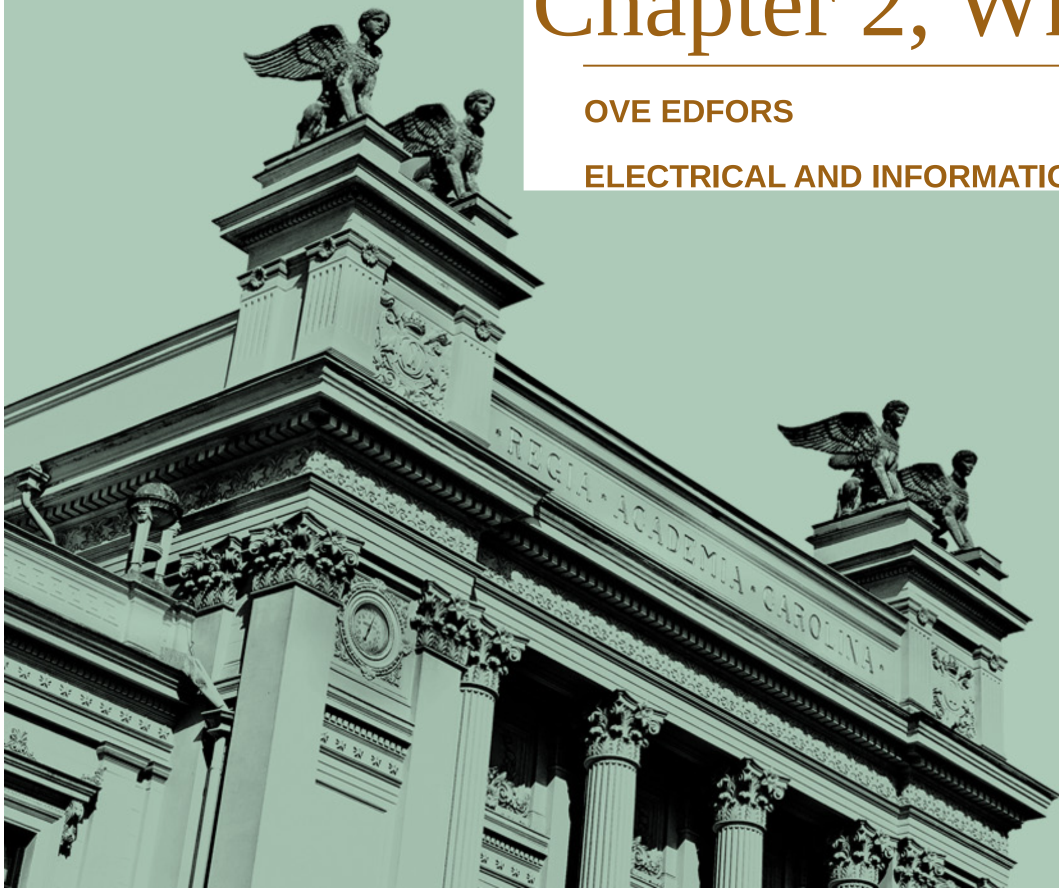
Lund University

# Information Transmission
# Chapter 2, What is bandwidth


**OVE EDFORS**

**ELECTRICAL AND INFORMATION TECHNOLOGY**

# Learning outcomes

- Afther this lecture, the student should
  - understand the basic principles of sampling, including
  - the concept of orthogonal basis functions,
  - the sampling theorem,
  - Nyquist rates/frequencies and Shannon bandwidths, and
  - be able to perform calculations on necessary sampling rates based on the characteristics of the sampled signals.

LUND UNIVERSITY

**“Certain factors affecting telegraph speed”, H. Nyquist,**

**The Bell System Technical Journal (Volume:3 , Issue: 2 ), 1924**

“This paper considers two fundamental factors entering into the maximum speed of transmission of intelligence by telegraph. These factors are signal shaping and choice of codes. The first is concerned with the best wave shape to be impressed on the transmitting medium so as to permit of greater speed without undue interference either in the circuit under consideration or in those adjacent….”

