Written Exam
Information Transmission - EITA30/EIT100

Department of Electrical and Information Technology
Lund University

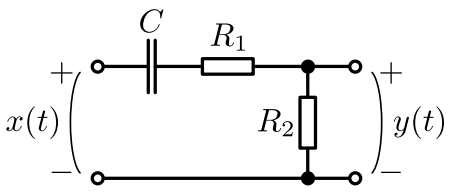
2018-05-28
8.00 – 13.00

The exam consists of five problems. 20 of 50 points are required to pass.

Permitted aids: Pocket calculator without any programs, scripts or files stored, formula collection without any notes.

- Write your personal identifier on each page.
- Each solution must be written on separate sheets.
- Your solutions must clearly reveal your method of solution.

1. Given this circuit (a filter attenuating signals at some frequencies more than others),



where $x(t)$ is the input signal and $y(t)$ the output signal, answer the following questions:

   (a) What is the frequency function $H(f)$ of the circuit, expressed in capacitance C and resistances R_1 and R_2? (3 p)

   (b) Assume that the input is a sinus signal with angular frequency ω and unit amplitude, i.e., $x(t) = \sin(\omega t)$. How large is the amplitude of the output $y(t)$ when the frequency is

      i. low, i.e., when $\omega \to 0$, and (1 p)

      ii. high, i.e., when $\omega \to \infty$? (1 p)

   (c) Assume that you want as much as possible of your signal to go through the filter at high frequencies and therefore set $R_1 = 0$ Ohm.

      i. At what frequency f (in Hz) is the amplitude of the output $1/\sqrt{2}$ of the input amplitude? (Note: This is called the *cut-off frequency* of the filter.) (3 p)

      ii. Given that you have a capacitor with $C = 10$ μF, what value on the resistor R_2 (in Ohms) do you need to set the above cut-off frequency to 100 Hz? (1 p)

   (d) Given what you now know about the circuit/filter above, would you call it a low-pass filter (letting through low-frequency signals) or a high-pass filter (letting through high-frequency signals)? Motivate your answer, in a single sentence. (1 p)

2. Sinc pulses are to be used to transmit a binary sequence of ± 1 at a rate of 500 kbit/s by means of 2-ASK. Transmission takes place in the base-band, i.e., *without* up-conversion to (modulation on) a carrier frequency.

   (a) Sketch as accurately as possible the transmitted signal produced by the transmitted sequence +1, +1, -1, +1. (3 p)

   (b) Sketch the spectrum of the transmitted signal as accurately as you can assuming that random independent symbols are transmitted. (3 p)

   (c) If 4-ASK (four amplitude levels, e.g. –3, –1, +1, and +3) is used instead of 2-ASK, also with sinc pulses, what is the symbol time given that the bit rate remains unchanged? How is the spectrum affected by this choice of modulation compared to the case of 2-ASK? (4 p)

3. Peter wants to send messages to Petra using as efficient source coding as possible. Peter sends one word at the time and knows only 6 words: Hello, Love, Banana, Me, Apple, Pen. The probability of transmission of the different words are the following:

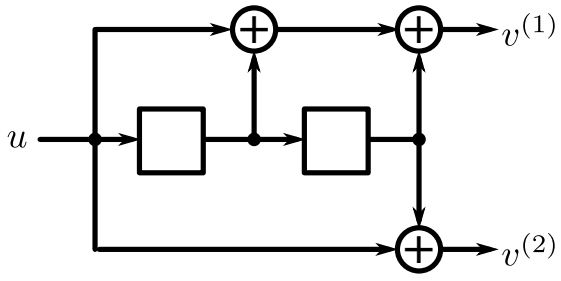
word	$P(word)$
Hello	0.12
Love	0.08
Banana	0.30
Me	0.33
Apple	0.10
Pen	0.07

(a) What is the uncertainty of each transmission? (3 p)

(b) Derive an efficient bit representation for the case when many words are sent after another using using as few bits as possible on the average for the transmission. Different number of bits lengths are allowed if this increases the efficiency. (5 p)

(c) What is the average number of bits per transmission using your bit representation and how far are you from the optimum representation? (2 p)

4. Consider the rate $R = 1/2$ convolutional encoder shown below.



Hint: After the problems, there are two pages with pre-drawn trellises (only transitions, no outputs) for the type of encoder shown in the figure. Using these may simplify the problem solving a bit.

