Fading Channels I: Characterization and Signaling

Digital Communications

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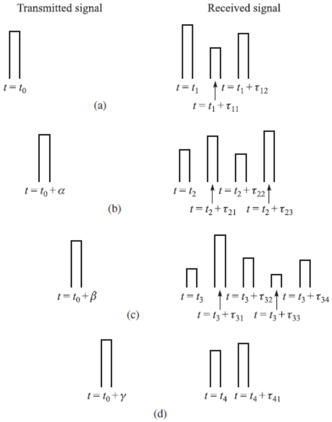
Characterization of Fading Multipath Channels



Characterization of Fading Multipath Channels

- In addition to the *time spread* introduced by the multipath medium, in this chapter we consider *variations in time* of the nature of the multipath.
- Each path n has an associated propagation delay τ_n and attenuation factor α_n.
- The time-variant impulse response of the equivalent lowpass channel is given by

$$c(\tau; t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \delta[\tau - \tau_n(t)]$$

for the discrete-time channel, and

$$c(\tau; t) = \alpha(\tau; t) e^{-j2\pi f_c t}$$

for the continuous-time channel.

- Note that $c(\tau; t)$ represents the response of the channel at time t due to an impulse applied at time $t - \tau$.

Characterization of Fading Multipath Channels

- The received signal to an unmodulated carrier transmission $s_l(t) = 1$ is given by

$$r_l(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}$$
$$= \sum_n \alpha_n(t) e^{j\theta_n(t)}$$

  where $\theta_n(t) = -2\pi f_c \tau_n(t)$ and f_c is the carrier frequency.
- $\theta_n(t)$ changes much faster than $\alpha_n(t)$.
- *Signal fading* is a result of the time variations in the phases $\{\theta_n(t)\}$.
- A statistical treatment of the channel is suitable. Some models are
  - Rayleigh fading channel.
  - Rice fading channel.
  - Nakagami-*m* fading channel.

Channel Correlation Functions and Power Spectra

- Assume that $c(\tau; t)$ is *wide-sense-stationary*. The associated autocorrelation function is

$$\begin{aligned} R_c(\tau_2, \tau_1; \Delta t) &= E[c^*(\tau_1; t)c(\tau_2; t + \Delta t)] \\ &= R_c(\tau_1; \Delta t)\delta(\tau_2 - \tau_1) \end{aligned}$$

  where, in the last step, *uncorrelated scattering* is assumed.
- $R_c(\tau) \equiv R_c(\tau; 0)$ is called the *multipath intensity profile* or the *power delay spectrum* of the channel.
- The support of $R_c(\tau)$ is called the *multipath spread of the channel* and is denoted by T_m.

Channel Correlation Functions and Power Spectra

- Equivalently, in the Fourier domain we have the time-variant transfer function $C(f;t)$ defined as

$$C(f;t) = \int_{-\infty}^{\infty} c(\tau;t)e^{-j2\pi f\tau}\,d\tau$$

- We define the autocorrelation function

$$\begin{aligned} R_C(f_2, f_1; \Delta t) &= E[C^*(f_1;t)C(f_2;t+\Delta t)] \\ &= R_C(\Delta f; \Delta t) \end{aligned}$$

- $R_C(\Delta f; \Delta t)$ is called the *spaced-frequency, spaced-time correlation function* of the channel.
- The *spaced-frequency correlation function* is defined as $R_C(\Delta f) \equiv R_C(\Delta f; 0)$, and can also be computed as $R_C(\Delta f) = \int_{-\infty}^{\infty} R_c(\tau;t)e^{-j2\pi\Delta f\tau}\,d\tau$
- $R_C(\Delta f)$ provides a measure of the *coherence bandwidth* of the channel, $(\Delta f)_c \approx \frac{1}{T_m}$.

Channel Correlation Functions and Power Spectra

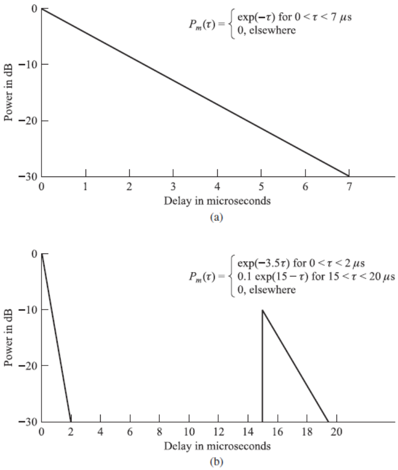


FIGURE 13.1–7
Cost 207 average power delay profiles: (a) typical delay profile for suburban and urban areas; (b) typical "bad"-case delay profile for hilly terrain. [*From Cost 207 Document 207 TD (86)51 rev 3.*]

Channel Correlation Functions and Power Spectra

- In order to characterize time variations we define

$$S_C(\Delta f; \lambda) = \int_{-\infty}^{\infty} R_C(\Delta f; \Delta t) e^{-j2\pi\lambda\Delta t} \, d\Delta t.$$

- Furthermore, $S_C(\lambda) \equiv S_C(0; \lambda)$ is called the *Doppler power spectrum* of the channel.
- The support of $S_C(\lambda)$ is called the *Doppler spread* B_d of the channel.
- The *coherence time* $(\Delta t)_c \approx \frac{1}{B_d}$ is related to the *spaced-time correlation function* $R_C(\Delta t) \equiv R_C(0; \Delta t)$.
- Finally, we define the *scattering function* of the channel $S(\tau; \lambda)$ as

$$S(\tau; \lambda) = \int_{-\infty}^{\infty} S_C(\Delta f; \lambda) e^{j2\pi\tau\Delta f} \, d\Delta f.$$

