Synchronization for OFDM systems

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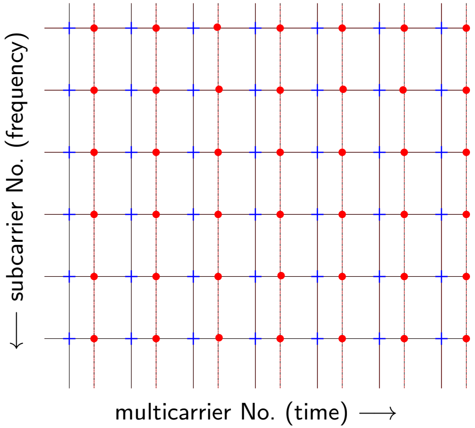
Synchronization: offset/errors and their cause

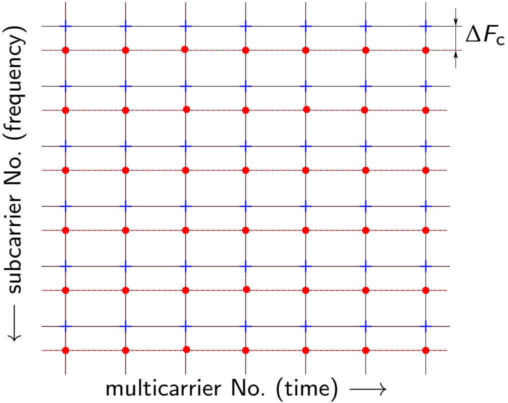
- *Symbol (timing)* offset (OFDM and DMT)
  - transmitter and receiver do not have a common time reference
  - receiver needs to find symbol boundaries to avoid ISI/ICI
- *Carrier frequency/phase* offset (OFDM)
  - carrier frequency/phase of transmitter's local oscillator (LO) and receiver's LO can be off by some ppm
  - resulting frequency difference ΔF_c Hz between transmitter's and receiver's carrier introduces the additional term $e^{j2\pi\Delta F_c/F_s n}$ in the baseband multiplex $\longrightarrow$ ICI
  - receiver needs to compensate for the frequency offset (and the phase offset for coherent detection)
- *Symbol clock (sampling frequency)* offset (DMT and OFDM)
  - frequency/phase of transmitter's local oscillator (LO) and receiver's LO for the sampling clock can be off by some ppm
  - resulting frequency difference ΔF_s Hz between transmitter's and receiver's clock causes
    - a gradually growing timing offset $\longrightarrow$ ISI/ICI
    - a slightly too large (or too small) subcarrier-spacing $\longrightarrow$ ICI
  - receiver needs to compensate for the clock offset

Symbol (timing) offset



- receiver assumes symbols on the dots in the time-frequency grid
- effect: severe ISI and ICI

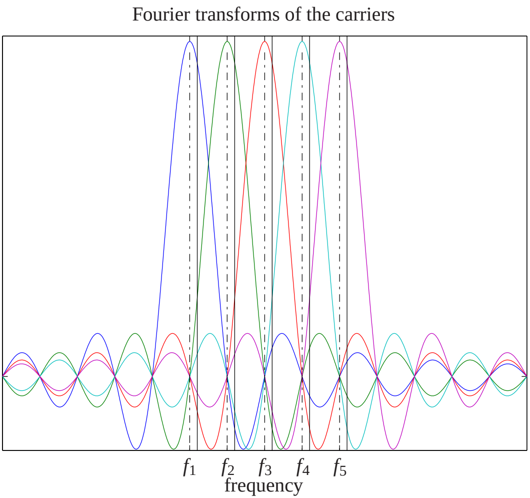
Carrier frequency/phase offset



- receiver assumes symbols on the dots in the time-frequency grid
- effect: ICI

Carrier frequency/phase offset cont'd

Assume a ΔF_c between the carrier frequency of transmitter and receiver:



Carrier frequency/phase offset cont'd

Consequence of $\Delta F_c \neq 0$:

- each subcarrier of a multicarrier symbol ends up on a frequency position that is shifted by ΔF_c Hz compared to the transmitter $\rightarrow$ orthogonality of the subcarriers is gone
- impact of a normalised frequency offset $\Delta_c = \Delta F_c / (F_s / N)$ (normalised by the subcarrier spacing) on the ℓth receive symbol is described by