LUND UNIVERSITY

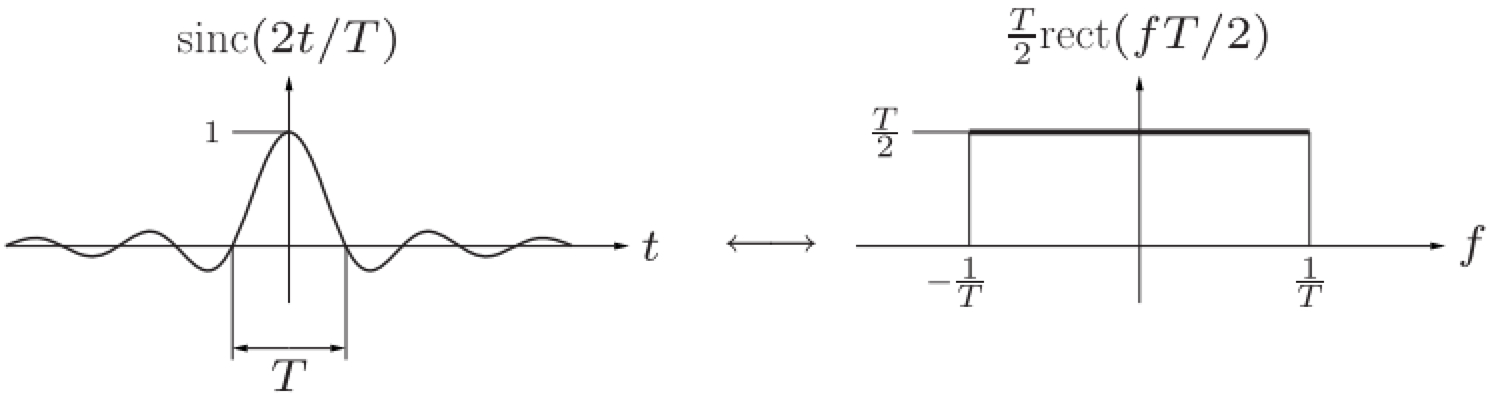
# An introductory example

Assume $x(t) = \mathrm{sinc}(2t/T)$

Combining $\mathrm{sinc}(t) \leftrightarrow \mathrm{rect}(f)$

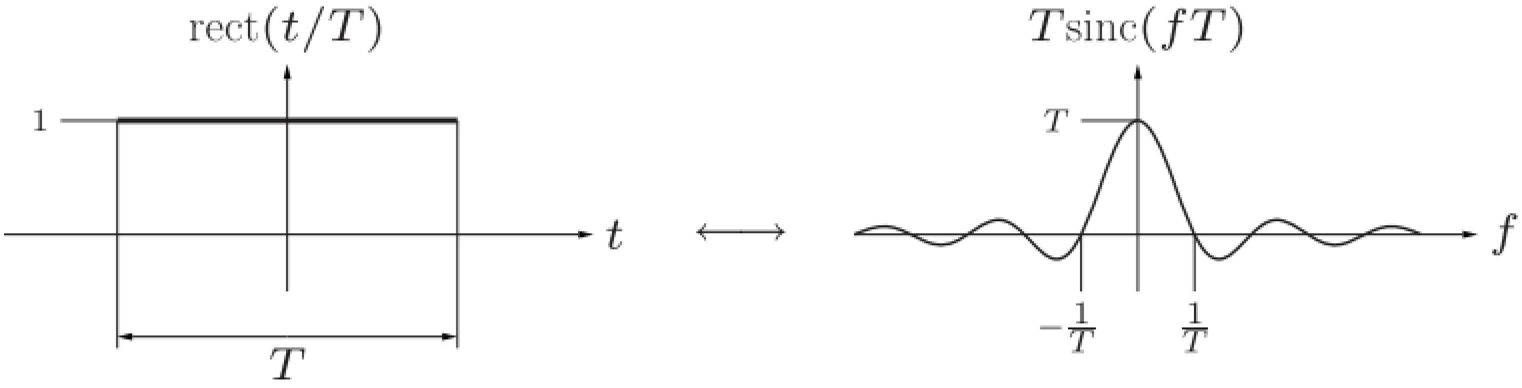
with the time scaling property, $x(at) \leftrightarrow |a|^{-1}X(f/a)$, yields

$$\mathrm{sinc}(2t/T) \leftrightarrow \frac{T}{2}\mathrm{rect}(fT/2) = \begin{cases} \frac{T}{2}, & |f| < \frac{1}{T} \\ 0, & |f| > \frac{1}{T} \end{cases}$$




What is the bandwidth of the signal?

# What is the bandwidth of a rect pulse?




The spectrum is not confined to a finite band

# Bandwidth of a signal

A softer restriction of bandwidth, that works for all signals, is that the signal *x(t)* has a certain fraction of its energy inside the frequency band [-*W*,*W*], i.e.

$$\frac{\int_{-W}^{W} |X(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df} = \eta$$

