Information Transmission
Appendix B, Circuit theory

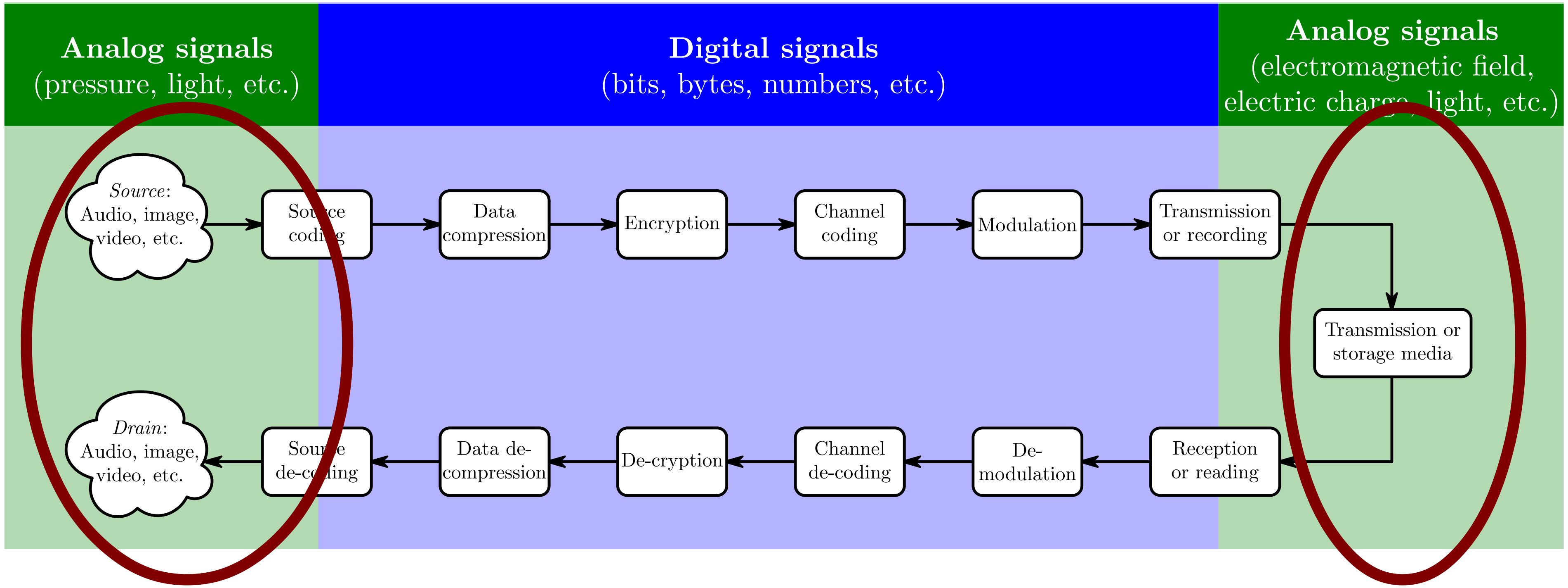
OVE EDFORS
ELECTRICAL AND INFORMATION TECHNOLOGY

Learning outcomes

After this lecture the student should

- Know the properties of, and be able to perform basic calculations with, resistors, inductors and capacitors
- Know how resistors, inductors, and capacitors, behave when sinusoidal signals are applied.
- Know Kirchhoff's voltage and current laws and understand how they are applied to perform basic calculations on electronic circuits.
- Understand the impedance concept and how to caclculate the total impedance of multiple impedances connected in serial or parallel

Where are we in the BIG PICTURE?

