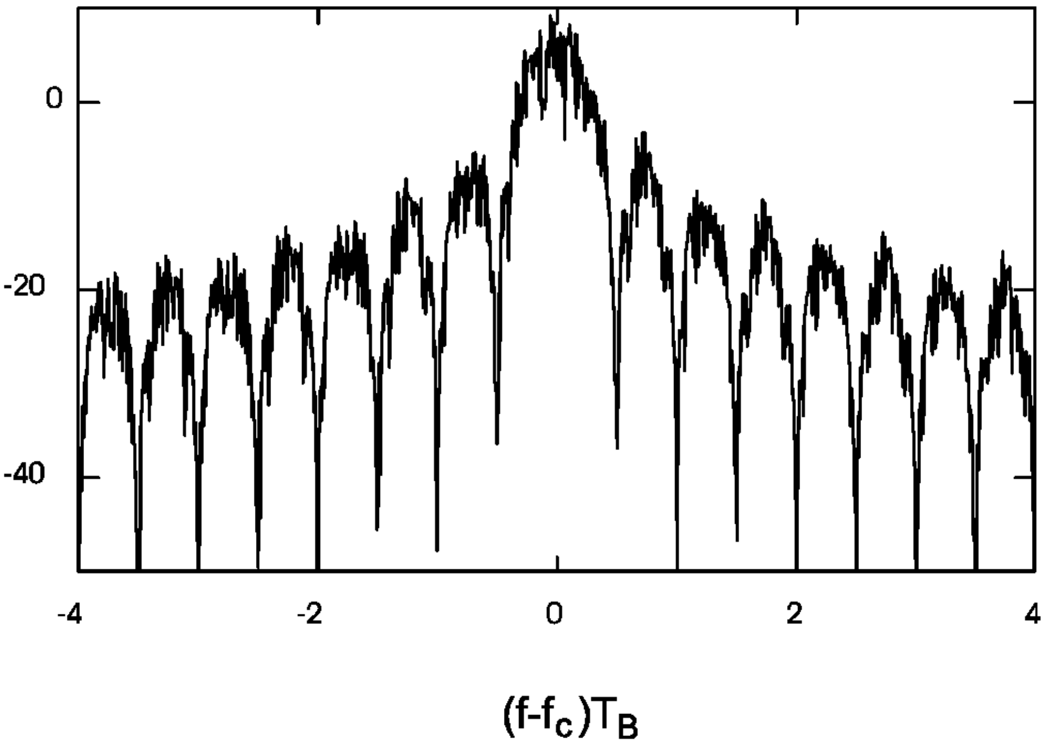
Radio signal



Complex representation

Quaternary PSK (QPSK or 4-PSK)
Rectangular pulses

- Power spectral density for QPSK



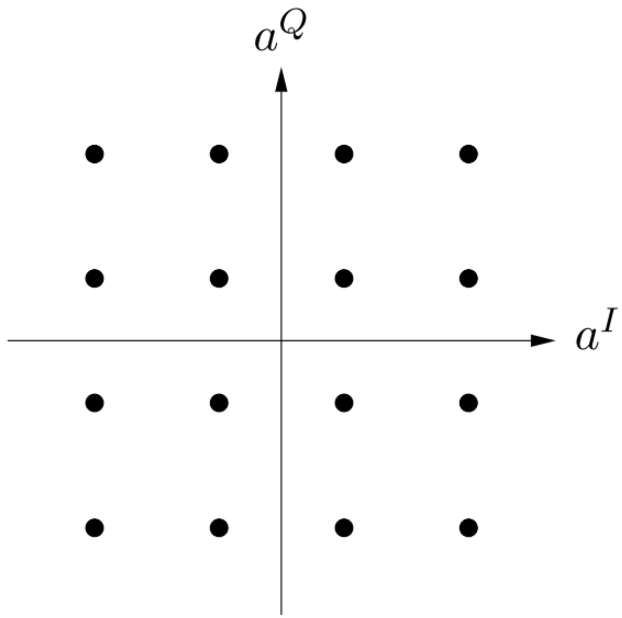
Twice the spectrum efficiency of BPSK (with rect. pulses). TWO bits/pulse instead of one.

