

Information Transmission

Chapter 5, Convolutional codes

OVE EDFORS

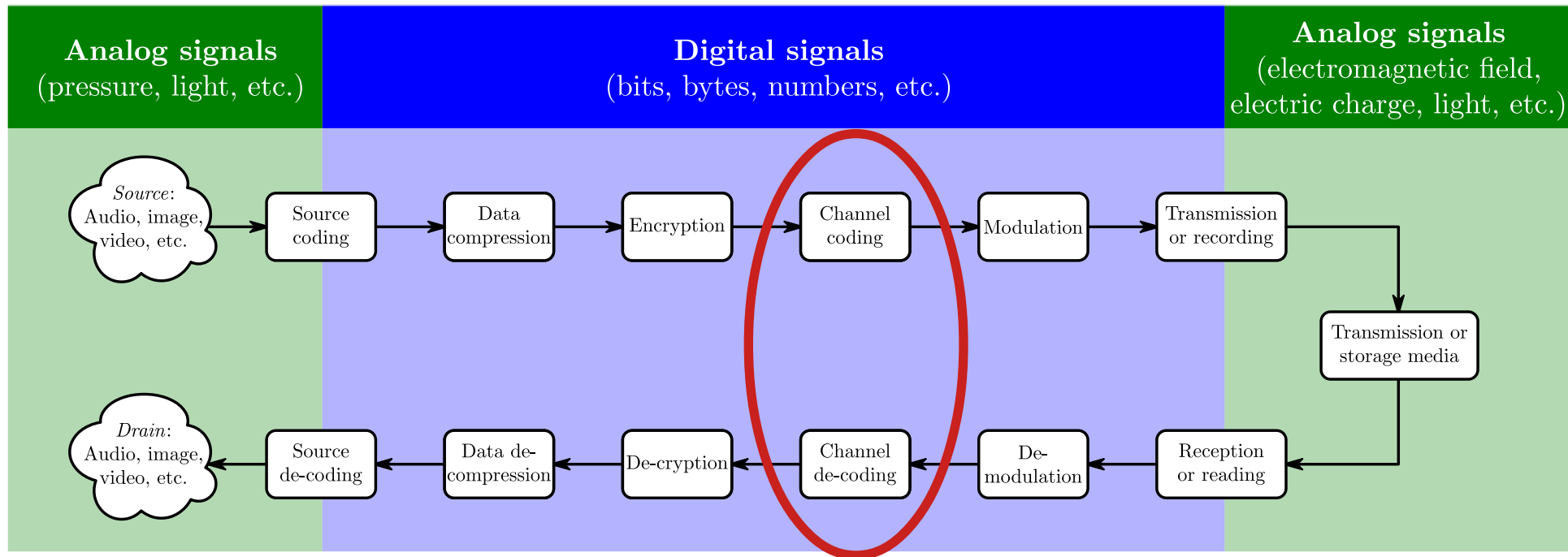
Electrical and information technology



Learning outcomes

- After this lecture, the student should
 - Understand what a convolutional code is
 - How convolutional encoding is done using a state machine or a trellis representation
 - Know how to set up a state transition diagram, draw a state machine and a corresponding trellis
 - Be able to execute the (optimal) Viterbi algorithm to decode a received sequence encoded using a convolutional code

Where are we in the BIG PICTURE?