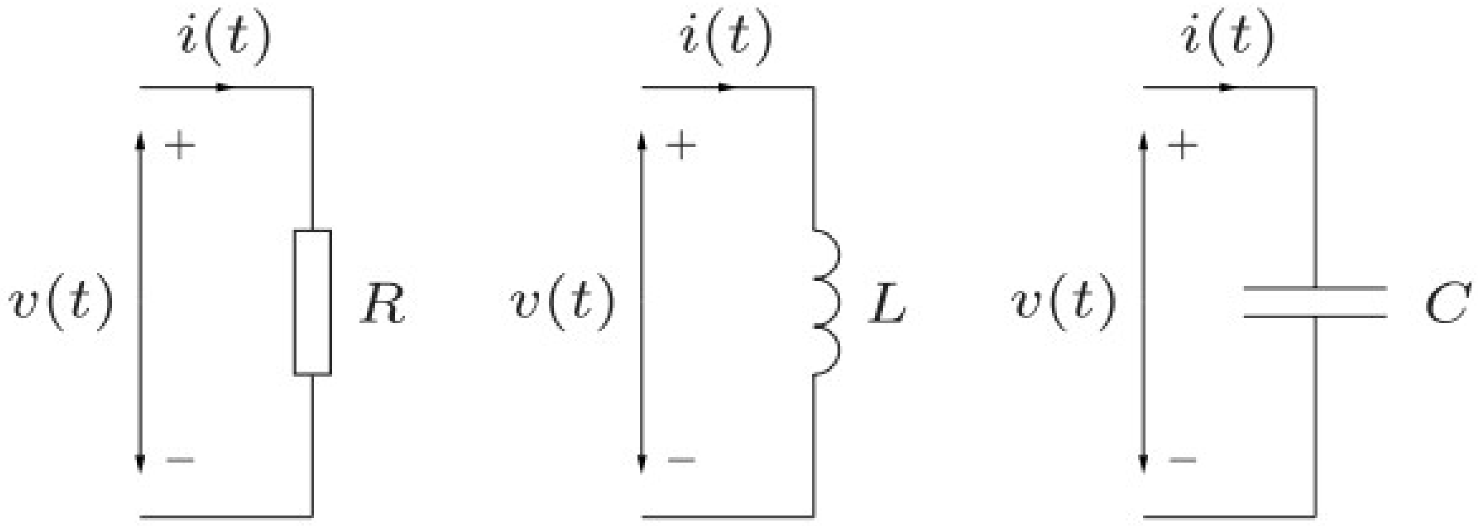


Electronics for analog input and output, including sampling and reconstruction.

Lecture relates to pages 281–293 in textbook.

Models of transmission and storage media.

Resistors, Inductors, Capacitors



Resistors

Ohm's law: the voltage $v(t)$ volt [V] across a resistor with resistance R ohm [Ω] is proportional to the current $i(t)$ ampére [A] through the resistance.

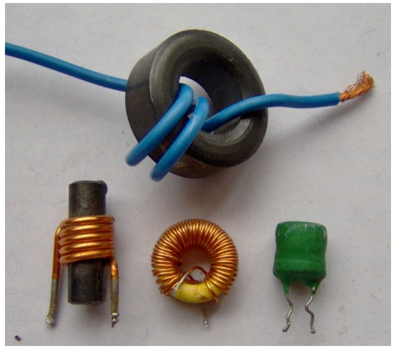
$$v(t) = Ri(t) \tag{B.1}$$

For $i(t) = e^{j\omega_0 t}$ we have

$$\boxed{v(t) = \underbrace{R}_{\text{Does not depend on frequency.}} e^{j\omega_0 t}} \tag{B.2}$$

Inductors

The voltage across an inductor with inductance L henry [H] is proportional to the derivative of the current $i(t)$ through the inductor.



$$v(t) = L\frac{di(t)}{dt} \tag{B.3}$$

For $i(t) = e^{j\omega_0 t}$ we have

$$\boxed{v(t) = \underbrace{j\omega_0 L}_{\text{Depends (increases) with frequency.}} e^{j\omega_0 t}} \tag{B.4}$$

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Capacitors

The charge $q(t)$ coulombs [C] of a capacitor with capacitans C farad [F] is proportional to the voltage across the capacitor:

$$q(t) = Cv(t)$$

Since $q(t) = \int_{-\infty}^{t} i(\tau)d\tau$, we have

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

For $i(t) = e^{j\omega_0 t}$ we have

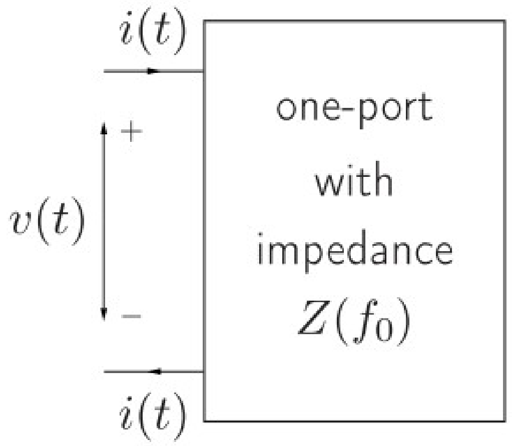
Depends on (decreases with) frequency.

$$\boxed{v(t) = \frac{1}{C}\int_{-\infty}^{t} e^{j\omega_0\tau}d\tau = \overbrace{\frac{1}{j\omega_0 C}}e^{j\omega_0 t}}$$

Impedance

The one-port shown in Fig. B.2 is a network consisting of resistors, inductors, and capacitors. The voltage across the one-port is $v(t) = e^{j\omega_0 t}$.