A golden bandwidth rule

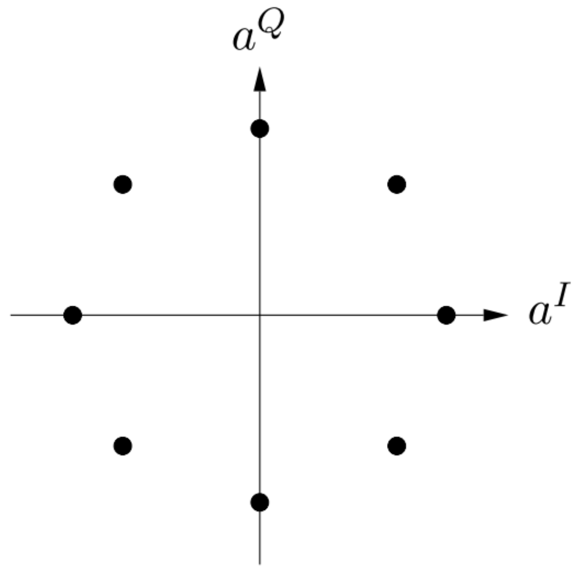
The narrowest bandwidth of any pulses that act independently is $[-1/2T, 1/2T]$ where T is the symbol interval

Other common signal constellations

- 16 QAM
  - Less bandwidth but higher SNR required



- 8 PSK



Detection, receivers

Detecting pulse waveforms

- Find the method that minimizes the error probability in white Gaussian noise
  - Correlation detector
- Correlate the received signal with a local copy of the ideal pulse alternatives

$$\mathrm{I}_{+} = + \int r(t)\sqrt{E_s}v(t)\,dt$$

$$\mathrm{I}_{-} = - \int r(t)\sqrt{E_s}v(t)\,dt$$

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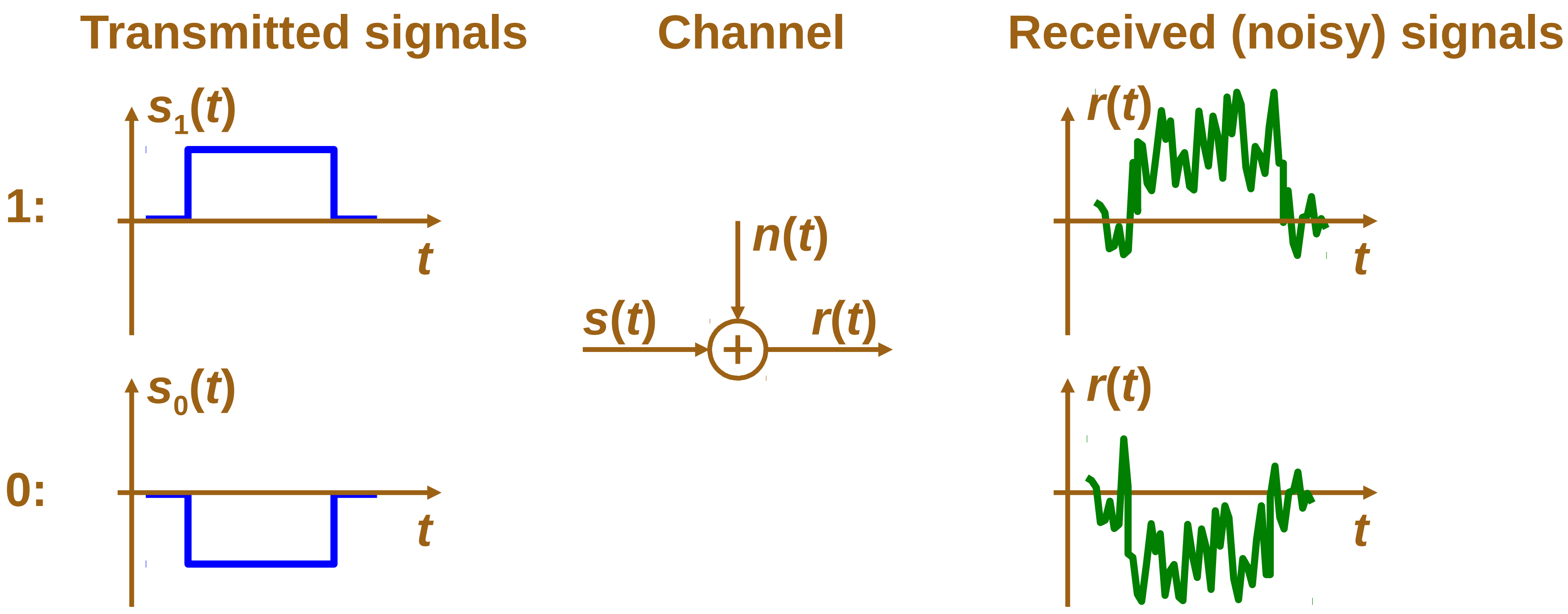
Optimal receiver
What do we mean by optimal?