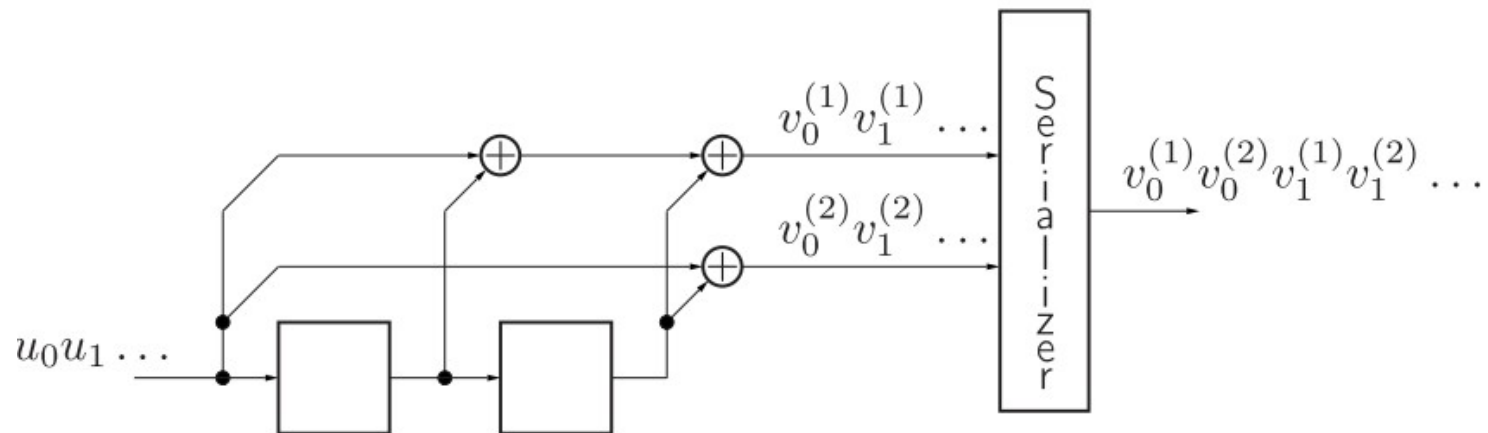


Convolutional codes

Lecture relates to pages 195-199 in textbook.

Convolutional codes

When we study convolutional codes we regard the *information sequence* $\mathbf{u} = u_0 u_1 \dots$ and the *code sequence* $\mathbf{v} = v_0^{(1)} v_0^{(2)} v_1^{(1)} v_1^{(2)} \dots$ as semi-infinite; they