OFDM—what, why, how?

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OFDM applications

- Terrestrial digital TV broadcasting. DVB-T (Digital Video Broadcasting - Terrestrial) EN 300 744
- Terrestrial radio broadcasting. DAB (Digital Audio Broadcasting), DRM (Digital Radio Mondial)
- Wireless high-speed local networking and wireless Internet access. WLAN (Wireless Local Area Network) or WiFi IEEE 802.11a/g, WiMAX IEEE 802.16e
- Inhome high-speed networking over home wiring (powerline, phone-line, coax). ITU-T G.hn
- High-speed Internet access via phone line. Digital Subscriber Line (ADSL, VDSL)
- Wireless personal area networks. WiMedia/ecma-368
- Mobile radio. LTE (Long Term Evolution) 3GPP Release 7,8
Elements of a generic communication system

- **Source**: Encoding of the message
- **Channel**: Transmission of the encoded message
- **Sink**: Decoding of the received message

**Encryption**
- Protection of the message from being understood or falsified by a third party

**Channel Encoding**
- Controlled introduction of redundancy to increase reliability of transmission

**Modulation**
- Conversion of symbols into waveforms appropriate for the given channel

**Synchronization**
- Retrieval of block boundaries

**Equalization**
- Mitigation of distortion introduced by the channel

**Source Encoding**: Compression of the message as close to its entropy as possible (lossless) or below (lossy)
Time dispersion in a wireless channel

Multipath propagation leads to dispersion of the receive signal in time

\[
\begin{align*}
\text{transmitted signal} & \\
\text{power} & = p_0 \\
\text{delay} & \\
\end{align*}
\]

\[
\begin{align*}
\text{power-delay profile} & \\
\text{power} & = p_0, p_1, p_2 \\
\text{delay} & = \tau_0, \tau_1, \tau_2, \tau_{\text{max}} \\
\end{align*}
\]
Time dispersion in a wireline channel

Different frequency-components experience different attenuation and different delay $\rightarrow$ dispersion (“smearing”) in time
Discrete-time linear time-invariant (LTI) dispersive channel

\[ s(n) = \sum_k s(k) \delta(n - k) \]

\[ r(n) = \sum_k s(k) h(n - k) \]
A discrete-time, linear, time-invariant, causal, time-dispersive channel is described by its impulse response $h(n) \neq 0, n = 0, 1, \ldots, M$ of length $M+1$.

The channel performs linear convolution:

$$r(n) = \sum_k s(k) h(n - k)$$

Tapped delay-line model
Discrete-time linear time-invariant (LTI) dispersive channel

- Linear convolution, denoted by $\ast$, of a length-$N$ input sequence $s(n), n = n_0, \ldots, n_0 + N - 1$ and $h(n)$ results in an output sequence

$$r(n) = h(n) \ast s(n) = \sum_k h(k) s(n - k) =$$

$$= s(n) \ast h(n) = \sum_k s(k) h(n - k), \quad n = n_0, \ldots, n_0 + N + M - 1$$

of length $N + M$.

- Since each input sample is “spread out” over $M = \lceil \tau_{\text{max}} / T_S \rceil$ additional samples, we refer to $M$ as the dispersion of the channel ($T_S$ is the sampling period)
Time-dispersion $\leftrightarrow$ frequency selectivity

Variation in frequency causes dispersion in time over $\tau_{\text{max}}$ seconds.
Communication: wideband versus narrowband

**Wideband:** $T_{\text{sym}} \ll \tau_{\text{max}}$

- **Amplitude:** $T_{\text{sym}} \ll \tau_{\text{max}}$
- **Delay:** $T_{\text{sym}} \ll \tau_{\text{max}}$
- **Magnitude:** $B$

**Narrowband:** $T_{\text{sym}} \gg \tau_{\text{max}}$

- **Amplitude:** $T_{\text{sym}} \gg \tau_{\text{max}}$
- **Delay:** $T_{\text{sym}} \gg \tau_{\text{max}}$
- **Magnitude:** $B$
Effect of dispersion on block transmission: ISI

- Length-$N$ symbol: $s(0), s(1), \ldots, s(N-1)$

Transmission of length-$N$ symbols over an LTI channel with dispersion $M$ (length $M+1$) causes inter-symbol interference (ISI).
Dilemma: data rate and intersymbol interference

- **Moderate symbol rate (with respect to dispersion)**
  - moderate data rate
  - moderate ISI

- **High symbol rate (with respect to dispersion)**
  - high data rate
  - severe ISI!
Real-world examples

Example

GSM

- Guess: max. delay spread $\approx$ ___.
- Guess: symbol duration $\approx$ ___.
- Compute: ISI affects ___.
- Conclusion: ISI is manageable (5-tap equalizer is standard).

Example

ADSL (assume: data rate = 8 Mbit/s, uncoded 16QAM, single-carrier)

- Guess: max. impulse response duration $\approx$ ___.
- Compute: symbol duration = ___.
- Compute: ISI affects ___ symbol(s).
- Conclusion: ISI is severe! (equalizer is not an option).
Single-carrier transmission

**Time domain**

- Graph showing oscillations over time intervals $T_{sym}$, $2T_{sym}$, and $3T_{sym}$.

**Frequency domain**

- Region $B$.