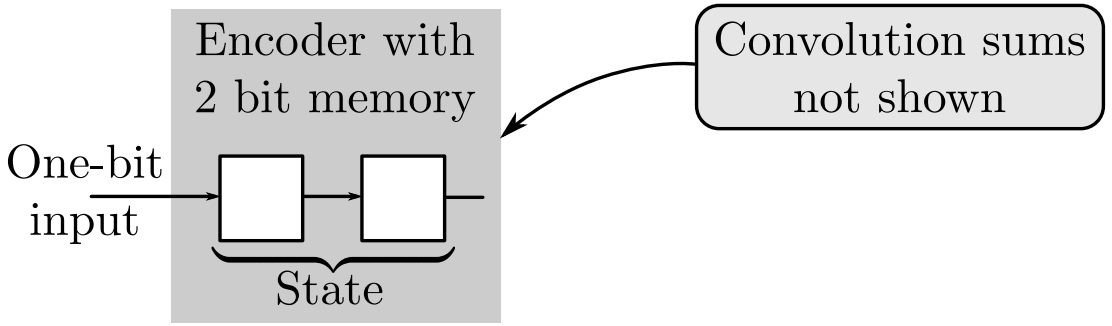
(a) Given the input sequence $\boldsymbol{u} = 1\ 0\ 1\ 1\ 0\ 0$, what is the corresponding codeword? (The encoder starts in the all-zero state and the last two zeros in the sequence are termination bits used to force the encoder back to the all-zero state.) (3 p)

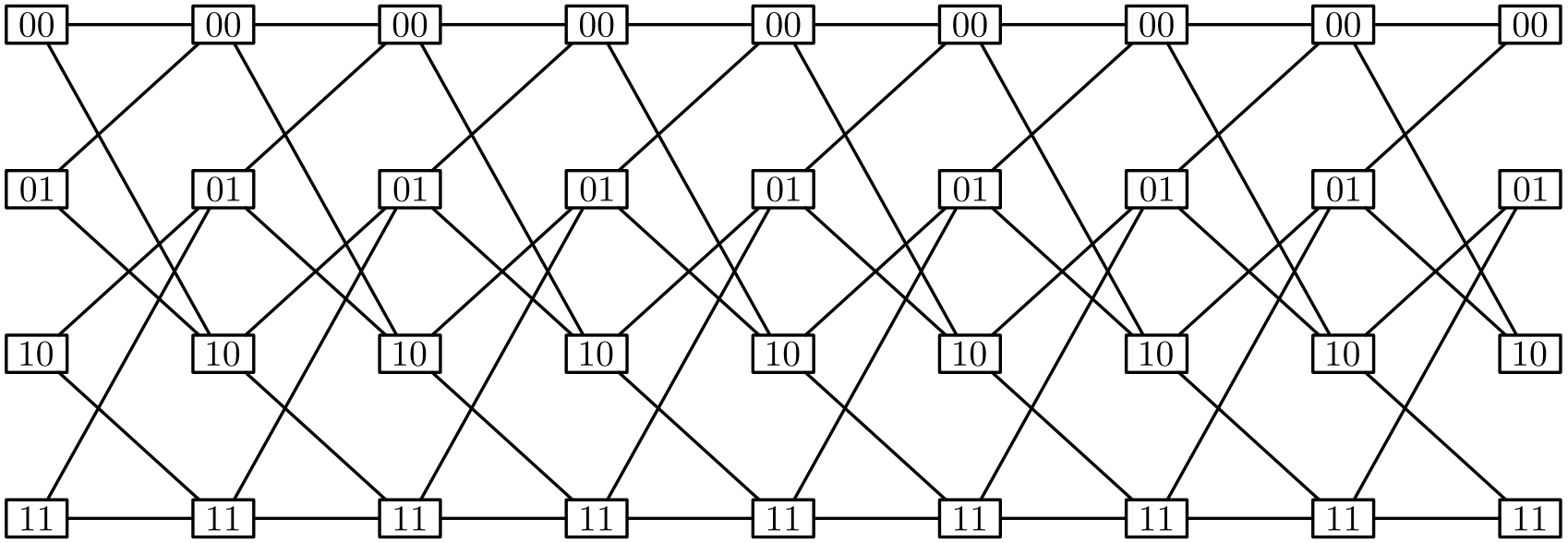
(b) Use the Viterbi algorithm to decode the received sequence $\boldsymbol{r} = 10\ 10\ 00\ 01\ 00\ 01$ and show clearly how you get your answer. (5 p)

(c) How many bit errors can for sure be corrected using this code? Motivate your answer. (2 p)

5. Consider an RSA public key crypto system with the public parameters $n = 697$ and $e = 27$. Find the plaintext P corresponding to the the ciphertext $C = 190$. *Hint:* One of the factors of n is $p = 17$. (10 p)

State-transition trellis for a one-bit input, memory two, feed-forward convolutional encoder.





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