Every receiver is optimal according to some criterion!

We would like to use optimal in the sense that we achieve a minimal probability of error.

In all calculations, we will assume that the noise is white and Gaussian – unless otherwise stated.

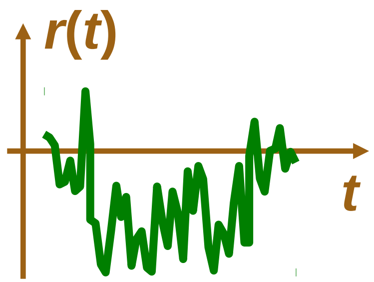
Optimal receiver
Transmitted and received signal



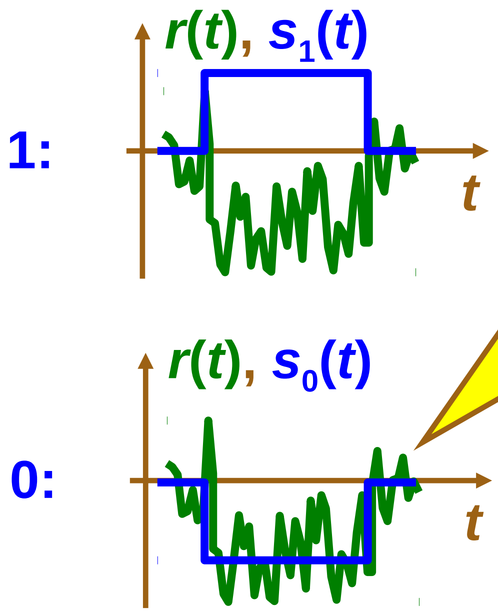
Optimal receiver
A first "intuitive" approach

"Look" at the received signal and compare it to the possible received noise free signals. Select the one with the best "fit".

Assume that the following signal is received:



Comparing it to the two possible noise free received signals:

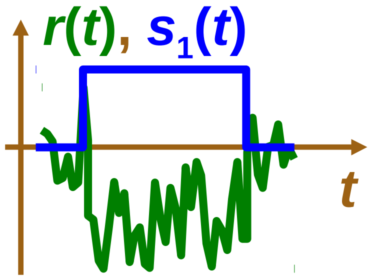


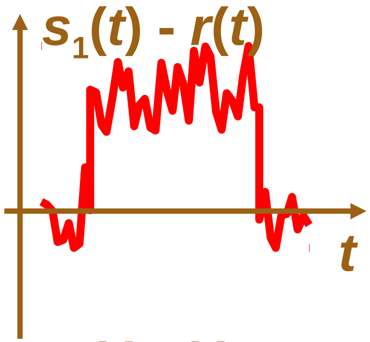
This seems to be the best "fit". We assume that "0" was the transmitted bit.

Optimal receiver
Let’s make it more measurable

To be able to better measure the “fit” we look at the energy of the residual (difference) between received and the possible noise free signals:

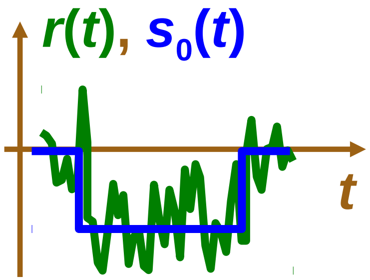
1:

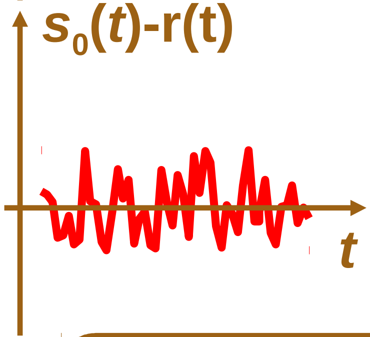




$$e_1 = \int |s_1(t) - r(t)|^2 dt$$

0:





$$e_0 = \int |s_0(t) - r(t)|^2 dt$$

This residual energy is much smaller. We assume that “0” was transmitted.