**Constellation diagram**

- Points in the constellation diagram labeled as $\{x_0^{(i)}, x_0^{(q)}\}$, $\{x_1^{(i)}, x_1^{(q)}\}$, and $\{x_2^{(i)}, x_2^{(q)}\}$.
Idea: transmit several longer symbols simultaneously

\[ T_{\text{sym}} = T_{\text{MC}} \]

\[ B \]
\[ \frac{2B}{3} \]
\[ \frac{B}{3} \]

\[ 3T_{\text{sym}} = T_{\text{MC}} \]
Idea (cont’d): \( N \) narrowband channels

- Multicarrier-symbol duration \( T_{MC} \gg \) delay spread \( \tau_{\text{max}} \)
- \( N \) narrowband channels with bandwidth \( \Delta f = B/N \) instead of one wideband channel with bandwidth \( B \)
- A multicarrier symbol in baseband is \( N = \lceil T_{MC}/T_S \rceil \) samples long
- Subcarrier (tone) spacing \( \Delta f = B/N = 1/T_{MC} \)
- Each subchannel is quasi frequency-flat
Idea (cont’d): cyclic extension

- We make each transmit symbol at least $\tau_{\text{max}}$ seconds (or $L \geq M$ samples, where $M = \lceil \tau_{\text{max}} / T_S \rceil$) longer
- At the receiver, we remove this extension, which eliminates the ISI
- If $T_{MC} \gg \tau_{\text{max}}$, the loss is bearable
Cyclic extension

- We use sinusoidal transmit signal components
- We extend the beginning of each multicarrier symbol by $L \geq M$ samples
- Since each transmit signal component extends over an integer number of periods, this extension corresponds to copying the last $L$ samples to the beginning
- The same holds for a linear combination of transmit signal components, i.e., for the complete transmit signal
- Cyclic extension is a cheap operation (copying)
We transmit a white image (black spots mark receive errors; noise-free channel, no dispersion)

Certain values of the carrier spacing do not lead to inter-carrier interference

Carrier waveforms with such a spacing in frequency are mutually orthogonal
Why sinusoids?

- Small reasons: easy to generate (simple hardware), offer an intuitively appealing control over the spectrum, etc.
- BIG reason: infinitely-long sinusoids are eigen-functions of every linear time-invariant (LTI) system!
- We exploit this when transmitting finite-length sinusoids over a dispersive channel:
Why sinusoids?

- Input: sine wave $\rightarrow$ output: transient ($\tau_{\text{max}} = MT_S$ seconds long) followed by shifted and scaled sine wave
- The transient is eliminated by removing the cyclic extension at the receiver
- The influence of the channel reduces to scaling and shifting in time (2 parameters $= 1$ complex coefficient per subcarrier)
- We need the cyclic extension to produce the transient so that the channel answers to our symbols as if they were infinitely long sinusoids
- If we use orthogonal transmit signal components, a cyclic extension of at least $\tau_{\text{max}}$ seconds ($L \geq M$) avoids not only ISI but also inter-carrier interference (ICI)
Choice of multicarrier symbol length $T_{MC}$ — limits and design criteria

$\tau_{\text{max}} \ll T_{MC} \ll T_{\text{coh}}$

- Lower limit: $T_{MC} \gg \tau_{\text{max}}$ keeps loss due to cyclic extension low
- Upper limit:
  - The channel properties should not change during a symbol $\rightarrow T_{MC} \ll T_{\text{coh}}$ (otherwise our linear \textit{time-invariant} channel model is invalid)
  - Latency
- The coherence time $T_{\text{coh}} = 1/B_{\text{dop}}$ is a measure for the duration the channel properties remain quasi constant
- Variation in time causes dispersion in frequency over $B_{\text{dop}}$ Hz
- Dispersion in frequency occurs for example due to motion: Doppler effect
Dispersion in frequency due to motion: Doppler effect

When the fixed terminal (FT) transmits a signal with frequency $f = c/\lambda$, the mobile terminal (MT) receives this signal at frequency $f + \nu = f + \nu' / \lambda$, where $\nu'$ is the relative velocity of the MT with respect to the FT and $c$ is the propagation speed (for light $c \approx 3 \cdot 10^8$ m/s).

- Note that $\nu'$ is a signed quantity

\[
\begin{align*}

v_1' &= v_1 \cos \alpha_1 \\
v_2' &= -v_2 \cos \alpha_2
\end{align*}
\]

- $\nu$ is called the Doppler shift (example GSM: maximum Doppler shift for 100 km/h and 900 MHz: $\nu = \nu' f / c \approx 83$ Hz)
Choice of multicarrier symbol length $T_{MC}$ — limits and design criteria

Example

HiperLAN2 (pedestrian mobility: $v \leq 15$ km/h, $f = 5.2$ GHz, $B = 20$ MHz, $N = 64$):

- Guess: max. dispersion in time $\tau_{\text{max}} \approx ____$
- Compute: max. dispersion in frequency (due to motion): $T_{\text{coh}} = ____$
- Compute: symbol duration $T_{MC} = ____$
- Conclusion: $\tau_{\text{max}} T_{MC} T_{\text{coh}}$ → HiperLAN2 should / should not work
Challenge: peaky transmit signal

- Problem at receiver: clip noise limits performance

- Problem at transmitter: clipping power-amplifier violates PSD specs
Challenge: synchronisation

“Rattling the grid”

- symbol timing offset (shown above)
- carrier frequency/phase offset
- sampling clock offset
Challenge: synchronisation

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"Rattling the grid"

\[(N + L)\Delta T_s\]

\[\Delta F_s\]

- symbol timing offset (shown above)
- carrier frequency/phase offset
- sampling clock offset
Summary

- Motivation: high data rate $\rightarrow$ high symbol rate $\rightarrow$ time-dispersive channel causes severe ISI
- Multicarrier modulation: simultaneous transmission of many, long, and mutually orthogonal (sinusoidal) symbols
- Cyclic extension
  - avoids ISI
  - avoids ICI
- Multicarrier-system design limits: $\tau_{\text{max}} \ll T_{\text{MC}} \ll T_{\text{coh}}$