start at time $t = 0$ and go on forever. Consider the convolutional encoder below:



The encoder state

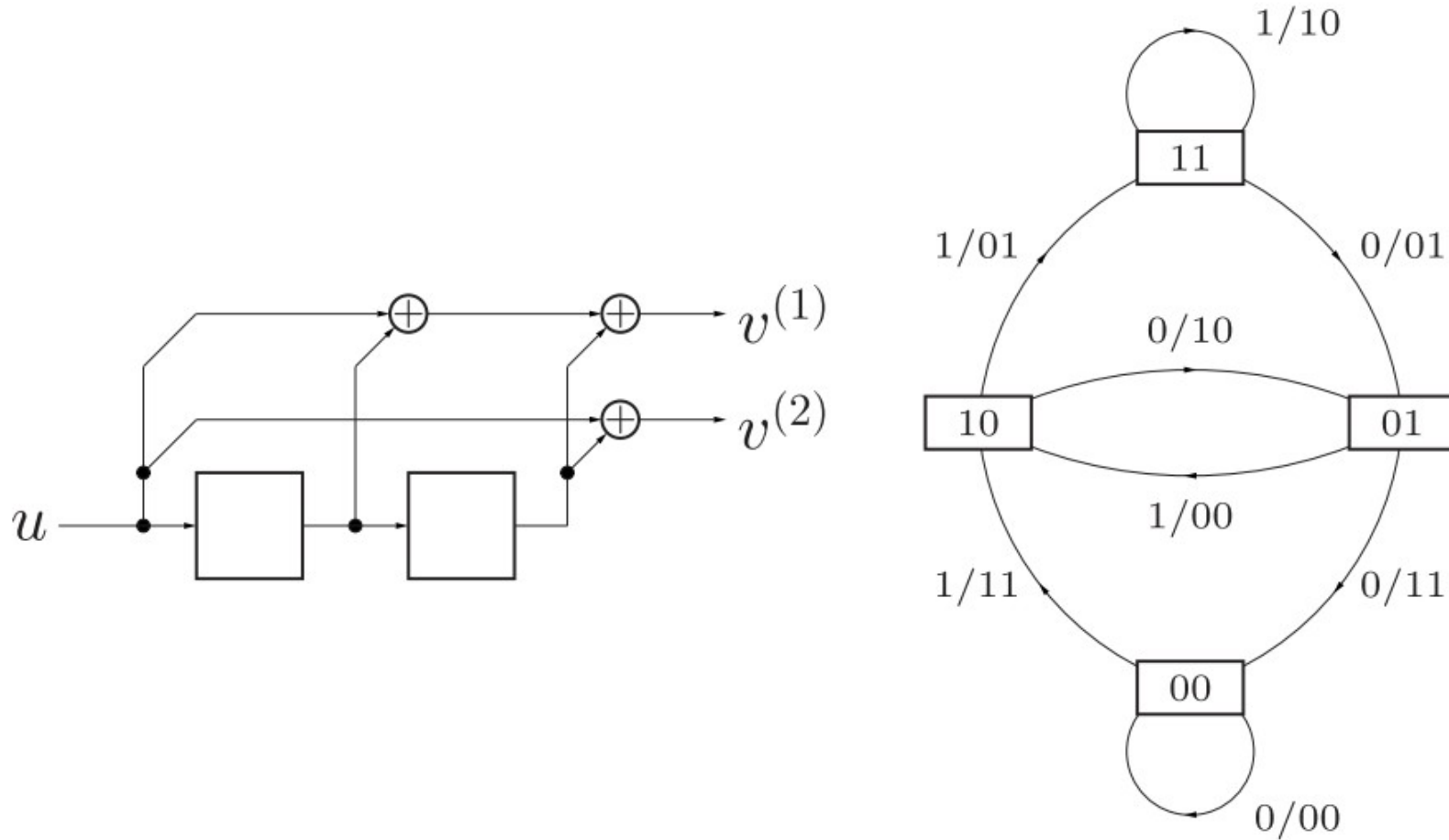
The *state* of a system is a compact description of its past history such that it, together with the present input, suffices to determine the present output and the next state.

