Reduction of PAR and out-of-band egress

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Multicarrier specific issues

The following issues are specific for multicarrier systems and deserve special attention:

- Peak-to-average (PAR) power ratio and its reduction
- Reduction of out-of-band emission
Peak-to-average (PAR) power ratio

Probably one of the biggest drawbacks of OFDM: the high peak-to-average power (PAR) ratio of the transmit signal multiplex

**Deterministic measure** to describe the peakiness of a given signal (or realisation of a random process):

\[
\text{PAR} = \frac{\max_n |s(n)|^2}{\frac{1}{N} \sum_{n=0}^{N-1} |s(n)|^2}
\]
PAR of different signals

- **Constant signal**: \( s^{(0)}(n) = 1 \), \( \text{PAR} = 1 \) (0 dB), \( (\frac{1}{N} \sum |s^{(0)}(n)|^2 = 1) \)

- **Sinusoidal signal**: \( s^{(1)}(n) \), \( \text{PAR} = 3 \) dB, \( \max |s^{(1)}(n)| = 1.41; (\frac{1}{N} \sum |s^{(1)}(n)|^2 = 1) \)

- **Uniform distribution**: \( s^{(2)}(n) \), samples are realisations of uniform distribution, \( \text{PAR} = 4.7 \) dB, \( \max |s^{(2)}(n)| = 1.71 (\frac{1}{N} \sum |s^{(2)}(n)|^2 = 1) \)

- **Gaussian distribution**: \( s^{(3)}(n) \), samples are realisation of Gaussian distribution, \( \text{PAR} = 15.3 \) dB, \( \max |s^{(3)}(n)| = 5.78, (\frac{1}{N} \sum |s^{(3)}(n)|^2 = 1) \)
Consequences of high PAR values

- high PAR-value requires *linearity* of the transmitter chain over a wide amplitude range
- analogue components must be designed to deliver the peak power while sustaining linearity
- high PAR-values thus cause a *large power consumption*, which is a problem for
  - power-aware devices
  - devices that suffer from the heat they produce themselves
Clip noise

- An example of a severe non-linearity is *clipping*: the signal amplitude $s(n)$ is simply limited to the certain value $s_{\text{max}}$

- Equivalently, a *distortion* $d(n)$ is added to the output signal

$$s_{\text{out}}(n) = s(n) + d(n),$$

where

$$d(n) = \begin{cases} 
\text{sign}\{s(n)\}(s_{\text{max}} - |s(n)|), & |s(n)| > s_{\text{max}} \\
0, & \text{otherwise}
\end{cases}$$

- The distortion $d(n)$ is often referred to as *clip noise*

- If clipping happens only rarely the *clip noise spectrum* is white
Where PAR hits hard

At which point(s) in the transmit-receive chain does a high PAR value cause troubles?

- First and foremost, in the *transmit power amplifier*
- Depending on the channel, the receive signal can also exhibit a high PAR-value
  - ISI-free channel: PAR value of the receive signal is comparable to the PAR-value of the transmit signal
  - ISI channel: averaging of the transmit signal reduces the PAR-value of a signal that has a high PAR
Observation: the continuous-time signal $s(t)$, obtained by discrete-time to continuous-time conversion, may exhibit larger values than the sequence $s(n)$.

Reducing the PAR-value of $s(n)$ to a certain level does not guarantee that the PAR of $s(t)$ will be reduced to the same value.

PAR-value re-grows in the transmit chain (interpolation filters, digital-to-analogue converter).

Level of re-growth depends strongly on the discrete-time to continuous-time conversion chain and is still a topic of ongoing research.
PAR of a multicarrier signal

- If we transmit random independent symbols $x_k$, the transmit multiplex

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j2\pi \frac{kn}{N}}$$

is a sum of many independent random variables for each time instant $n$

- In practice, $x_k$ are never truly mutually independent

- However, given that
  - the number $N$ of subcarriers is large, and ...
  - ... large QAM constellations are used,

it is reasonable to assume that each sample $s(n)$ has Gaussian distribution according (central limit theorem)

- A Gaussian signal is comparably peaky (cf. $s^{(3)}(n)$)
Stochastic measure to describe peakiness (PAR)

- Assuming $s(n) \sim \mathcal{N}(0, \sigma^2)$, the probability that $|s(n)|$ exceeds a certain value $p$ is given by

$$\Pr(|s(n)| > p) = 2 \frac{1}{\sqrt{2\pi\sigma}} \int_{x=p}^{\infty} e^{-\frac{x^2}{2\sigma^2}} dx = 2Q\left(\frac{p}{\sigma}\right), \quad (1)$$

where the Q-function $Q(x)$ is the complementary Gaussian $\mathcal{N}(0, 1)$ cumulative distribution function.