Channel Correlation Functions and Power Spectra

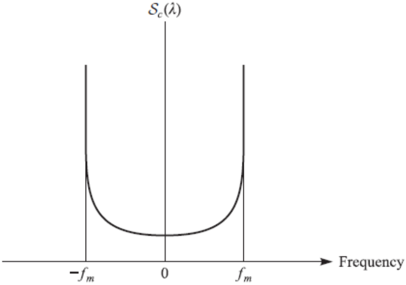


FIGURE 13.1–8
Model of Doppler spectrum for a mobile radio channel.

Channel Correlation Functions and Power Spectra

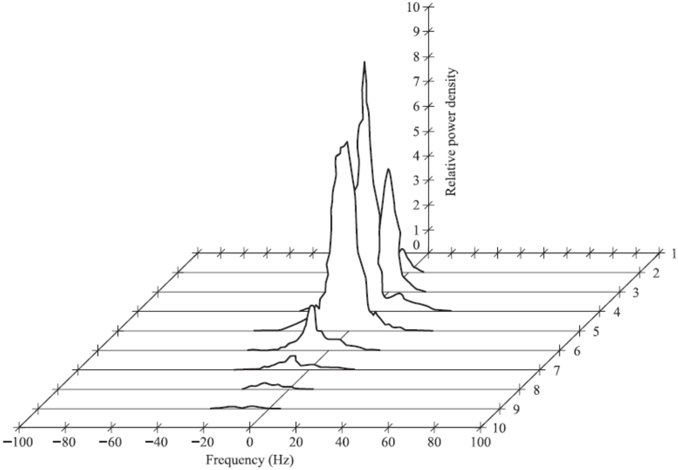


FIGURE 13.1–6
Scattering function of a medium-range tropospheric scatter channel. The taps delay increment is 0.1 μs.

Channel Correlation Functions and Power Spectra

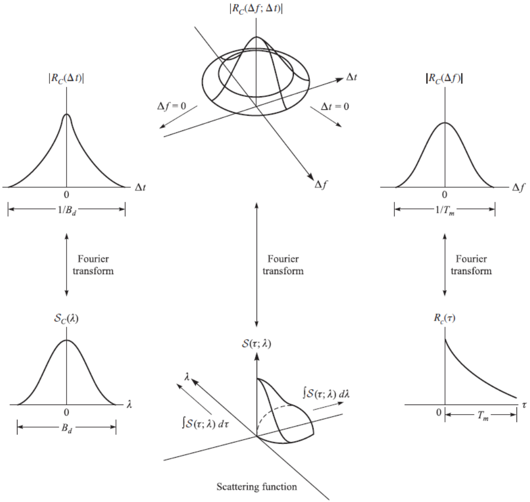


FIGURE 13.1–5
Relationships among the channel correlation functions and power spectra. [*From Green (1962), with permission.*]

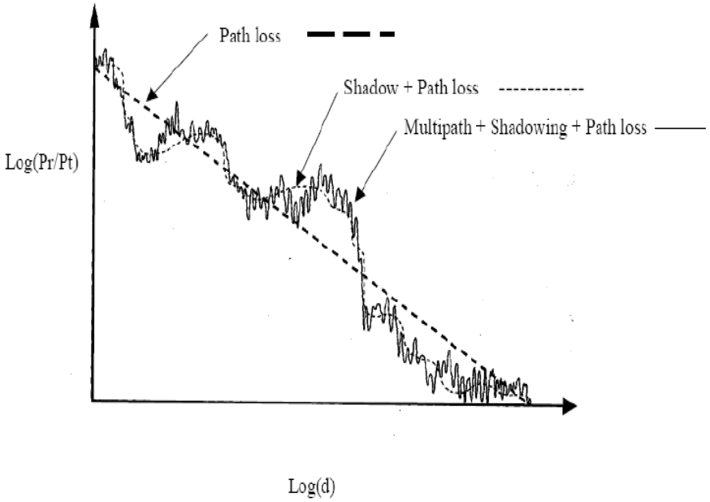
Statistical Models for Fading Channels

- Large number of scatterers contributing $\sum_n X_n$, with $X_n \sim N(0, \sigma^2)$ i.i.d, then, by the central limit theorem, phase uniformly distributed in $[0, 2\pi]$ and envelope follows a Rayleigh probability distribution

$$p_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, \quad r \geq 0$$

  where $\Omega = E(R^2)$.
- Nakagami-m distribution, which includes the Rayleigh distribution as a special case $(m = 1)$.
- Rice distribution, which includes the power of the non-fading, *specular components*.

Propagation Models for Mobile Radio Channels



The Effect of Signal Characteristics on the Choice of a Channel Model

- The effect of the channel $c(\tau; t)$ on the transmitted signal $s_l(t)$ is a function of our choice of signal bandwidth W and signal duration T.

- $W \gg \frac{1}{T_m} \approx (\Delta f)_c$, *frequency-selective* channel.
- $W \ll \frac{1}{T_m} \approx (\Delta f)_c$, *frequency-nonselective* channel or *flat fading*.