For our convolutional encoder we can simply choose the state to be the contents of its memory element; that is, at time t we have the state

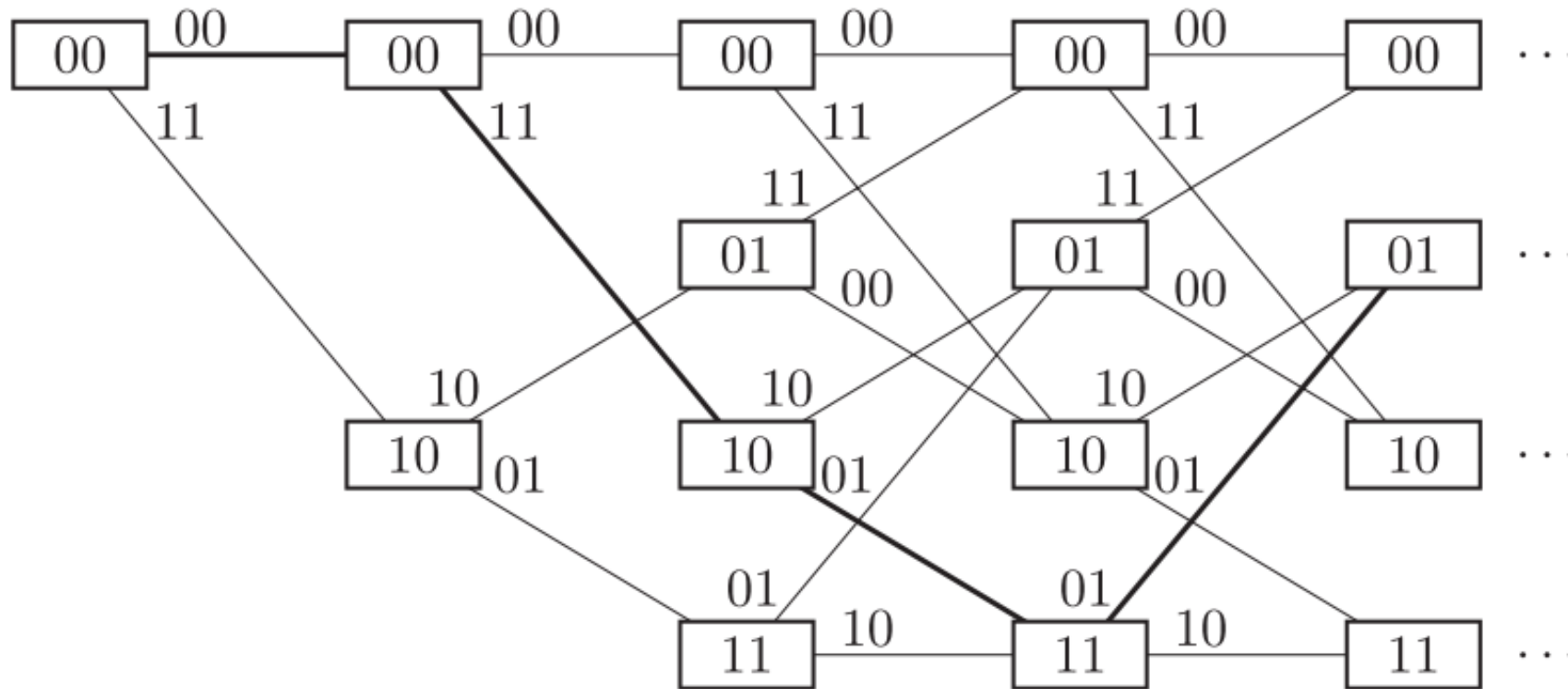
$$\sigma_t = u_{t-1}u_{t-2}$$

How many states does our encoder have?

The state transition diagram



The trellis description



Optimal decoder – the Viterbi algorithm

At each state we compare subpaths leading to it and discard the one that is not closest (measured in Hamming distance) to the received sequence.