The current $i(t)$ will also be a complex exponential signal;
but, in general, with different amplitude and phase.



We have

$$\boxed{v(t) = Z(f_0)i(t)}$$

where $Z(f_0)$ is called the impedance.

Impedance

The impedance is in general complex and dependent on the frequency f_0, but if the one-port consists only of resistors, then its impedance will always be real and independent of the frequency f_0; that is, it is a resistance.

Ohm's law for alternating current holds only for stationary sinusoidal voltages and currents (including the special case when $f_0 = 0$).

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Kirchhoff's current law

Kirchhoff's current law (KCL): The algebraic sum of the currents entering any node is identically zero for all instants of time.

OR:

KCL: Sum of currents flowing into a node
= sum of currents leaving the node

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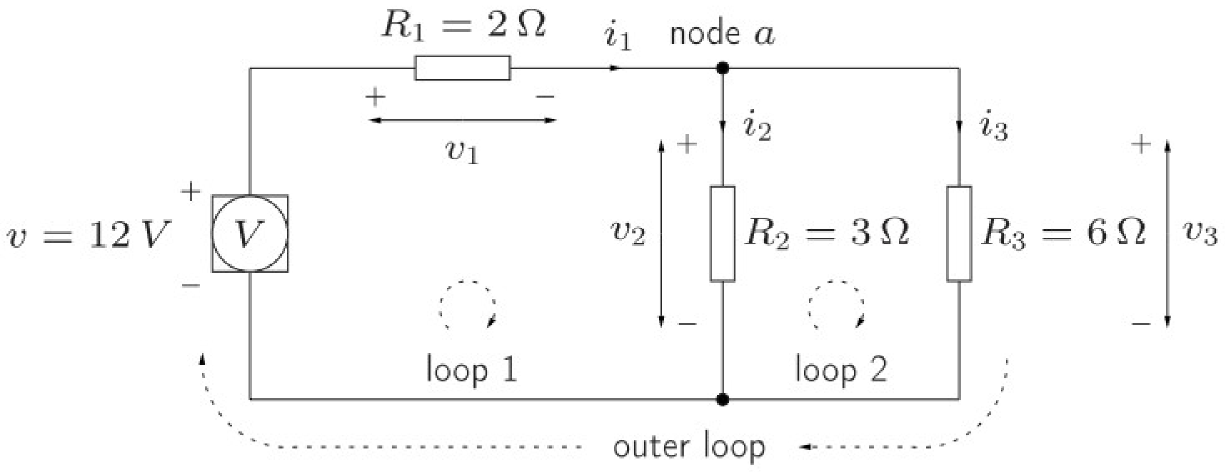
Kirchhoff's voltage law

Kirchhoff's voltage law (KVL): *The algebraic sum of the voltages around any closed path, or loop, in a circuit is identically zero for all instants of time.*

Example 1 (Example B.1 in textbook)



What is i_1, i_2, i_3, v_1, v_2, v_3?

Six unknowns –
we need six equations!

One of these are
redundant, so we need
one more.

Ohms law:

$$v_1 = 2i_1 \text{ (resistor } R_1\text{)}$$
$$v_2 = 3i_2 \text{ (resistor } R_2\text{)}$$
$$v_3 = 6i_3 \text{ (resistor } R_3\text{)}$$

Kirchoff's voltage law:

$$12 - v_1 - v_2 = 0 \text{ (loop 1)}$$
$$v_2 - v_3 = 0 \text{ (loop 2)}$$
$$12 - v_1 - v_3 = 0 \text{ (outer loop)}$$

Kirchoff's current law:

$$i_1 - i_2 - i_3 = 0 \text{ (node a)}$$

$\boxed{\text{Solution: } i_1 = 3 \text{ A}, i_2 = 2 \text{ A}, i_3 = 1 \text{ A}, v_1 = v_2 = v_3 = 6 \text{ V}}$