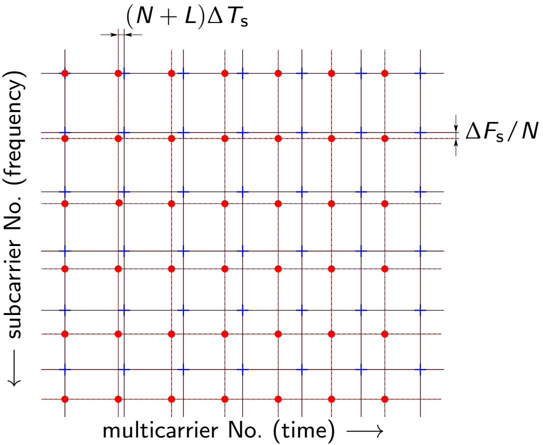
$$\underline{y}_\ell \overset{N\gg}{\approx} \underline{x}_\ell \underbrace{\mathrm{sinc}\,(\Delta_c)\, e^{j\pi\Delta_c}}_{\text{scaling, I/Q cross-coupling}} + \underbrace{\sum_{k\neq\ell} \underline{x}_k\, \mathrm{sinc}\,(k-\ell+\Delta_c)\, e^{j\pi(k-\ell+\Delta_c)}}_{\text{ICI caused by carrier offset}} \tag{1}$$

From (1) it is clear why carrier synchronization in OFDM is critical: interference from all the other subcarriers occurs if $\Delta F_c \neq 0$.

Carrier frequency/phase offset cont'd

- regarding the carrier offset correction, we distinguish
  - controlled oscillator; the oscillator of the down-converter (mixer) is adaptively tuned such that it corrects the carrier offset right away;
  - freely running oscillator followed by correction; the oscillator is not tunable, which simplifies its implementation; the resulting carrier offset is corrected by a multiplication of the receive signal multiplex with $e^{-j2\pi\Delta F_c/F_s n}$ before the DFT block;
- parameter estimation in many practical implementations is based on correlation
- most schemes operate in stages
  - coarse lock ("acquisition")
  - fine tuning of the offset ("tracking")
- most robust methods, especially in fading environments, employ pilot symbols

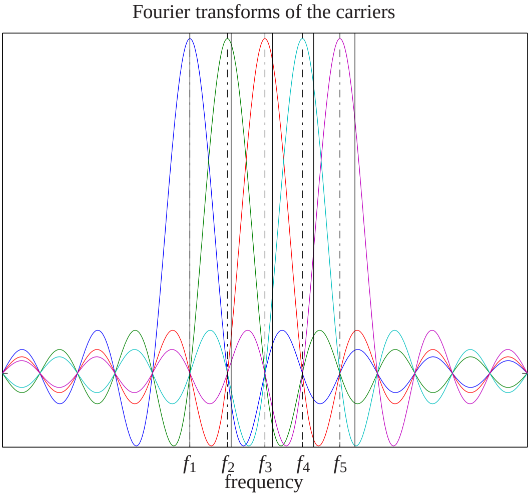
Clock (sampling frequency) offset



- receiver assumes symbols on the dots in the time-frequency grid
- effects: gradually increasing ICI (with carrier No.) due to mismatch in carrier spacing, gradually increasing symbol-timing offset

Clock (sampling frequency) offset cont'd

Assume a difference $\Delta F_s = F_s^{(\text{transmitter})} - F_s^{(\text{receiver})}$ between the sampling clock of the transmitter and the receiver:



Clock (sampling frequency) offset cont'd

Consequence of $\Delta F_s \neq 0$:

- frequency spacing of receive signal's DFT is larger or smaller than the spacing of the transmit signal's DFT $\rightarrow$ orthogonality is lost
- impact of a normalised sampling clock offset $\Delta_s = \Delta F_s / F_s^{(\text{receiver})}$ on the ℓth receive symbol is described by

$$
\begin{aligned}
\underline{y}_\ell &\overset{N\gg}{\approx} \underline{x}_\ell \frac{e^{j2\pi\ell\Delta_s}-1}{j2\pi\ell\Delta_s} + \sum_{k\neq\ell} \underline{x}_k \frac{e^{j2\pi(k(1+\Delta_s)-\ell)}-1}{j2\pi(k(1+\Delta_s)-\ell)} \\
&\overset{\Delta_s\ll}{\approx} \underline{x}_\ell \underbrace{(1-j\pi\ell\Delta_s)}_{\text{scaling, I/Q cross-coupling}} + \underbrace{\sum_{k\neq\ell} \underline{x}_k \frac{k\Delta_s}{k-\ell}}_{\text{ICI caused by sampling clock offset}}
\end{aligned} \tag{2}
$$

- many terms contribute to the inter-carrier interference
- tolerable distortion for a subchannel depends on constellation size and desired bit error rate performance
- in practice, system clock is often locked to carrier-frequency clock

Synchronization

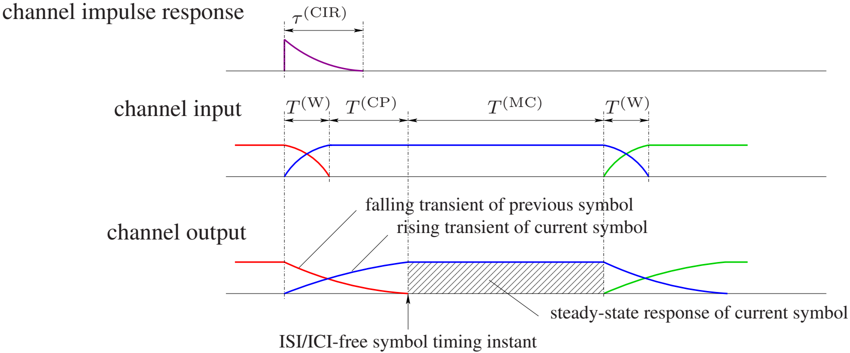
synchronization consists of two tasks:

- estimation of an appropriate parameter (frequency offset, phase offset, time offset)
- actual offset correction based on the estimate

regarding the estimation task, we distinguish

- methods that are supported by deliberately inserted synchronization-assisting signals (pilot signals, synchronization symbols)
- methods that operate without this assistance (often referred to as “blind” estimation algorithms)

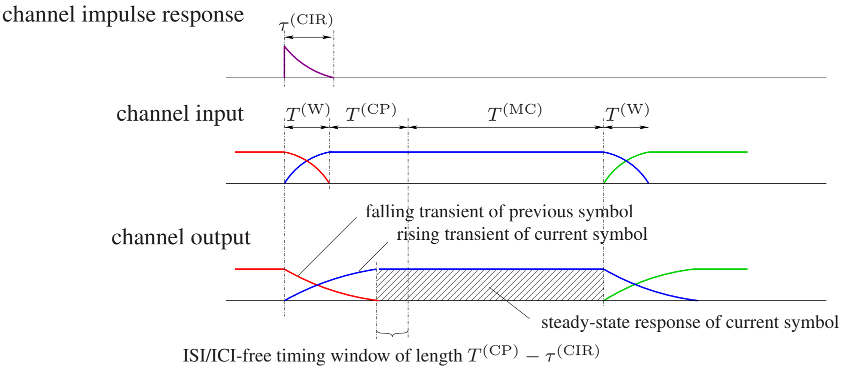
Symbol timing offset—CP-length=CIR-length



- unique ISI-free timing instant

Prefix and postfix of length $T^{(W)}$ are windowed cyclic extensions to improve out-of-band spectrum

Symbol timing offset—CP-length>CIR-length



- latest timing-instant (right-most in picture) yields largest delay-spread immunity

Symbol timing offset—CP-length>CIR-length cont'd

- captured n samples are cyclically shifted by n samples
- cyclic time-domain of $r(n), n = 0, \ldots, N-1$ by s samples corresponds to phase rotation

  $$r'(n) = r((n-s)\text{mod } N) \longleftrightarrow R'(k) = R(k)\mathrm{e}^{-j2\pi sk/N}, k = 0, \ldots, N-1,$$

  where $R(k)$ and $R'(k)$ denote the DFT of $r(n)$ and $r'(n)$
- differential modulation in time: inherently immune to time-invariant phase rotations
- absolute modulation: phase rotations are taken care of by frequency-domain equalization