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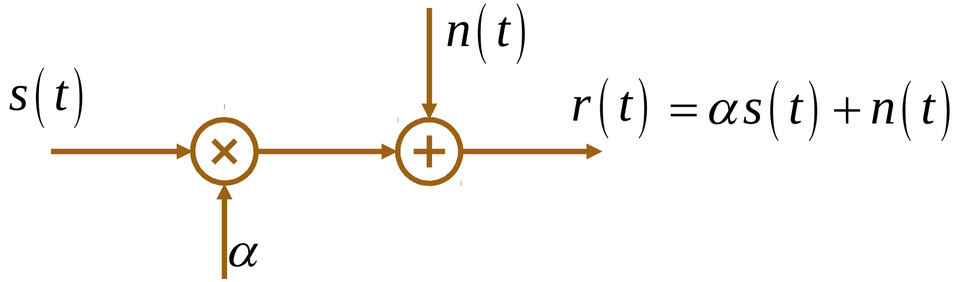
Optimal receiver
The AWGN channel

The additive white Gaussian noise (AWGN) channel



$s(t)$ **- transmitted signal**

α **- channel attenuation**

$n(t)$ **- white Gaussian noise**

$r(t)$ **- received signal**

In our digital transmission system, the transmitted signal $s(t)$ would be one of, let's say M, different alternatives $s_0(t), s_1(t), \ldots, s_{M-1}(t)$.

Optimal receiver
The AWGN channel, cont.

It can be shown that finding the minimal residual energy (as we did before) is the optimal way of deciding which of $s_0(t)$, $s_1(t)$, ... , $s_{M-1}(t)$ was transmitted over the AWGN channel (if they are equally probable).

For a received $r(t)$, the residual energy e_i for each possible transmitted alternative $s_i(t)$ is calculated as

$$e_i = \int \left| r(t) - \alpha s_i(t) \right|^2 dt = \int \left(r(t) - \alpha s_i(t) \right) \left(r(t) - \alpha s_i(t) \right)^* dt$$

$$= \int \left| r(t) \right|^2 dt - 2\,\mathrm{Re}\left\{ \alpha^* \int r(t)\, s_i^*(t)\, dt \right\} + \left| \alpha \right|^2 \int \left| s_i(t) \right|^2 dt$$

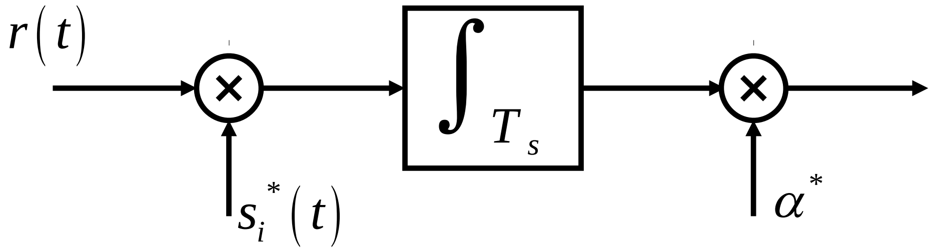
Same for all *i*

The residual energy is minimized by maximizing this part of the expression.

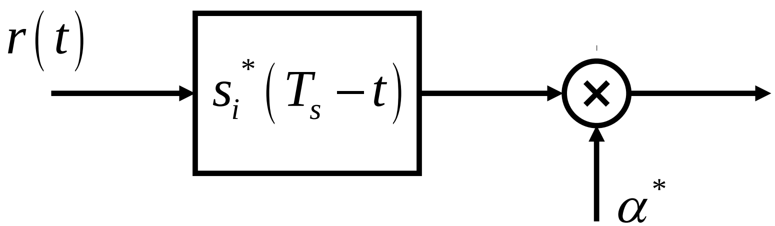
Same for all *i*, if the transmitted signals are of equal energy.

Optimal receiver
The AWGN channel, cont.

The central part of the comparison of different signal alternatives is a correlation, that can be implemented as a correlator:



or a matched filter



The real part of the output from either of these is sampled at $t = T_s$

where T_s is the symbol time (duration).