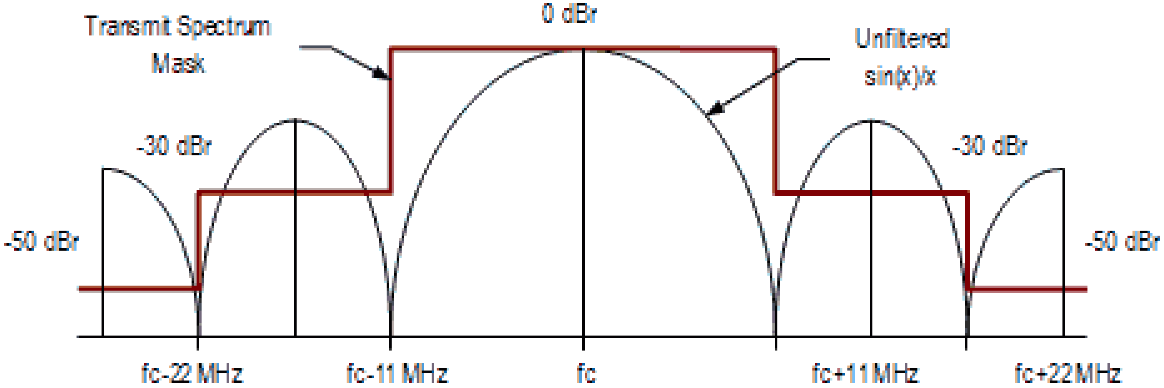
With this it seems reasonable to say that the signal $\mathrm{rect}(t/T)$ has a Fourier bandwidth of *W = 1/T* Hz.

To be more precise, we could say that the “90%-energy Fourier bandwidth" of the signal is *W = 1/T* Hz, but we will usually not have need for such precision.

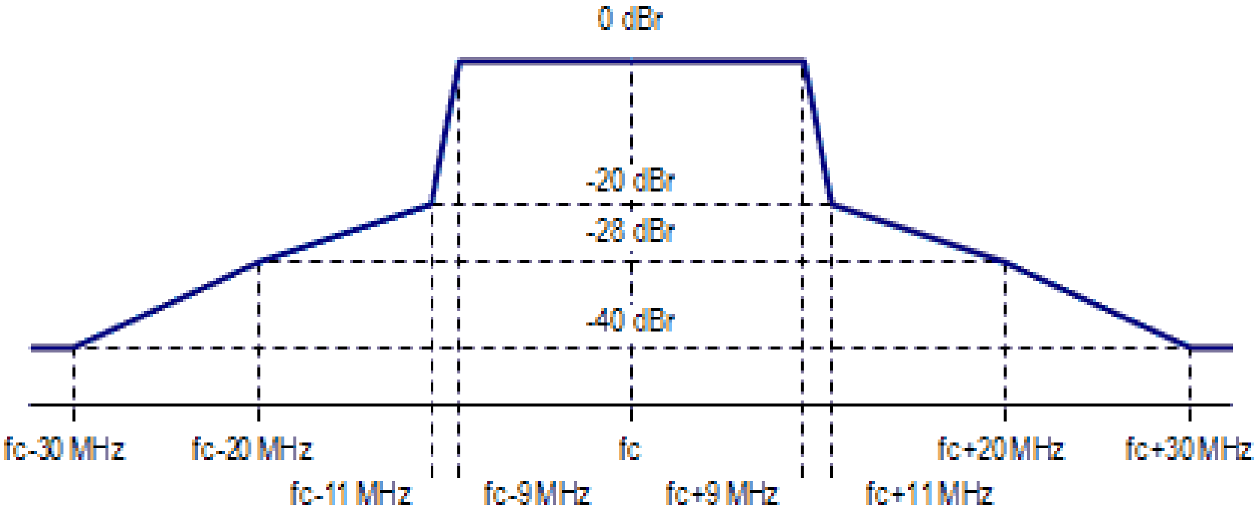
LUND UNIVERSITY

# WLAN - IEEE 802.11 spectrum mask

802.11b




802.11a




Source: http://www.rfcafe.com/references/electrical/wlan-masks.htm

# The sampling theorem

# “Certain Topics in Telegraph Transmission Theory”, H. Nyquist,

**Transactions of the American Institute of Electrical Engineers (Volume:47 , Issue: 2 ), 1928**

“The most obvious method for determining the distortion of telegraph signals is to calculate the transients of the telegraph system... The present paper attacks the same problem from the alternative standpoint…This method of treatment necessitates expressing the criteria of distortionless transmission in terms of the steady-state characteristics. Accordingly, a considerable portion of the paper describes and illustrates a method for making this translation. A discussion is given of the minimum frequency range required for transmission at a given speed of signaling….”

# Basis functions for sampling

Consider the function

$$\phi(t) = \sqrt{2W}\,\frac{\sin 2\pi W t}{2\pi W t} = \sqrt{2W}\,\mathrm{sinc}(2Wt)$$

with Fourier transform

$$\Phi(f) = \frac{1}{\sqrt{2W}}\,\mathrm{rect}\left(\frac{f}{2W}\right) = \begin{cases} \frac{1}{\sqrt{2W}}, & |f| < W \\ 0, & |f| > W \end{cases}$$

$\phi(t)$ is confined to the frequency band [-*W*,*W*].