Pilot-based timing/frequency synchronization: time-offset

- time-domain pilot symbol with 2 identical halves (period $P = N/2$)
- key idea: if CP is long enough, both halves are identical (up to noise) at channel output
- normalized correlation measure

$$M_{\text{time}}(d) = \frac{\left| \sum_{m=d}^{d+N/2-1} r^*(m) r(m+N/2) \right|}{\sum_{m=d}^{d+N-1} |r(m)|^2}$$

- timing instant

$$\widehat{d} = \arg\max_{d} M_{\text{time}}(d)$$

pinpoints the first sample of the pilot symbol's steady-state part

Pilot-based timing/frequency synchronization: frequency-offset

- receive signal: $r(n) = r'(n)\,\mathrm{e}^{j2\pi(n_0+n)\Delta F_\mathrm{c}/F_\mathrm{s}}$
- any two samples $r(n)$ and $r(n+P)$ are identical up to a constant phase difference

$$\arg(r(n+P)) - \arg(r(n)) = \arg(r(n+P)r^*(n)) = 2\pi P\Delta F_\mathrm{c}/F_\mathrm{s} + 2\pi i$$

  and noise
- fractional part of frequency offset:

$$\widehat{\Delta F_\mathrm{c}} = \frac{F_\mathrm{s}}{2\pi P}\arg\Big(\sum_n r(\widehat{d}+n+P)r^*(\widehat{d}+n)\Big)$$

- if $|\Delta F_\mathrm{c}| \leq \frac{F_\mathrm{s}}{2P}$, ΔF_c yields correct offset
- fractional frequency-offset is corrected before FFT processing through counter-rotation yielding $r'(n) = r(n)\,\mathrm{e}^{-j2\pi n\widehat{\Delta F_\mathrm{c}}/F_\mathrm{s}}$

Pilot-based timing/frequency synchronization: frequency-offset cont'd

- for integer part of frequency offset: second training symbol R_2 with differentially modulated data with respect to the pilot symbol R_1
- pilot symbols are ICI-free but shifted by iF_s/P
- differential modulation: $R_2 = R_1 P$, where P is pseudo-random number (PN) vector
- similarity between $R_2(k)/R_1(k)$ and a shifted version $P(k - iN/P)$ of P is measured by[1]

$$M_{\text{freq}}(i) = \sum_k \left|\frac{R_2(k)}{R_1(k)} P(k - iN/P)\right| = \sum_k \frac{|R_2(k) R_1^*(k) P(k - iN/P)|}{|R_1(k)|^2}$$

and its maximum indicates the integer part

$$\widehat{i} = \arg\max_i M_{\text{freq}}(i)$$

of the frequency offset

[1]Note that the sum index k includes only subcarriers for which $R_1(k) \neq 0$ and $k - iN/P \in [1, N]$.

CP-based timing/frequency synchronization: time-offset

- a longer-than-necessary CP provides repetitive pattern of receive stream
- correlation measure

$$\gamma(d) \hat{=} \sum_{m=d}^{d+L-1} r^*(m) r(m+N)$$

- power measure

$$\Phi(d) \hat{=} \frac{1}{2} \sum_{m=d}^{d+L-1} |r(m)|^2 + |r(m+N)|^2$$

- joint ML estimation of both timing offset and frequency offset yields the estimate

$$\widehat{d} = \arg\max\{|\gamma(d)| - \rho\Phi(d)\} + L,$$

  which pinpoints the first sample of the pilot symbol's steady-state part
- ρ is the magnitude of correlation coefficient between two N-spaced samples
- performance degrades with stronger time dispersion and with increasing channel noise

CP-based timing/frequency synchronization: frequency-offset

- same principle as before: repetitive signal parts are identical up to a phase difference
- joint ML estimation of offsets in time and frequency yields

$$\widehat{\Delta F_{\mathrm{c}}} = \frac{F_{\mathrm{s}}}{2\pi N} \arg(\gamma(\widehat{d}))$$

