LUND UNIVERSITY
Electrical and Information Technology

Exam in
Information Theory, EIT080

May 28, 2011, kl 8-13

- Write your name on each paper.
- Start a new solution on a new sheet of paper. Use only one side of the paper.
- Solutions should clearly show the line of reasoning.
- Aid:
  - Course book.
  - Handed out material.
  - Lecture slides and articles from the home page.
  - Calculator.

Good luck!

Problem 1

The two random variables X and Y take values in $x \in \{0, 1\}$ and $y \in \{0, 1, 2\}$, respectively. Their joint distribution function can be written as

$$P(x, y) = K \cdot (x + y)$$

(a) What is K?
(b) Find $H(X)$ and $H(Y)$
(c) Find $I(X; Y)$

(2+4+4=10p)

Problem 2

A memory-less source produces symbols in $x \in \{0, 1, 2, 3, 4\}$ according to the probabilities

x	0	1	2	3	4
$p(x)$	$1/12$	$1/12$	$1/6$	$1/6$	$1/2$

(a) Construct an optimal binary source code for the source. What is the expected codeword length? Compare with the entropy of the source.
(b) How many different (binary) Huffman codes can be found?[1]

(5+5=10p)

[1] Notice that the same tree can give several codes depending on the mapping of the branches.

Problem 3

In Figure 3.1 the plan for a house is given. Some of the doors is one-way while others can be used both ways, this is marked in the plan by arrows. A cat walks around in the house at random. When the cat is in a room it can either stay or leave by some of the outgoing doors. It chooses among the alternatives randomly with equal probability. Since there are ants in room 3, the cat will leave that room directly and not stay.

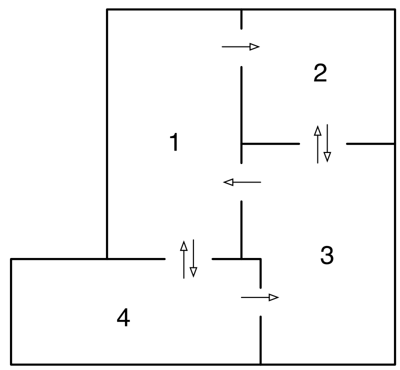

Figure 3.1: Plan of a house.

(a) What is the steady state distribution for what room the cat is in.
(b) Find the entropy rate for the cat's walk.

(5+5=10p)

Problem 4

In Figure 4.1 a continuous distribution is shown.

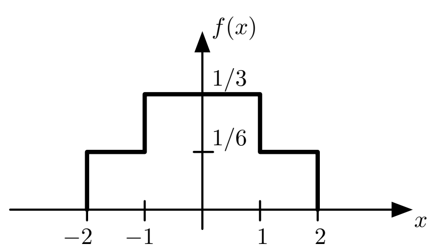

Figure 4.1: A distribution for a continuous random variable X.

(a) Calculate the differential entropy for the distribution $f(x)$.
(b) Consider an i.i.d. vector $\boldsymbol{x} = (x_1, x_2, \dots, x_{10})$ of length 10, and let $\epsilon = 0.1$. Determine if the all-zero vector $\boldsymbol{x_0} = (0, 0, \dots, 0)$ is a typical vector. If it is not typical, give an example of a typical vector.

(5+5=10p)

Problem 5

In Figure 5.1 a communication system is shown. The source produces independent binary symbols, $x \in \{0, 1\}$, that is transmitted over a memory-less additive channel. The noise added on the channel can take values $z \in \{-1, 0, 1\}$ with equal probability, and the received symbol is $y = x + z$. Furthermore, there is a possibility to get knowledge about the magnitude of the noise added on the channel, $w = |z|$, by closing the switch A.

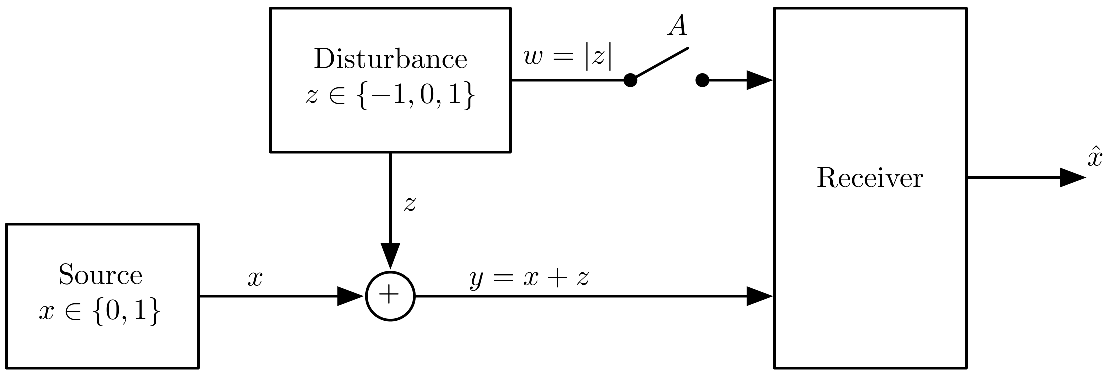

Figure 5.1: A channel with possibility of having information about the noise.

(a) What is the capacity of the channel when the switch A is open, i.e. when no information about the noise is received?

(b) What is the capacity of the channel when the switch A is closed, i.e. when information about the magnitude of the noise is received?

Hint: Start by showing that

$$I(X; Y|W) = \sum_{w} I(X; Y|W = w)P(W = w)$$

(5+5=10p)