- Signaling interval $T \ll \frac{1}{B_d} \approx (\Delta t)_c$, *slowly fading* channel.
- Signaling interval $T \gg \frac{1}{B_d} \approx (\Delta t)_c$, *fast fading* channel.

- For a frequency-nonselective channel $r_l(t) = C(0; t)s_l(t)$, with $C(0; t) = \alpha(t)e^{j\phi(t)}$. E.g. $C(0; t)$ zero-mean complex-valued Gaussian stochastic process.
- Define the *spread factor* of the channel $T_m B_d$. Channel is *underspread* if $T_m B_d < 1$.

Frequency-Nonselective, Slowly Fading Channel

- Channel modeled as a one-tap filter, with filter coefficient $C(0; t) = \alpha(t)e^{j\phi(t)}$ constant for at least T. Therefore,

$$r_l(t) = \alpha e^{j\phi} s_l(t) + z(t), \quad 0 \leq t \leq T.$$

- Assume phase shift ϕ estimated withour error, hence, coherent detection at the output of a matched filter demodulator applies. For PSK (4.3-13),

$$P_b(\gamma_b) = Q(\sqrt{2\gamma_b})$$

where $\gamma_b = \alpha^2 \mathcal{E}_b / N_0$ and α is fixed. Similarly, for FSK (4.2-42),

$$P_b(\gamma_b) = Q(\sqrt{\gamma_b}).$$

- Finally,

$$P_b = \int_0^\infty P_b(\gamma_b) p_b(\gamma_b)\, d\gamma_b.$$

Frequency-Nonselective, Slowly Fading Channel, with Rayleigh Statistics

- Assume α is Rayleigh-distributed. Thus, γ_b is chi-square-distributed with

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\gamma_b/\bar{\gamma}_b}, \quad \gamma_b \geq 0$$

  and $\bar{\gamma}_b = \frac{\mathcal{E}_b}{N_0} E(\alpha^2)$ is the average signal-to-noise ratio.
- The bit error probability is then (see problem 4.44-1)

$$P_b = \begin{cases} \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right), & \text{for BPSK} \\ \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right), & \text{for BFSK} \end{cases}$$

Frequency-Nonselective, Slowly Fading Channel, with Rayleigh Statistics

- The error probabilities for high SNR, i.e. $\bar{\gamma}_b \gg 1$ become

$$P_b \approx \begin{cases} 1/4\bar{\gamma}_b, & \text{for coherent PSK} \\ 1/2\bar{\gamma}_b, & \text{for coherent, orthogonal FSK} \\ 1/2\bar{\gamma}_b, & \text{for DPSK} \\ 1/\bar{\gamma}_b, & \text{for noncoherent, orthogonal FSK} \end{cases}$$

- The error rate P_b **decreases only inversely with SNR**. Compare to a nonfading channel (exponential decrease with SNR).
- DPSK performs in pair with coherent, orthogonal FSK.

Frequency-Nonselective, Slowly Fading Channel, with Rayleigh Statistics

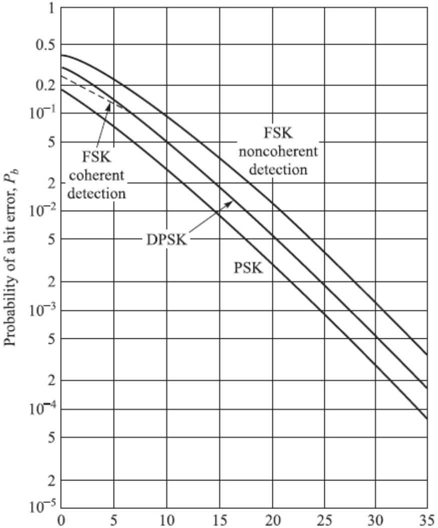


FIGURE 13.3–1
Performance of binary signaling on a Rayleigh fading channel.

Frequency-Nonselective, Slowly Fading Channel, with Nakagami-m Statistics

- When envelope α is characterized by the Nakagami-m distribution, the distribution of $\gamma = \alpha^2 \mathcal{E}_b / N_0$ is

  $$p(\gamma) = \frac{m^m}{\Gamma(m)\bar{\gamma}^m} \gamma^{m-1} e^{-m\gamma/\bar{\gamma}},$$

  where $\bar{\gamma} = \mathcal{E}/N_0 E(\alpha^2)$.
- $m = 1$ corresponds to Rayleigh fading.
- $m > 1$ corresponds to fading less severe than Rayleigh.
- $m < 1$ corresponds to fading more severe than Rayleigh.

Diversity Techniques for Fading Multipath Channels

- The **idea** is to supply to the receiver several replicas of the same information transmitted over independently fading channels. For L **independently fading replicas** with probability p, the probability of all replicas being in a fading deep is p^L.
- How to achieve diversity?
  - *Frequency diversity*, by transmitting on L carriers with separation between successive carriers greater than $(\Delta f)_c$.
  - *Time diversity*, by transmitting on L time slots with separation between successive time slots greater than $(\Delta t)_c$.
  - *Space diversity*, by transmitting on L antennas with minimum separation between antennas greater than $(\Delta d)_c$.
  - *Angle-of-arrival diversity*.
  - *Polarization diversity*.
- Frequency diversity can also be obtained by using a wideband signal such that $W > (\Delta f)c$. The achievable diversity order is $L \approx W/(\Delta f)c$, which corresponds to the number of *resolvable* signal components.