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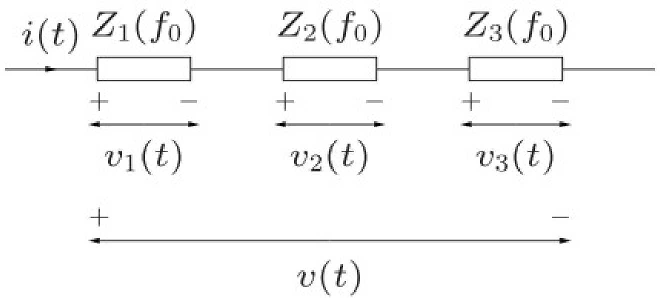
Serial impedance

Consider three impedances connected in a serial manner.
Assuming that the current $i(t)$ is sinusoidal with frequency f_0, let $Z_s(f_0)$ denote the impedance of this serial circuit.

$$v(t) = Z_s(f_0)i(t)$$

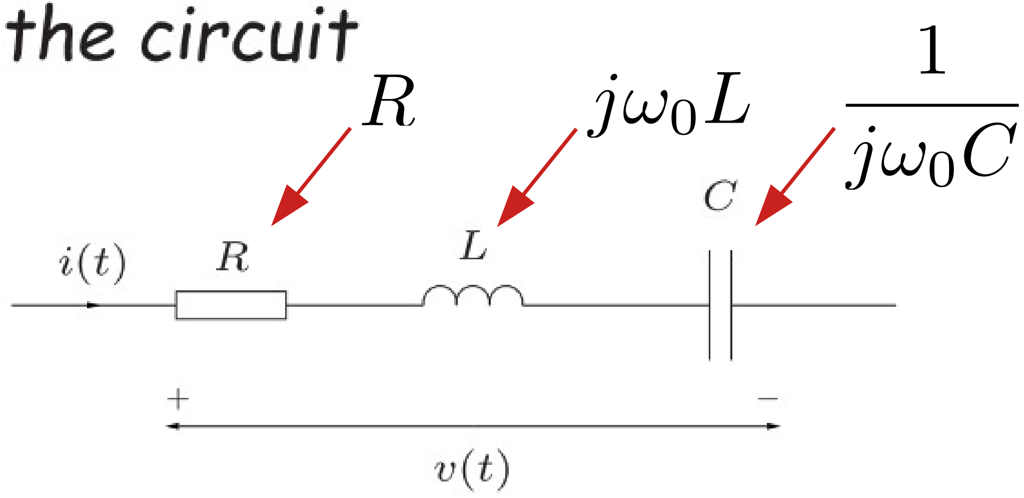


The impedance for the serial circuit is

$$\boxed{Z(f_0) = Z_1(f_0) + Z_2(f_0) + Z_3(f_0)}$$

Example 2 (Example B.2 in textbook)

Consider the circuit



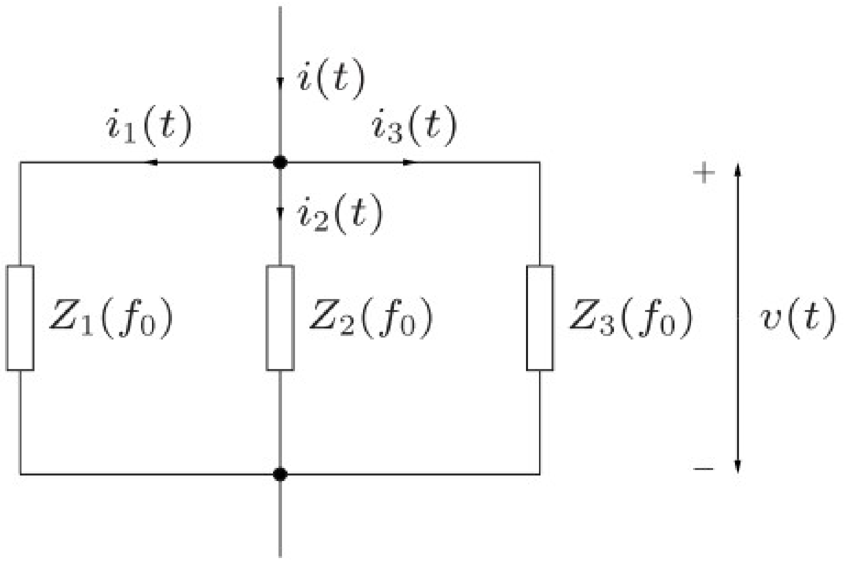
The current $i(t)$ is a sinusoid of frequency f_0 Hz. What is *v(t)*?