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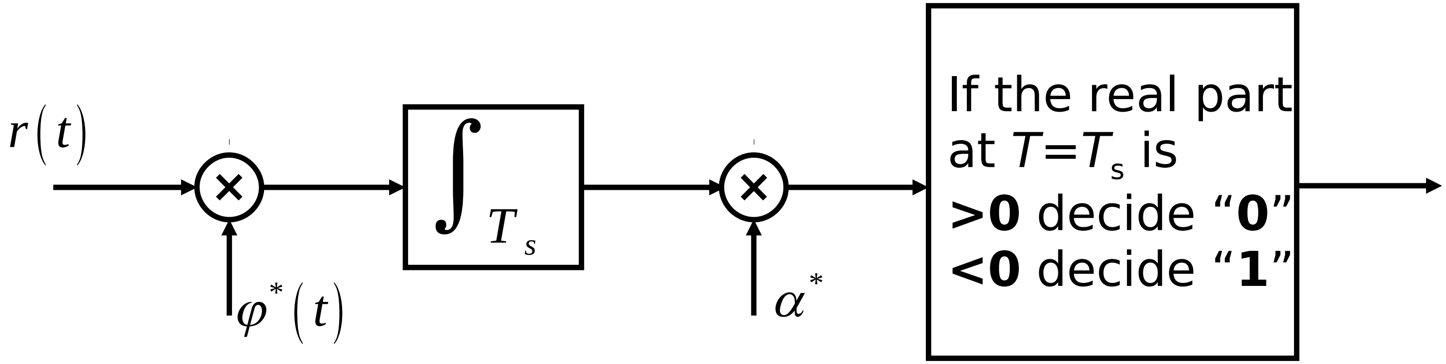
Optimal receiver
Antipodal signals

In antipodal signaling, the alternatives (for "0" and "1") are

$$s_0(t) = \varphi(t)$$
$$s_1(t) = -\varphi(t)$$

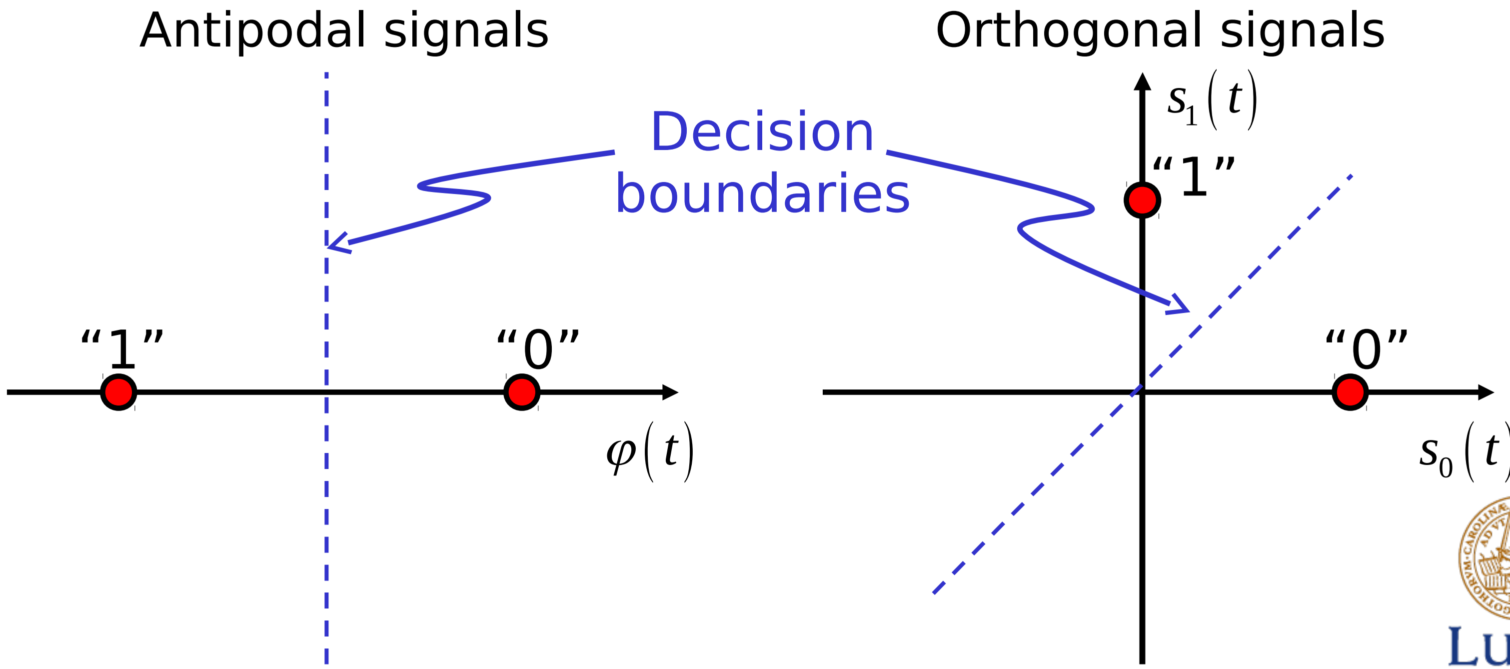
This means that we only need ONE correlation in the receiver for simplicity:



Optimal receiver
Interpretation in signal space

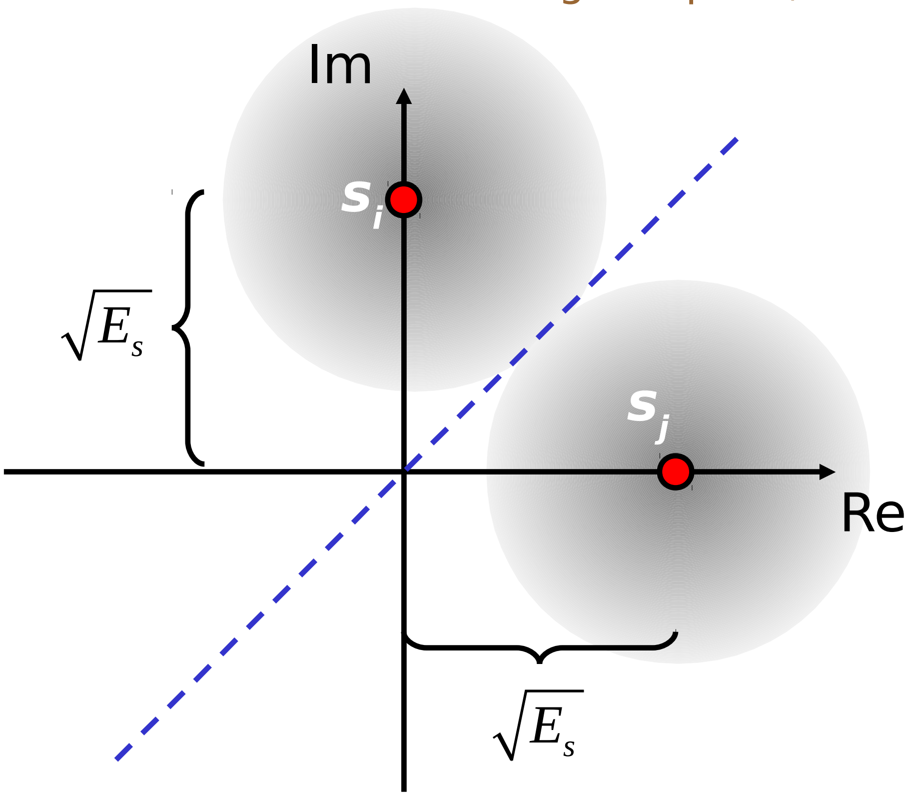
The correlations performed on the previous slides can be seen as inner products between the received signal and a set of basis functions for a signal space.

The resulting values are coordinates of the received signal in the signal space.



Optimal receiver
The noise contribution

Assume a 2-dimensional signal space, here viewed as the complex plane



This normalization of axes implies that the noise centered around each alternative is complex Gaussian

$$\mathrm{N}\left(0,\sigma^2\right) + j\mathrm{N}\left(0,\sigma^2\right)$$

with variance $\sigma^2 = N_0/2$ in each direction.

Fundamental question: What is the probability that we end up on the wrong side of the decision boundary?