Diversity Techniques for Fading Multipath Channels with Binary Signals

- Determine the bit error probability P_b for a binary digital communication system with L diversity channels, each frequency-nonselevtive and slowly fading. The fading processes $\{C_k(0;t)\}$ are assumed mutually independent. The noise processes $\{z_k(t)\}$ are assumed mutually independent, with identical autocorrelation functions.

$$r_{lk}(t) = \alpha_k e^{j\phi_k} s_{km}(t) + z_k(t), \quad k = 1, 2, \ldots, L, m = 1, 2$$

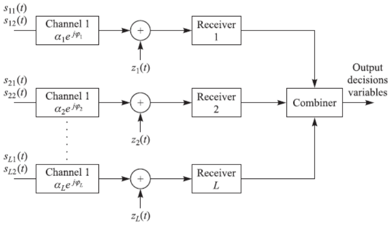


FIGURE 13.4–1
Model of binary digital communication system with diversity.

Diversity Techniques for Fading Multipath Channels with Binary Signals

- The optimum demodulator consists of a bank of matched filters and a *maximal ratio combiner* (MRC).

$$b_{k1}(t) = s_{k1}^*(T - t)$$
$$b_{k2}(t) = s_{k2}^*(T - t)$$

  - Compensate for the phase shift in the channel.
  - Weight the signal by a factor proportional to the signal strenght.

- For PSK, at the output of the MRC we have

$$U = \mathrm{Re}\left(2\mathcal{E}\sum_{k=1}^{L}\alpha_k^2 + \sum_{k=1}^{L}\alpha_k N_k\right) = 2\mathcal{E}\sum_{k=1}^{L}\alpha_k^2 + \sum_{k=1}^{L}\alpha_k \mathrm{Re}(N_k)$$

and $N_k = e^{-j\phi_k}\int_0^T z_k(t)s_k^*(t)\,dt$.

Diversity Techniques for Fading Multipath Channels

- For PSK modulation and a fixed set of $\{\alpha_k\}$ we have that

$$P_b(\gamma_b) = Q(E^2(U)/\sigma_U^2) = Q(\sqrt{2\gamma_b}),$$

where the SNR per bit, γ_b, is given as

$$\gamma_b = \frac{\mathcal{E}}{N_0}\sum_{k=1}^{L}\alpha_k^2 = \sum_{k=1}^{L}\gamma_k$$

and $\gamma_k = \mathcal{E}\alpha_k^2/N_0$ is the instantaneous SNR on the kth channel.

- The probability density function $p(\gamma_b)$ is that of a chi-square-dsitributed r.v. with $2L$ degrees of freedom:

$$p(\gamma_b) = \frac{1}{(L-1)!\bar{\gamma}_c^L}\gamma_b^{L-1}e^{-\gamma_b/\bar{\gamma}_c}$$

where $\bar{\gamma}_c = E(\alpha_k^2)\mathcal{E}/N_0$ is the average SNR per channel.

Diversity Techniques for Fading Multipath Channels

- Finally, we need to average over the fading channel statistics

$$P_b = \int_0^\infty P_b(\gamma_b) p_b(\gamma_b)\, d\gamma_b.$$

- The closed-form solution to this integral is

$$P_b = [\frac{1}{2}(1-\mu)]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} [\frac{1}{2}(1+\mu)]^k$$

where, by definition $\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}$

- We note that

$$P_{b,\mathrm{PSK}} = \frac{1}{2}(1-\mu), \quad 1 - P_{b,\mathrm{PSK}} = \frac{1}{2}(1+\mu)$$

where $P_{b,\mathrm{PSK}}$ is the probability error for a single fading channel.

Diversity Techniques for Fading Multipath Channels

- When the average SNR per channel $\bar{\gamma}_c$ is greater that 10 dB, we have that

$$P_b \approx (\frac{1}{4\bar{\gamma}_c})^L \binom{2L-1}{L}.$$

- The error rate decreases **inversely** with the ***L*th power** of the **SNR**.
- Actually

$$P_b \approx \begin{cases} (\frac{1}{4\bar{\gamma}_c})^L \binom{2L-1}{L}, & \text{for BPSK} \\ (\frac{1}{2\bar{\gamma}_c})^L \binom{2L-1}{L}, & \text{for BFSK} \\ (\frac{1}{2\bar{\gamma}_c})^L \binom{2L-1}{L}, & \text{for DPSK} \\ (\frac{1}{\bar{\gamma}_c})^L \binom{2L-1}{L}, & \text{for noncoherent BFSK} \end{cases}$$

Diversity Techniques for Fading Multipath Channels

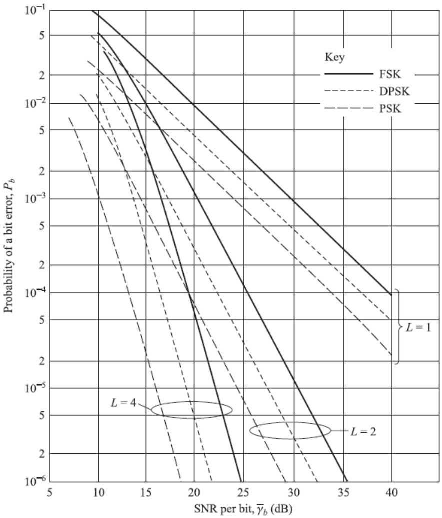


FIGURE 13.4–2
Performance of binary signals with diversity.