Total (serial) impedance at frequency $f_0 = \frac{\omega_0}{2\pi}$: $Z(\omega_0) = R + j\omega_0 L + \frac{1}{j\omega_0 C}$

Ohm's law gives: $v(t) = Z(\omega_0) i(t) = (R + j\omega_0 L + \frac{1}{j\omega_0 C}) e^{j\omega_0 t}$

Parallel impedances

Consider three impedances connected in parallel:



What is the equivalent parallel impedance?

Parallel impedances

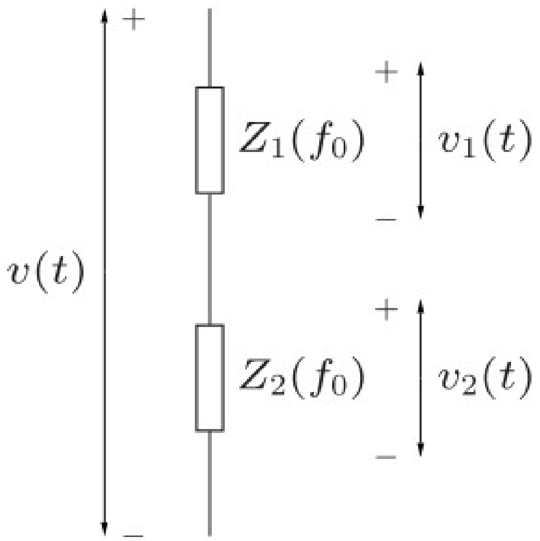
The impedance for the parallel circuit is written

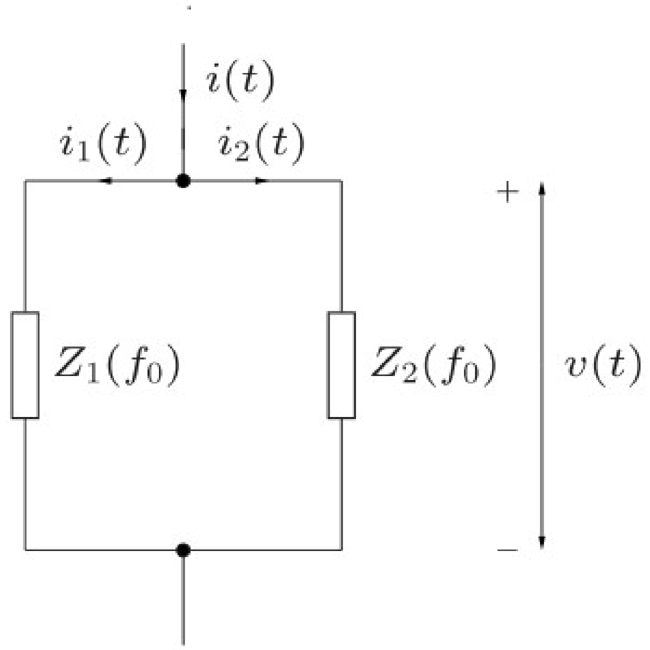
$$\frac{1}{Z_p(f_0)} = \frac{1}{Z_1(f_0)} + \frac{1}{Z_2(f_0)} + \frac{1}{Z_3(f_0)}$$

Often we have only two impedances in parallel and get

$$Z_p(f_0) = \frac{Z_1(f_0)Z_2(f_0)}{Z_1(f_0) + Z_2(f_0)}$$

Split of voltage and current





$$v_1(t) = \frac{Z_1(f_0)}{Z_1(f_0) + Z_2(f_0)} v(t)$$

$$i_1(t) = \frac{Z_2(f_0)}{Z_1(f_0) + Z_2(f_0)} i(t)$$

Summary

- Resistors, inductors & capacitors
  - We can apply Ohm's law, also for inductors and capacitors, by using $j\omega_0 L$ and $1/j\omega_0 C$ in place of ”resistance”.
  - Impedance is the relation between voltage and current (often from a combination of resistors, inductors and capacitors).
- Kirchoff's laws (general)
  - **Current law**: At all times, the sum of all currents into a node most equal the sum of currents leaving the node. (Charge can't accumulate in a node.)
  - **Voltage law**: At all times, the sum of voltages around any closed loop in a circuit must be zero.
- Typical circuits (special cases)
  - Serial and parallel impedance
  - Split of voltage and current

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