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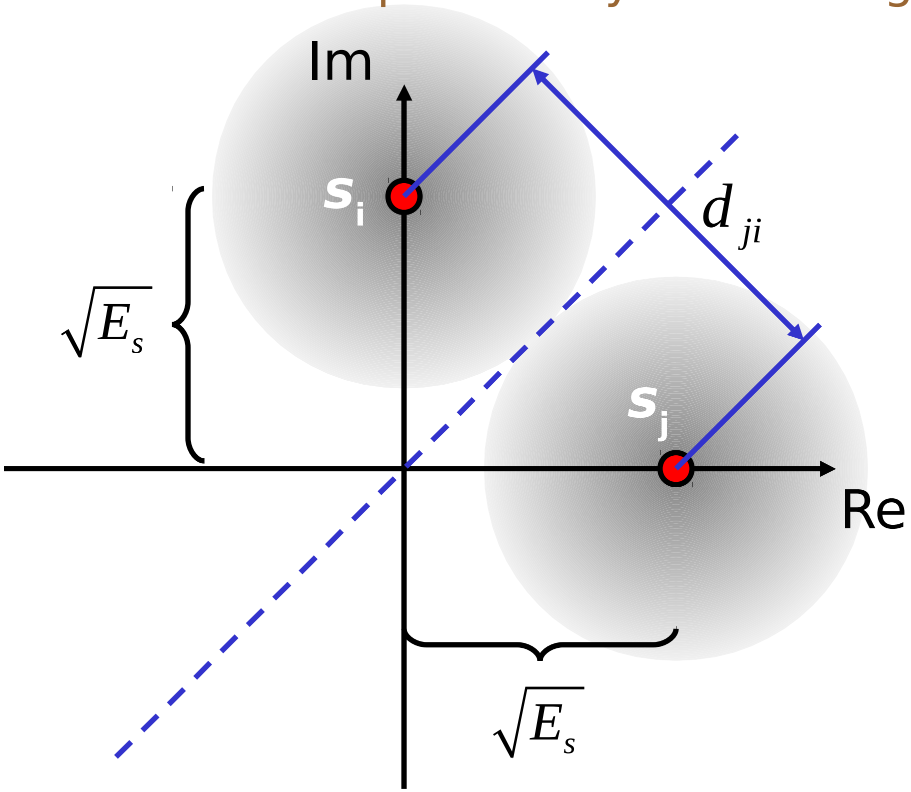
Optimal receiver
Pair-wise symbol error probability

What is the probability of deciding s_i if s_j was transmitted?



We need the distance between the two symbols. In this orthogonal case:

$$d_{ji} = \sqrt{\sqrt{E_s}^2 + \sqrt{E_s}^2} = \sqrt{2E_s}$$

The probability of the noise pushing us across the boundary at distance d_{ji} / 2 is

$$\Pr(s_j \to s_i) = Q\left(\frac{d_{ji}/2}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

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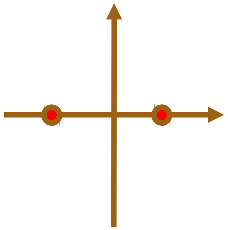
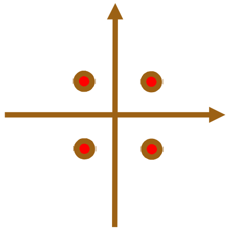
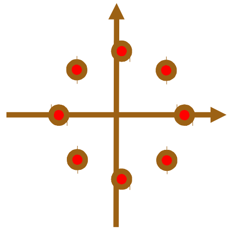
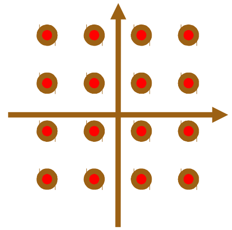
Optimal receiver
Bit-error rates (BER)

EXAMPLES:	2PAM	4QAM	8PSK	16QAM
				
Bits/symbol	**1**	**2**	**3**	**4**
Symbol energy	E_b	$2E_b$	$3E_b$	$4E_b$
BER	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\sim\frac{2}{3}Q\left(\sqrt{0.87\frac{E_b}{N_0}}\right)$	$\sim\frac{3}{2}Q\left(\sqrt{\frac{E_{b,\max}}{2.25N_0}}\right)$