Diversity Techniques for Fading Multipath Channels, Nakagami Fading

A K-channel system transmitting in a Nakagami fading channel with independent fading is equivalent to an $L = Km$ channel diversity in a Rayleigh fading channel.

A Tapped Delay Line Model

- The W-bandlimited, time-variant frequency-selective channel can be modeled as a tapped delay line with tap spacing $1/W$ and tap weight coefficients $c_n(t)$

$$c(\tau; t) = \sum_{n=-\infty}^{\infty} c_n(t)\delta(\tau - n/W) \quad (13.5-8)$$

with $c_n(t) = \frac{1}{W} c(\frac{n}{W}; t)$, and the corresponding time-variant transfer function is

$$C(f; t) = \sum_{n=-\infty}^{\infty} c_n(t) e^{-j2\pi fn/W}.$$

- For all practical purposes truncation can be applied at $L = \lfloor T_m W \rfloor + 1$ taps.
- $c_n(t)$ are complex-valued stationary, mutually uncorrelated (US) random processes.

A Tapped Delay Line Model

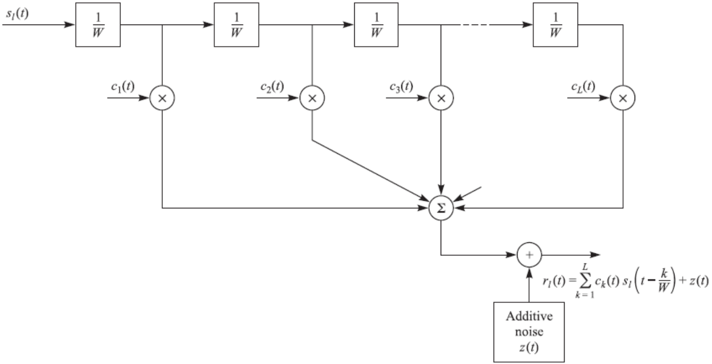


FIGURE 13.5–1
Trapped delay line model of frequency-selective channel.

The RAKE Demodulator

- Assume binary signaling with $T \gg T_m$, then

$$\begin{aligned} r_l(t) &= \sum_{k=1}^{L} c_k(t) s_{li}(t - k/W) + z(t) \\ &= v_i(t) + z(t), \quad 0 \le t \le T, \quad i = 1, 2. \end{aligned}$$

- The optimal demodulator consists of two filters matched to $v_1(t)$ and $v_2(t)$, and has decision variables

$$\begin{aligned} U_m &= \mathrm{Re}\left[\int_0^T r_l(t) v_m^*(t)\, dt\right] \\ &= \mathrm{Re}\left[\sum_{k=1}^{L} \int_0^T r_l(t) c_k^*(t) s_m^*(t - k/W)\, dt\right], \quad m = 1, 2. \end{aligned}$$

- The tapped delay line demodulator attempts to collect the signal energy from all received signal paths that fall within the span of the delay line and carry the same information.

The RAKE Demodulator

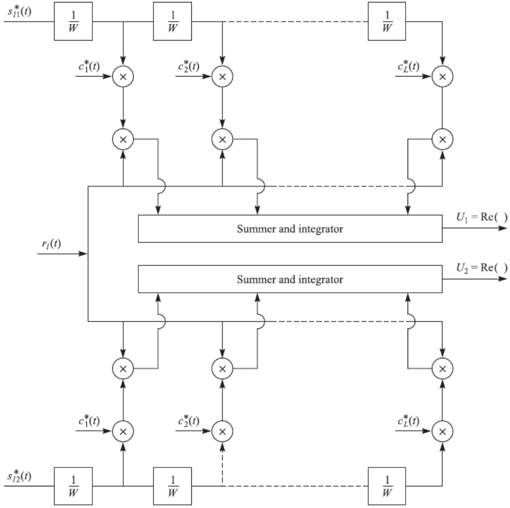


FIGURE 13.5–2
Optimum demodulator for wideband binary signals (delayed reference configuration).

The RAKE Demodulator

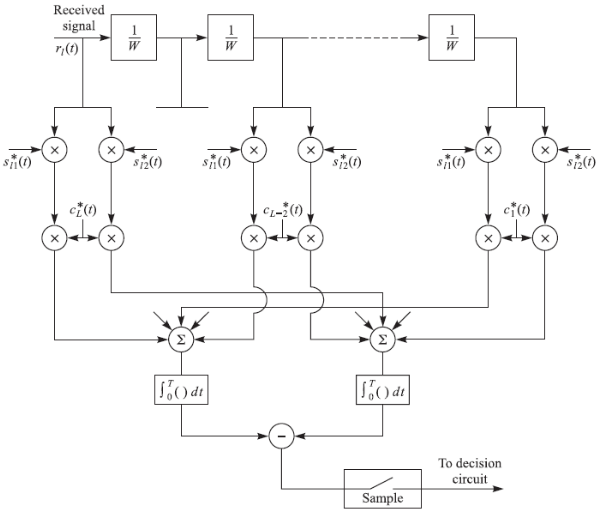


FIGURE 13.5–3
Optimum demodulator for wideband binary signals (delayed received signal configuration).