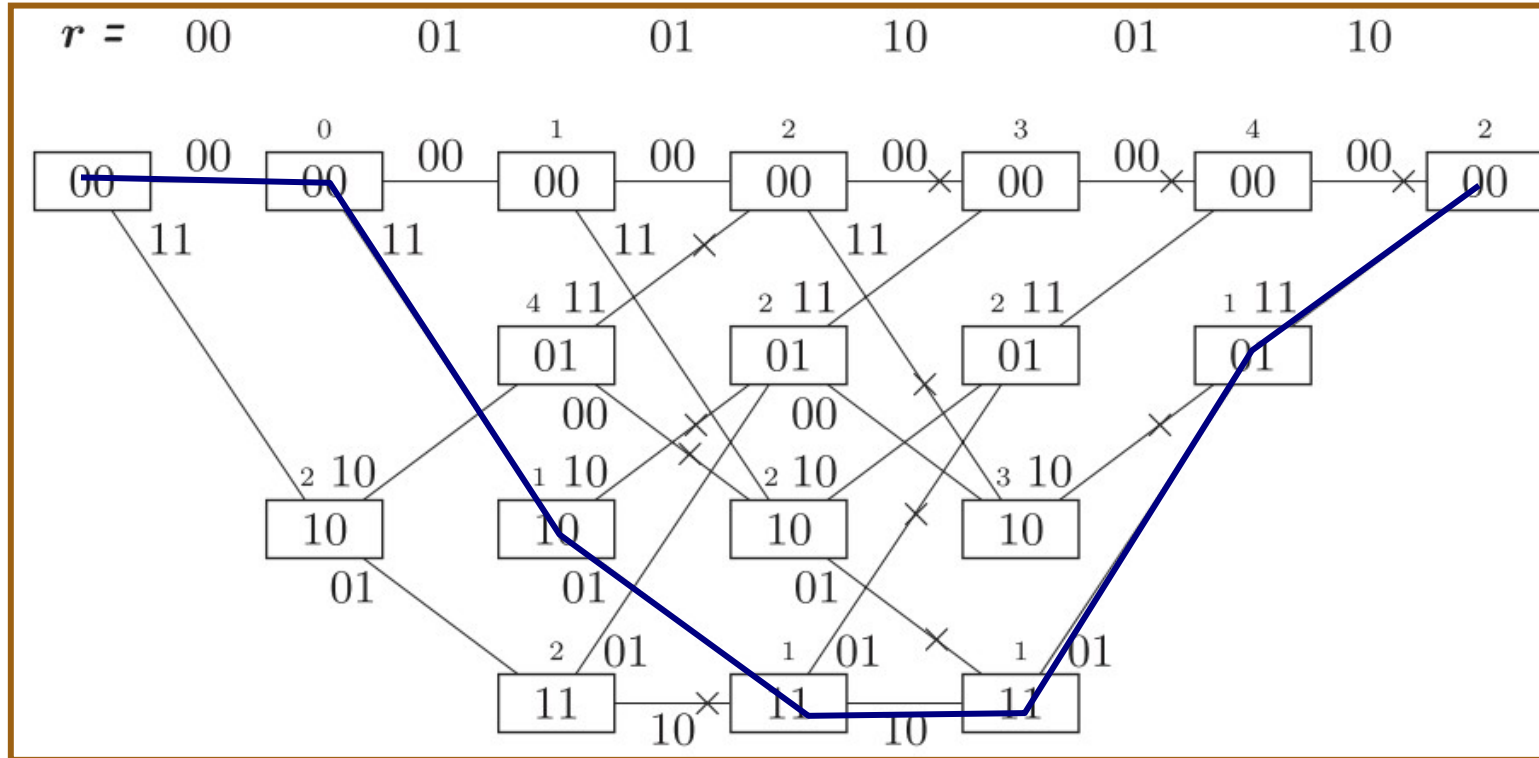
$\hat{\mathbf{v}}$

The discarded path cannot possibly be the initial part of the path that minimizes the Hamming distance between the \mathbf{r} sequence and the codeword \mathbf{v} .

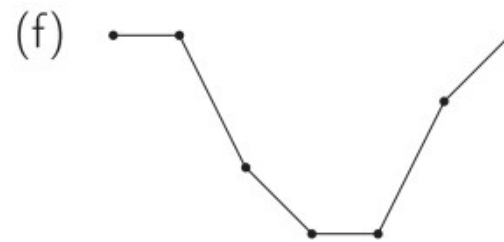
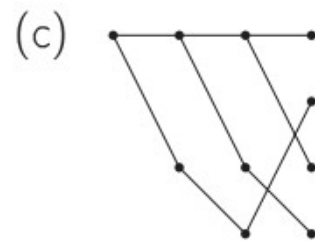
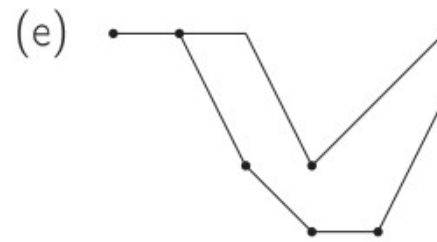
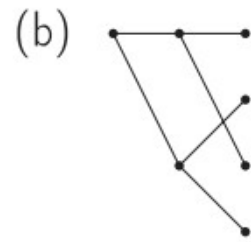
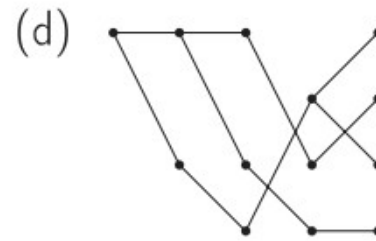
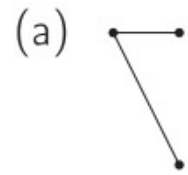
This is the *principle of nonoptimality*.

Let's consider the previous (7,5) $R=1/2$ code and a received sequence $\mathbf{r}=00\ 01\ 01\ 10\ 01\ 10$.

An example



Evolution of sub-paths



Error correction capabilities

We could correct a certain pattern of two errors, namely,

$$\begin{aligned}\hat{\mathbf{e}} = \mathbf{r} + \hat{\mathbf{v}} &= (00\ 01\ 01\ 10\ 01\ 10) + (00\ 11\ 01\ 10\ 01\ 11) \\ &= 00\ 10\ 00\ 00\ 00\ 01\end{aligned}$$

where $\hat{\mathbf{e}}$ is the estimated error pattern.