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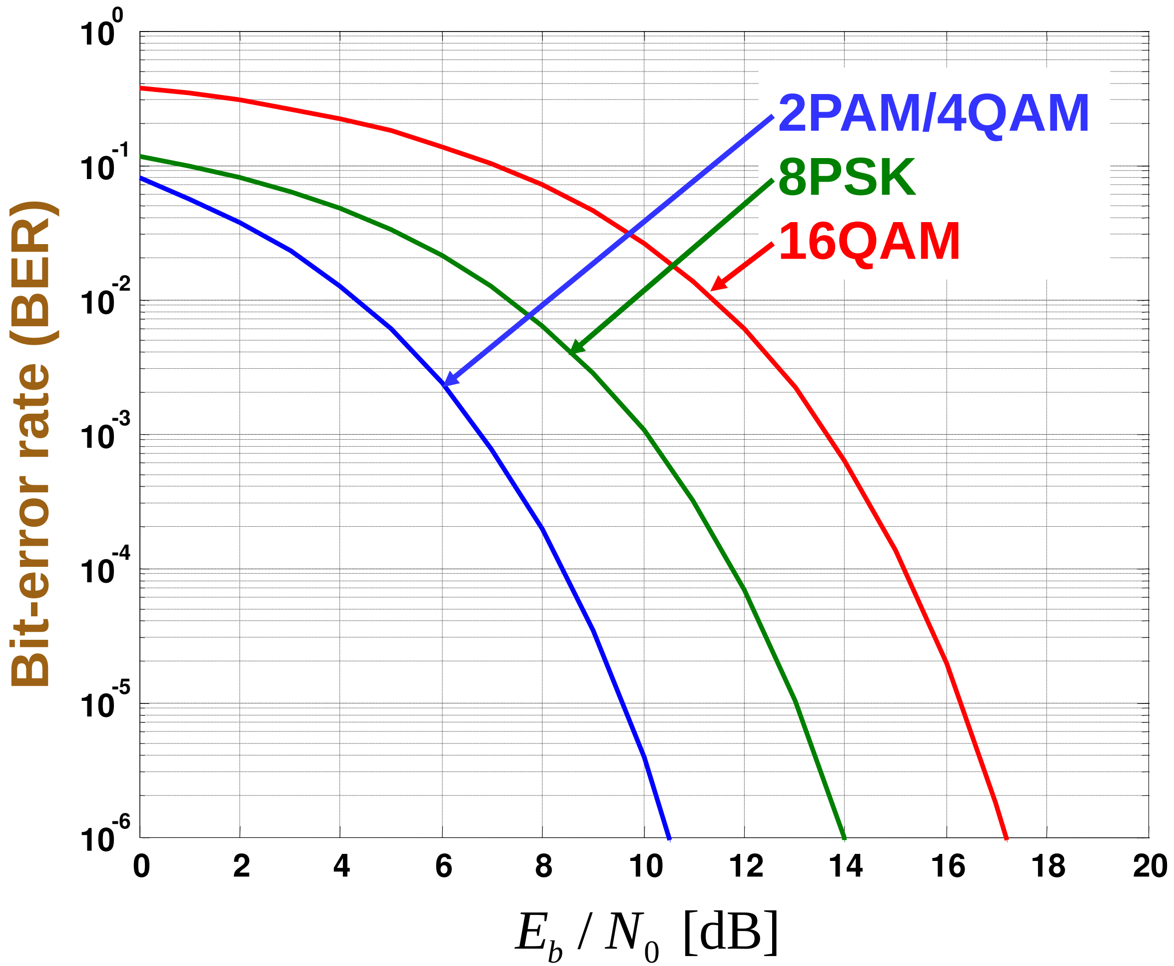
Optimal receiver
Bit-error rates (BER), cont.



Quadrature modulation, why is it working?

Any carrier digital modulation can be expressed as

$$s(t) = \sqrt{2E_s}\, I(t) \cos 2\pi f_0 t - \sqrt{2E_s}\, Q(t) \sin 2\pi f_0 t$$

The sine and cosine ”channels” are independent/orthogonal

$$\int_{-T/2}^{T/2} g_1(t) \cos 2\pi f_0 t \;\, g_2(t) \sin 2\pi f_0 t \;\, dt = 0$$

Therefore we can send two pulses at the same time without interference

$$\begin{aligned} s(t) \;\; &= \;\; \sqrt{2E_s} \left(\sum a_n^I v(t-nT) \right) \cos 2\pi f_0 t \\ &\qquad - \sqrt{2E_s} \left(\sum a_n^Q v(t-nT) \right) \sin 2\pi f_0 t \end{aligned}$$

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SUMMARY

- Bits/symbols are carried on analog signals by altering their amplitude/phase/frequency.
- Modulation basics, basis pulses
- Relation between data rate and bandwidth
- IQ modulator
- Basic modulation formats
- Detection of data at receiver - optimal receiver in AWGN channels
- Interpretation of received signal as a point in a signal space
- Euclidean distances between symbols determine the probability of symbol error
- Bit error rate (BER) calculations for some signal constellations

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