Performance of RAKE Demodulator

- Assume that $\{c_k(t)\}$ are estimated perfectly, and are constant within any one signaling interval. The decision variables are

$$U_m = \text{Re}\left[\sum_{k=1}^{L} c_k^* \int_0^T r_l(t) s_{lm}(t - k/W)\, dt\right], \quad m = 1, 2$$

with the received signal (assume, say, $s_{l1}(t)$ is transmitted)

$$r_l(t) = \sum_{n=1}^{L} c_n s_{l1}(t - n/W) + z(t), \quad 0 \le t \le T.$$

- This gives

$$U_m = \text{Re}\left[\sum_{k=1}^{L} c_k^* \sum_{n=1}^{L} c_n \int_0^T s_{l1}(t - n/W) s_{lm}^*(t - k/W)\, dt\right] + \text{Re}\left[\sum_{k=1}^{L} c_k^* \int_0^T z(t) s_{lm}^*(t - k/W)\, dt\right], \quad m = 1, 2.$$

Performance of RAKE Demodulator

- For pseudorandom sequences $s_{l1}(t)$ and $s_{l2}(t)$, the decission variables simplify to

$$U_m = \text{Re}\left[\sum_{k=1}^{L} |c_k|^2 \int_0^T s_{l1}(t - k/W) s_{lm}^*(t - k/W)\, dt\right] + \text{Re}\left[\sum_{k=1}^{L} c_k^* \int_0^T z(t) s_{lm}^*(t - k/W)\, dt\right], \quad m = 1, 2.$$

- When the binary signals are antipodal, a single decision variable suffices

$$U_1 = \text{Re}\left(2\mathcal{E}\sum_{k=1}^{L} \alpha_k^2 + \sum_{k=1}^{L} \alpha_k N_k\right)$$

where $\alpha_k = |c_k|$ and $N_k = e^{-j\phi_k} \int_0^T z(t) s_l^*(t - k/W)\, dt$.

- We have already seen this expression!

Performance of RAKE Demodulator

- The **RAKE** demodulator, with perfect (noiseless) estimates of the channel tap weights is **equivalent** to a **maximal ratio combiner** in a system with Lth order diversity.
- Let's consider this time binary antipodal signals subjected to **distinct** $\{E(\alpha_k^2)\}$. The error probability is given by

$$P_b = \frac{1}{2} \sum_{k=1}^{L} \pi_k \left[1 - \sqrt{\frac{\bar{\gamma}_k (1-\rho_r)}{2 + \bar{\gamma}_k (1-\rho_r)}} \right]$$

with $\pi_k = \prod_{i=1}^{L} \frac{\bar{\gamma}_k}{\bar{\gamma}_k - \bar{\gamma}_i}$ and $\bar{\gamma}_k = \frac{\mathcal{E}}{N_0} E(\alpha_k^2)$.

- For large values of the average SNR for all k taps, i.e., $\bar{\gamma}_k \gg 1$

$$P_b \approx \binom{2L-1}{L} \prod_{k=1}^{L} \frac{1}{2\bar{\gamma}_k (1-\rho_r)}.$$

Performance of RAKE Demodulator

- How to estimate $\{c_k(t)\}$? Assume sufficiently slow channel fading, e.g. $(\Delta t)_c / T \geq 100$.
- See Fig. 13.5-4 to Fig. 13.5-7.

Performance of RAKE Demodulator

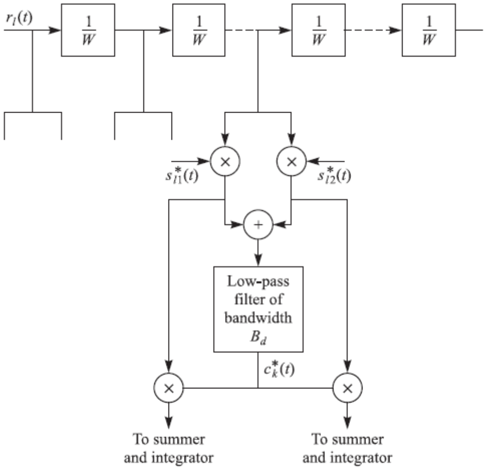


FIGURE 13.5–4
Channel tap weight estimation with binary orthogonal signals.

Performance of RAKE Demodulator

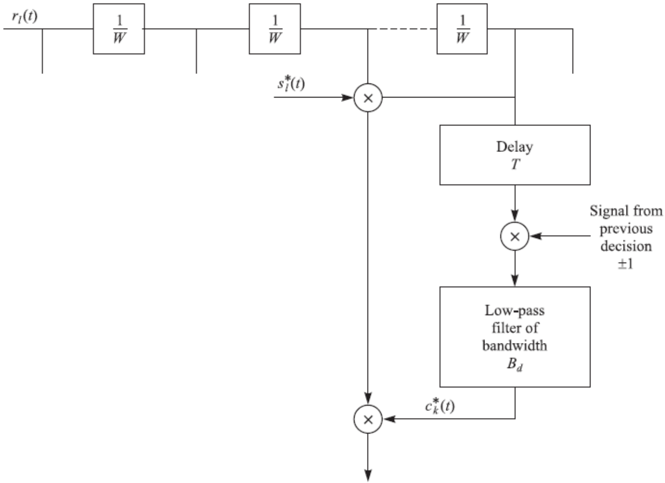


FIGURE 13.5–5
Channel tap weight estimation with binary antipodal signals.