- performance degrades with stronger time dispersion and with increasing channel noise

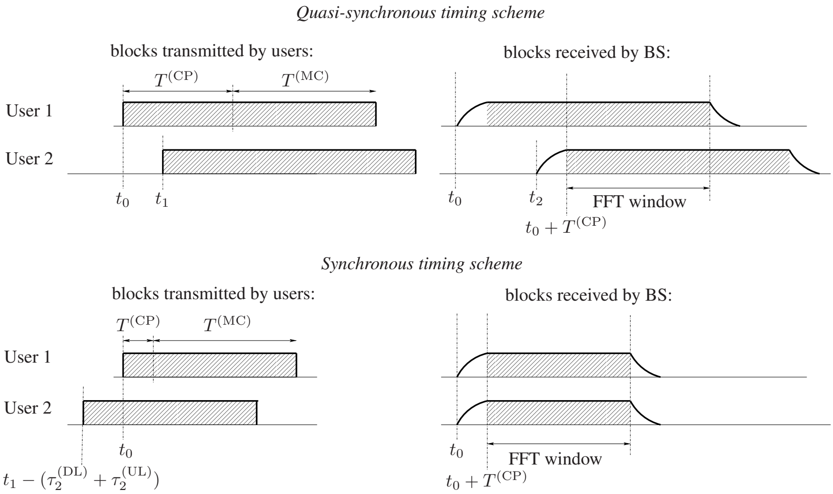
Tracking

- after acquisition, time offsets and frequency offsets are updated on a regular basis
- dedicated tracking pilots are used
- timing offset: linearly increasing phase offset across subcarriers
- frequency offset: linearly increasing phase offset over time

Synchronization in OFDMA-UL

- OFDMA idea: separate users via subcarrier-assignment (FDMA)
- subcarrier allocation: subband allocation, interleaved allocation, general (unstructured) allocation
- signals from different users arrive with different timing-offsets and frequency-offsets
- chicken-egg problem
  - synchronization is required for orthogonality
  - orthogonality is required to correct offsets

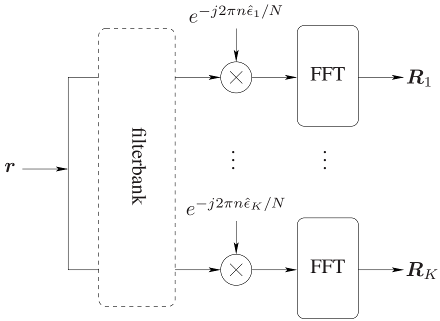
OFDMA-UL timing schemes



- Quasi-synchronous scheme: $T^{(\mathrm{CP})} \geq \max_k \tau_k^{(\mathrm{UL})} + \max_k \tau_k^{(\mathrm{DL})} + \max_k \tau_k^{(\mathrm{CIR})}$
- Synchronous scheme: $T^{(\mathrm{CP})} \geq \max_i \tau_i^{(\mathrm{CIR})}$

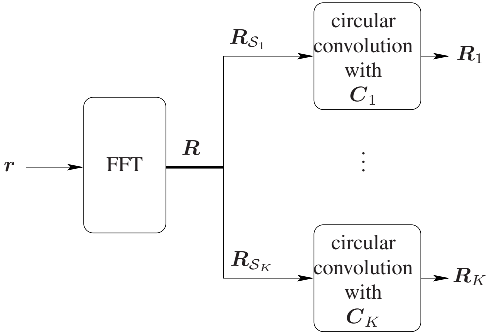
OFDMA-UL with subband allocation

- time offset: synchronous or quasi-synchronous timing scheme
- frequency-offset estimation and correction: separation in frequency
- direct frequency-offset correction:



OFDMA-UL with subband allocation

- time offset: synchronous or quasi-synchronous timing scheme
- frequency-offset estimation and correction: separation in frequency
- offset correction in FFT-domain (lower complexity):



Summary

- Different types of physical-layer offsets:
  - symbol (timing) offset
  - carrier frequency/phase offset
  - clock (sampling frequency) offset
- Single-user pilot-based synchronization
- Single-user CP-based synchronization
- Synchronization in OFDMA UL: challenge; approach for subband allocation