- (1) is a probabilistic measure for random signals.
PAR reduction techniques

- Clipping
- Clip modification (windowing, noise shaping) and reconstruction
- Coding
- Selected mapping
- Partial transmit sequences
- Tone reservation
- Tone injection
- “Constellation distortion”
Tone reservation

- idea: add a peak-annihilating signal $c(n)$ to the transmit multiplex $s(n)$
- $c(n)$ is chosen such that it has counter-peaks (peaks of opposite sign at the same positions as the peaks of $s(n)$)
- goal: peaks of the sum $s(n) + c(n)$ are lower than the peaks of $s(n)$
- $c(n)$ is generated by sacrificing a few subcarriers specified by the set $\mathcal{T}_{tr} = t_1, \ldots, t_T$ (→ tone reservation)
Tone reservation: optimal $c(n)$

- chose values $x_k, k \in \mathcal{I}_{tr}$ such that
  
  $$c(n) = \frac{1}{\sqrt{N}} \sum_{k \in \mathcal{I}_{tr}} x_k e^{j2\pi \frac{kn}{N}}$$

  is a “good” peak-annihilator

- Minimisation problem:

  $$\begin{bmatrix} x_{t_1}^{opt} \\ \vdots \\ x_{t_T}^{opt} \end{bmatrix} = \min_{x_i, i \in \mathcal{I}_{tr}} \max_n |s(n) + c(n)| \quad \text{subject to} \quad \frac{1}{N} \sum_{n=0}^{N-1} |s(n) + c(n)|^2 \leq \sigma^2,$$

  (2)

  where $\sigma^2$ is the average power per signal sample

- (2) can be cast as
  - QCQP for OFDM (complex transmit multiplex)
  - linear program for DMT (real transmit multiplex)

- (2) must be solved once per multicarrier symbol
Tone reservation: gradient-based approach

- A less complex approach that, although suboptimal, performs very well in practice
- Idea:
  - the symbols $x_i, i \in \mathcal{T}_{tr}$ span a $T$-dimensional space
  - determine the gradient of the clip noise power with respect to the symbols $x_i, i \in \mathcal{T}_{tr}$
  - the clip noise power is a (scalar-valued) convex function (quadratic) of $T$ variables
  - the gradient specifies the direction and the magnitude for every dimension such that the minimum of the error surface (clip noise power surface) is approached via the path of steepest descent
Tone reservation: gradient-based approach cont’d

Successive update of counter-peak:

\[
\mathbf{c}^{(i+1)} = \mathbf{c}^{(i)} - \mu \sum_{n: |s(n) + c^{(i)}(n)| > s_{\text{max}}} \text{sign}(s(n) + c^{(i)}(n)) \left( |s(n) + c^{(i)}(n)| - s_{\text{max}} \right) \mathbf{p}(n)
\]

Example of a precomputed counter-peak component \( \mathbf{p}(n_{\text{max}}) \) for \( n_{\text{max}} = 40 \) \((N = 512, \mathcal{T}_{\text{tr}} = \{10, 40, 100, 200, 180, 225\})\):
Tone reservation: application aspects

- Tone reservation is frequently used in practice and yields large reduction of PAR.
- A set of $T$ tones has to be sacrificed.
- The choice of the tones in the set $\mathcal{I}_{tr}$ is crucial:
  - often determined by system constraints
  - finding the optimal (in whichever sense) set is expensive
  - a random selection of $T$ tones yields good results in practice
- No coordination between the transmitter and the receiver required.
Tone injection

- Idea: like in tone reservation a peak-annihilating signal is added to the transmit signal multiplex
- However, tone injection does **not reserve tones**
- Tone injection is based on the modulo-principle: instead of transmitting a symbol $x_k$ from, e.g., a 16-QAM constellation, a corresponding symbol from the extended constellation can be chosen
Tone injection, cont’d