Generalized RAKE Demodulator

- Addresses communication scenarios in which additive interference from other users of the channels results in **coloured** additive Gaussian noise.
- See Fig. 13.5-8 and Fig. 13-5.9.
- Assumes knowledge of the channel coefficients $\{c_i\}$ and the time delays $\{\tau_i\}$. In CDMA systems, an unmodulated spread spectrum signal is used.
- Our problem is to estimate the weights $\{w_i\}$ at the L_g fingers, with $L_g > L$.

$$U = \mathbf{w}^H \mathbf{y}$$

$$\mathbf{y} = \mathbf{g} b + \mathbf{z}$$

  and $\mathbf{z}$ contains additive Gaussian noise plus interference from other users plus ISI from channel multipath.
- The ML detection solution is given by (linear MMSE estimator)

$$\mathbf{w} = \mathbf{R}_z^{-1} \mathbf{g}$$

Performance of RAKE Demodulator

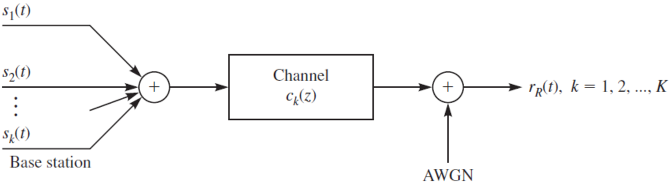


FIGURE 13.5–8
Model for the downlink transmission of a CDMA cellular communication system.

Performance of RAKE Demodulator

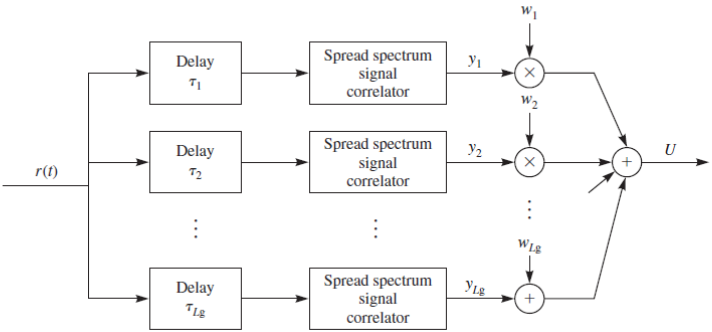


FIGURE 13.5–9
Structure of generalized RAKE demodulator.

Receiver Structures for Channels with Intersymbol Interference

- In the event that $T_b \gg T_m$ does not hold, the RAKE demodulator output will be corrupted by ISI.
- An equalizer in needed.
  - RAKE sampled at bit rate T_b, followed by equalizer, i.e., MLSE or DFE (Fig. 13.5-10).
  - Chip equalizer at chip rate T_c, with $LT_c = T_b$ (Fig. 13.5-11).

Receiver Structures for Channels with Intersymbol Interference

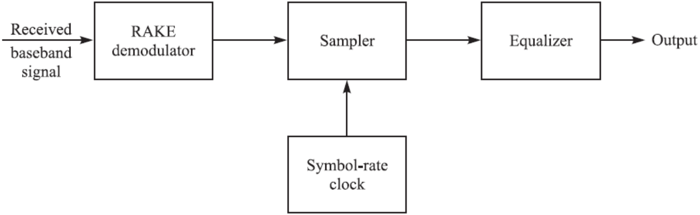


FIGURE 13.5–10
Receiver structure for processing wideband signal corrupted by ISI.

Receiver Structures for Channels with Intersymbol Interference

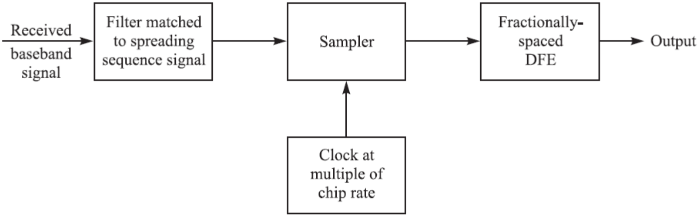


FIGURE 13.5–11
Alternative receiver structure for processing wideband signal corrupted by ISI.

Multicarrier Modulation (OFDM)

- OFDM is especially vulnerable to Doppler spread, which results into intercarrier interference (ICI).
- We analyze the performance degration due to Doppler spread in such a system, and some ICI suppression techniques.

Performance Degradation of an OFDM System due to Doppler Spreading

- OFDM **system** with N subcarriers $\{e^{j2\pi f_k t}\}$, M-ary QAM or PSK, symbol duration T and $f_k = k/T$, $k = 1, 2, \ldots N$ with

$$\frac{1}{T}\int_0^T e^{j2\pi f_i t} e^{-j2\pi f_k t}\, dt = \begin{cases} 1 & k = i \\ 0 & k \neq i \end{cases}$$

- Frequency-selective time-varying **channel** with impulse response $c(\tau; t)$, but non-selective for within each subcarrier band, i.e.

$$c_k(\tau; t) = \alpha_k(t)\delta(t), k = 0, 1, \ldots, N-1,$$

and $\{\alpha_k(t)\}$ complex-valued, jointly stationary, Gaussian stochastic processes with zero mean and cross-covariance

$$R_{\alpha_k \alpha_i}(\tau) = E[\alpha_k(t+\tau)\alpha_i^*(t)].$$

Performance Degradation of an OFDM System due to Doppler Spreading

- Furthermore
$$R_{\alpha_k \alpha_i}(\tau) = R_1(\tau) R_2(k - i).$$
- $R_1(\tau) = J_0(2\pi f_m \tau)$, with $J_0(x)$ the zero-order Bessel function of the first kind. Equivalenty,
$$S(f) = \begin{cases} \dfrac{1}{\pi f_m \sqrt{1 - (f/f_m)^2}} & |f| \leq f_m \\ 0 & \text{otherwise} \end{cases}$$
as is Jakes (1974).
- $R_2(k) = R_C(k/T)$ and,
$$R_C(f) = \frac{\beta}{\beta + j2\pi f} \longleftrightarrow R_c(\tau) = \beta e^{-\beta \tau}.$$