How many errors can we correct in general?

The answer is related to the minimum Hamming distance between any two codewords in the trellis. Since a convolutional code is linear this value is equal to the least number of ones in any nonzero codeword, this is called *the free distance*, d_{free} .

Error correction capabilities

Clearly we can correct all error patterns with $(d_{free} - 1)/2$ or fewer errors.

What about error patterns with more errors?

The answer is that it depends; if the errors are sparse enough we can correct many more! That is why convolutional codes are so powerful.

Concluding remarks, Chapter 5

Information theory

In this chapter we first gave a brief introduction to Claude E. Shannon's information theory which is the basis for modern communication technology. It provides guidelines for the design of digital communication systems. We then looked at some practical methods of source and channel coding.

Source coding

Shannon modeled sources as discrete stochastic processes and showed that a source is characterized by the uncertainty of its output, $H(U)$, in the sense that the source output sequence can be compressed arbitrarily close to $H(U)$ binary digits per source symbol but not further.

The *uncertainty* or *entropy* of a discrete random variable U is defined by the quantity

$$H(U) \stackrel{\text{def}}{=} - \sum_{i=1}^L P_U(u_i) \log P_U(u_i)$$

The unit of the uncertainty is called *bit*.

Channel capacity

Shannon's most remarkable result concerns transmission over a noisy channel. It is possible to encode the source at the channel input and decode the possibly corrupted signal at the receiver side such that:

if the source uncertainty is less than the channel capacity, $H(U) < C$, the source sequence can be reconstructed with arbitrary accuracy; This is impossible if the source uncertainty exceeds the channel capacity.

It can be shown that the capacity of a Gaussian channel with energy constraint E and noise variance $N_0/2$ is

$$C = \frac{1}{2} \log \left(1 + \frac{2E}{N_0} \right) \quad \text{bits/channel use}$$

Huffman coding

Huffman coding is a fixed-to-variable length optimal source coding procedure that uses the probability of the source symbols to obtain an encoding for single (or blocks of) source symbols into code words that consist of variable length strings of binary digits such that the average codeword length is minimum.

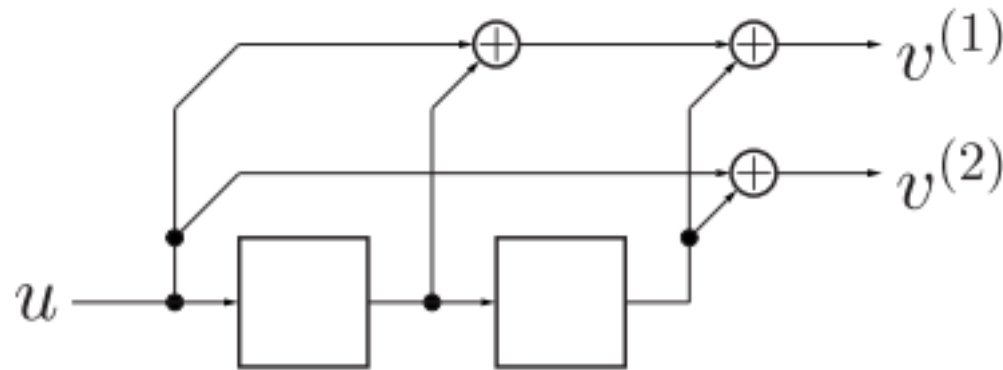
The LZW algorithm

The LZW algorithm is a procedure that does not need to know the source statistics beforehand. It parses the source output sequence, recognizes fragments that have appeared before, and refers to the addresses of these fragments in a dynamical dictionary. This algorithm is asymptotically optimum, easy to implement, and widely used in practice.