# Orthonormal functions

Versions of $\phi(t)$ delayed by $\frac{k}{2W}$ ,

$$\phi_k(t) = \phi\left(t - \frac{k}{2W}\right) = \sqrt{2W}\,\mathrm{sinc}\left(2W\left(t - \frac{k}{2W}\right)\right)$$

where *k* is an integer, form a set of *orthonormal functions*.

The term orthonormal means that the functions are ***orthogonal*** and ***normalized****.*

LUND UNIVERSITY

# Orthogonal functions

Orthogonality is an important notion in signal analysis. it means that

$$\int_{-\infty}^{\infty} \phi_k(t)\phi_l^*(t)dt = \begin{cases} e_k, & l = k \\ 0, & l \neq k \end{cases}$$

where $e_k$ is the energy of $\phi_k(t)$

LUND UNIVERSITY

# Orthogonality of the basis functions

$\phi_k(t)$, where *k* is an integer, are *orthogonal* functions since

$$\int_{-\infty}^{\infty} \phi_k(t)\phi_l^*(t)dt = \int_{-\infty}^{\infty} \Phi_k(f)\Phi_l^*(f)df$$

$$= \frac{1}{2W}\int_{-W}^{W} e^{j\omega(l-k)/2W} df$$

$$= \frac{2}{2W}\int_{0}^{W} \cos\left(\frac{\omega(l-k)}{2W}\right) df = \begin{cases} 1, & l = k \\ 0, & l \neq k \end{cases}$$

What does this mean?

LUND UNIVERSITY

# Normalized functions

Furthermore, since

$$\int_{-\infty}^{\infty} |\phi_k(t)|^2 dt = \int_{-\infty}^{\infty} |\phi(t)|^2 dt = 1$$

for all *k*, these functions are *normalized* (energy $e_k = 1$).

A set of orthogonal and normalized functions is called an *orthonormal set of functions*.

LUND UNIVERSITY

# The sampling theorem

If *x(t)* is a signal whose Fourier transform is identically zero for $|f| \geq W$, then *x(t)* is completely determined by its samples taken every $\frac{1}{2W}$ seconds in the manner

$$x(t) = \sum_{k=-\infty}^{\infty} x\left(\frac{k}{2W}\right) \operatorname{sinc}\left(2W\left(t - \frac{k}{2W}\right)\right)$$

See the book for a proof

LUND UNIVERSITY

# Nyquist rate/Nyquist frequency

The sample points $\{x(\frac{k}{2W})\}$ are taken at the rate *2W* samples per second.

If *W* is the smallest frequency such that the Fourier transform of *x(t)* is identically zero for $|f| \geq W$ then the sampling rate 2W is called the *Nyquist rate* or Nyquist frequency.

# Shannon bandwidth

Let $T_N$ denote the smallest $\tau$ such that $\psi(t)$ is orthogonal to every time-shift of itself $\psi(t - k\tau)$ by a nonzero multiple of $\tau$

We call $T_N$ the *Nyquist-shift* of the basis signal $\psi(t)$

The Nyquist-shift of the signal $\phi_k(t) = \operatorname{sinc}\left(2W\left(t - \frac{k}{2W}\right)\right)$ is $T_N = \frac{1}{2W}$.

The *Shannon bandwidth B* of the basis signal $\psi(t)$ is $B = 1/2T_N$ or equivalently $2B = 1/T_N$ basis functions per second

LUND UNIVERSITY

# The fundamental theorem of bandwidth

The Shannon bandwidth $B$ of a basis signal is at most equal to its Fourier bandwidth $W$; equality holds when the signal is a sinc function.

The Shannon bandwidth can be thought of as the amount of bandwidth a signal *needs* and the Fourier bandwidth as the amount of bandwidth a signal *uses*.

# Summary

- Under certain conditions, ”arbitrary” continuous-time signals can be perfectly described by discrete samples.
- Orthogonal basis functions are an important tool for translating between continuous-time sigbals and their disctrete-time sampled equivalents.
- According to the sampling theorem, a real time-continuous signal without frequency components from $W$ Hz and above, can be perfectly represented by samples taken at a rate $2W$ samples/sec, or $2W$ Hz. This sampling rate (frequency) is called the *Nyquist rate* or *Nyquist frequency*.
- A signal with highest frequency component at $W$ Hz (Nyquist bandwidth W) may have a characteristic that allows it to be represented by fewer samples than what is given by the Nyquist sampling rate. This is characterized by the Shannon bandwidth, which is then said to be smaller than the Nyquist bandwidth.

SIGILLVM · VNIVERSITATIS · GOTHORVM · CAROLINÆ
AD VTRVMQVE
1666

LUND
UNIVERSITY