Performance Degradation of an OFDM System due to Doppler Spreading

- Use the two-term Taylor series expansion on $\{\alpha_k(t)\}$, i.e.

$$\alpha_k(t) = \alpha_k(t_0) + \alpha'_k(t_0)(t - t_0), \quad t_0 = \frac{T}{2}, 0 \leq t \leq T.$$

- Results in channel

$$c_k(\tau; t) = \alpha_k(t)\delta(t) = \alpha_k(t_0)\delta(t) + (t - t_0)\alpha'_k(t_0)\delta(t).$$

- The baseband signal

$$s(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} s_k e^{j2\pi f_k t}, \quad 0 \leq t \leq T$$

with $E[|s_k|^2] = 2\mathcal{E}_{\text{avg}}$ is filtered with $c(\tau; t)$.

Performance Degradation of an OFDM System due to Doppler Spreading

- The received signal is

$$r(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} \alpha_k(t_0) s_k e^{j2\pi f_k t} + \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} (t-t_0) \alpha_k'(t_0) s_k e^{j2\pi f_k t} + n(t)$$

- The output of the *i*th correlator at the sampling instant is

$$\begin{aligned} \hat{s}_i &= \frac{1}{\sqrt{T}} \int_0^T r(t) e^{-j2\pi f_i t}\, dt \\ &= \alpha_i(t_0) s_i + \frac{T}{2\pi j} \sum_{k=0, k\neq i}^{N-1} \frac{\alpha'(t_0) s_k}{k-i} + n_i, \end{aligned}$$

where the terms are the desired signal, ICI and additive noise, resp.

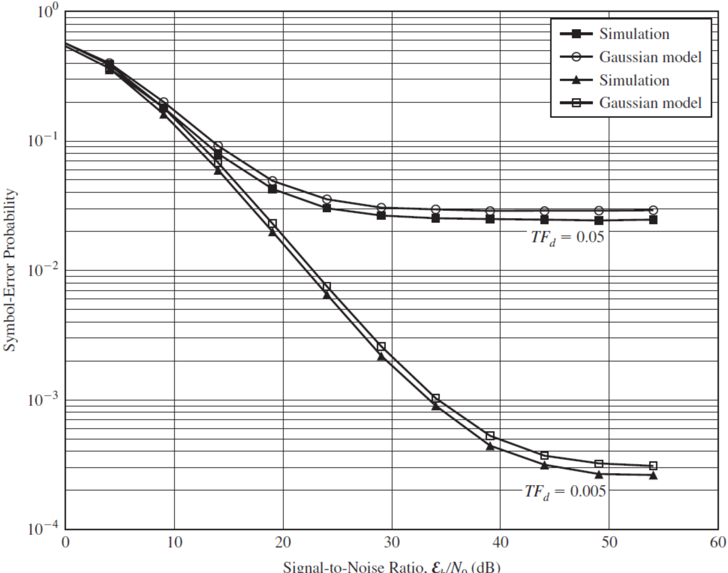
Performance Degradation of an OFDM System due to Doppler Spreading

- After some computation we arrive at

$$\frac{S}{I} = \frac{1}{\frac{(Tf_m)^2}{2} \sum_{k=0,k\neq i}^{N-1} \frac{1}{(k-i)^2}}.$$

- For a large number of subcarriers N the distribution of ICI is approximately Gaussian.
- **ICI severely degrades the performance of an OFDM system.**

Performance Degradation of an OFDM System due to Doppler Spreading



Suppression of ICI in OFDM Systems

- ICI in an OFDM system analogous to ISI in a single-carrier system: apply (linear) MMSE criterion.
- Estimate symbol $s_k(m)$ as

$$\hat{s}_k(m) = \mathbf{b}_k^H(m)\mathbf{R}(m), \quad k = 0, 1, \ldots, N-1$$

in order to minimize

$$E[|s_k(m) - \hat{s}_k(m)|^2] = E[|s_k(m) - \mathbf{b}_k^H(m)\mathbf{R}(m)|^2]$$

where $\mathbf{R}(m)$ denotes the output of the DFT processor.

Suppression of ICI in OFDM Systems

- The optimum coefficient vector is

$$\mathbf{b}_k(m) = [\mathbf{G}(m)\mathbf{G}^H(m) + \sigma^2\mathbf{I}_N]^{-1}\mathbf{g}_k(m), \quad k = 0, 1, \ldots, N-1$$

  where

$$E[\mathbf{R}(m)\mathbf{R}^H(m)] = \mathbf{G}(m)\mathbf{G}^H(m) + \sigma^2\mathbf{I}_N$$
$$E[\mathbf{R}(m)s_k^*(m)] = \mathbf{g}_k(m)$$

  and $\mathbf{G}(m) = \mathbf{W}^H\mathbf{H}(m)\mathbf{W}$, and $\mathbf{W}$ is the orthonormal IDFT transformation matrix.
- Knowledge of the channel impulse response is required.