Extended 16-QAM constellation: for each point of the master constellation there are 8 alternatives
Tone injection, cont’d

- Receiver has to know that the transmitter may choose to send points from the extended constellation (coordination required)
- Modifying the decision device such that it copes with the extended constellation is simple
- Selecting symbols from the extended constellation increases the average transmit power
- If peaks to be combated occur rarely, the average excess power remains low
- Tone injection attempts to reduce time-domain peaks at the expense of frequency-domain peaks
- Choice of
  - equivalent symbol in the extended constellation
  - tones to be used
    is done by limited-search algorithms
- No tones that could be used for information transmission are “wasted”
- Performance of tone injection is worse compared to tone reservation
Reduction of out-of-band emission

- Assigned bandwidth is usually limited (the channel is often shared with other users or the receive data stream (frequency division duplexing))

- **Power spectral density masks** specify the allowed out-of-band emission

- Limitation of a symbol to $N$ or $N + L$ samples, corresponds to a multiplication of the symbol in continuous-time domain with a rectangular window

- In frequency domain this corresponds to a convolution of each Dirac pulse, representing a subcarrier, with the Fourier transform of a rectangular pulse

- The magnitude of the sidelobes of the Fourier transform of a rectangular pulse decay only slowly with frequency

- In practice, often a stronger suppression of the out-of-band components is required to meet the masks
Windowing

- **windowing** denotes that a signal is multiplied by a finite-length weighting function—the so called window (or window function)
- Observation: sharp transitions in time domain cause high-frequency components in the Fourier domain
- A good window function should ‘smoothen’ the transitions
- In fact, it can be shown that the quality of a continuous-time window function with respect to its out-of-band emission is proportional to its number of continuous derivatives
- → A rectangular window, which is not even continuous itself, has rather high out-of-band components
- A window function for which all derivatives are continuous is the raised-cosine window

\[ s_w(n) = \begin{cases} 
\frac{1}{2} (1 - \cos(\pi \frac{n+L+W+1}{W+1})), & n = -L - W, \ldots, -L - 1 \\
1, & n = -L, \ldots, N - 1 \\
\frac{1}{2} (1 + \cos(\pi \frac{n-N+1}{W+1})), & n = N, \ldots, N + W - 1 
\end{cases} \]

where \( W \) is the length of the 'raised-cosine' transition interval.
In order to maintain orthogonality of the receive signal multiplex, the transmit signal has to be extended by twice the length $W$.

The symbols needs to be cyclically extended by $L + W$ samples at the beginning and by $W$ samples at the end before windowing.

**Definition of excess interval:** $\frac{2W}{(N + L)} \times 100\%$
There is a tradeoff between out-of-band emission and excess interval:

\[ N = 512, \ L = 40, \ f_s = 1 \]
Filtering

- Straightforward filtering of the transmit multiplex $s(n)$ causes a total dispersion of $M + F$, where $F$ is the dispersion of the transmit filter.
- In order to preserve orthogonality (avoid ISI and ICI), the cyclic prefix has to be extended to length $L = M + F$.
- Hence, one goal of the filter design for this purpose is clearly to keep $F$ low.
- Apart from the desired suppression in the stopband, the filter may introduce an undesired ripple in the passband, which has a negative impact on the bit error rate performance.
- A predistortion in frequency domain (before the IDFT) can be employed to mitigate this ripple.
Spectral compensation

- Idea: like in tone reservation, a set $\mathcal{I}_{sc}$ of subcarriers close to the band edge is used for transmitting symbols $x_k, k \in \mathcal{I}_{sc}$ such that the out-of-band power spectral density is reduced.

- Like in tone reservation, the subcarriers in the set $\mathcal{I}_{sc}$ do not carry information.

- The symbols $x_k, k \in \mathcal{I}_{sc}$ depend on the information carrying symbols $x_k, k \notin \mathcal{I}_{sc}$.

- However, the dependence is fixed and does not change from symbol to symbol like in the case of tone reservation.

- Hence, this dependence can be determined offline (before the transmission begins).
Summary

**PAR**
- problem: $s(n)$ has peaks with large power compared to average power
- remedies exist (tradeoff: throughput versus PAR)

**Out-of-band emission**
- problem: the out-of-band energy of a DFT/IDFT-based multicarrier is often too high
- remedies exist (tradeoff: throughput versus out-of-band egress):