Coding methods

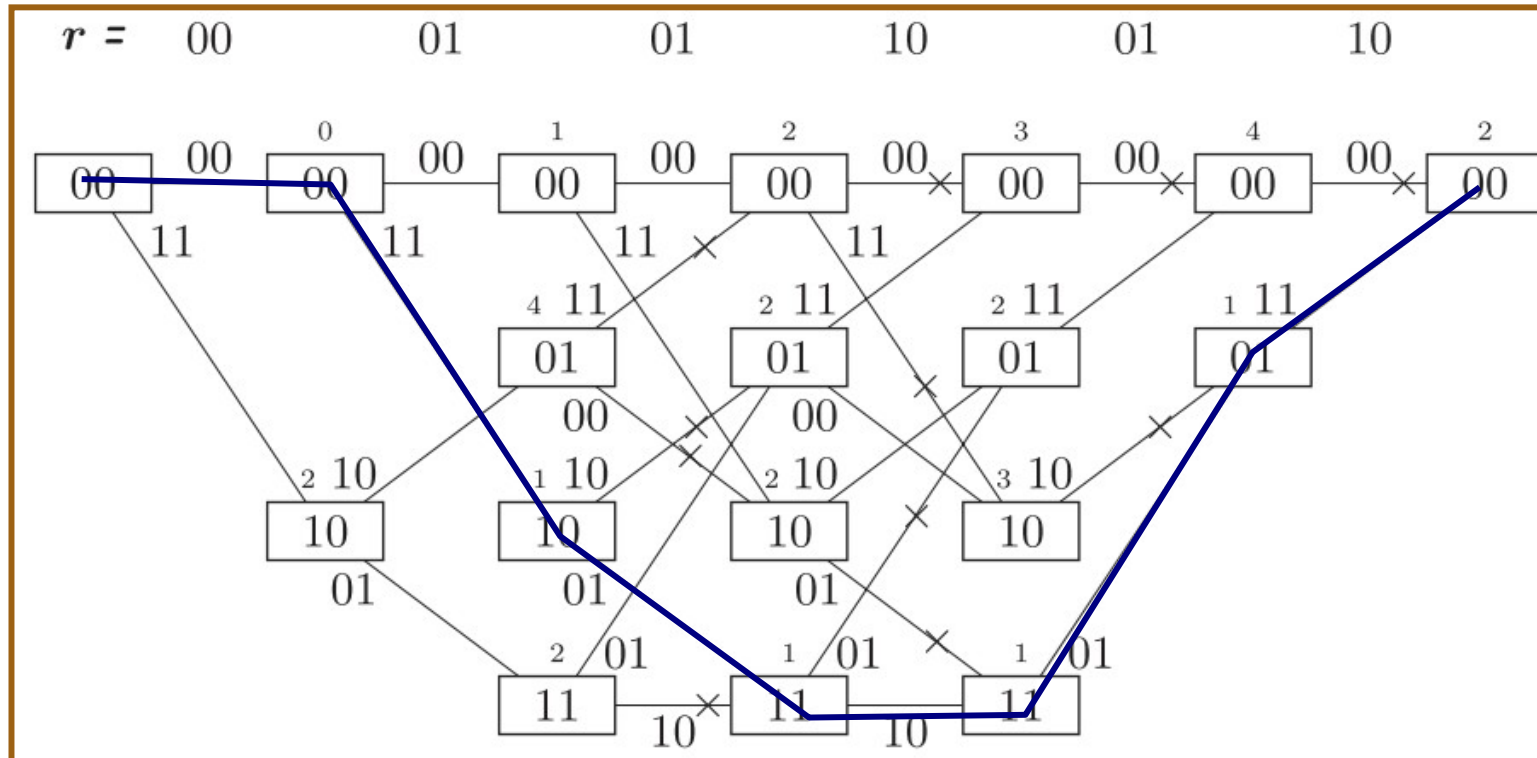
Hamming codes constitute the most celebrated class of block codes. Their minimum distance is $d_{min}=3$ and, thus, they correct all error patterns of single errors.

Convolutional codes are more powerful and often used in practice, either by themselves, as concatenated codes or for parallel codes.



The Viterbi decoder.

Viterbi decoding is a both optimum and practical method for decoding convolutional codes. It is widely used in mobile telephony and high-speed modems.





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