LUND UNIVERSITY
Electrical and Information Technology

Exam in
Information Theory, EIT080

May 28, 2009, kl 14-19

- Write your name on each paper.
- Start a new solution on a new sheet of paper. Use only one side of the paper.
- Solutions should clearly show the line of reasoning.
- Aid:
  - Course book.
  - Handed out material.
  - Lecture slides and articles from the home page.
  - Calculator.

Good luck!

Problem 1

The stochastic variables X and Y describe the outcome of two tosses with a dice. Let $Z = X + Y$ be the sum of the results. Determine

- $H(X)$, $H(Y)$, $H(Z)$
- $H(Y|X)$, $H(Z|X)$, $H(X|Z)$
- $H(X,Z)$, $H(X,Y,Z)$
- $I(Z;X)$

(10p)

Problem 2

A binary information source gives the symbols 0 and 1 with probabilities $P(X=0)=\frac{1}{4}$ and $P(X=1)=\frac{3}{4}$. The sequence from the source is blocked into triplets that should be source encoded.

(a) Construct a Huffman code that encodes a three tuple at a time. What is the average length of a code symbol? Compare with the entropy of the source symbols.

(b) Use the encoder in part a of the problem to encode the sequence

$$\boldsymbol{x} = 111011101110111100111\ldots$$

What is the average codeword length in the encoded sequence?

(6+4=10p)

Problem 3

A communication link is modelled by the discrete memoryless channel i Figure 3.1.

(a) derive the channel capacity.

(b) For the communication over the channel a length three repetition code is used,

$$\mathcal{B} = \{000, 111\}$$

Construct an ML-decoder. (Notice that Δ_0 and Δ_1 have the same probabilities and, thus, need not be treated separately). Give an upper bound for the block error probability P_B.

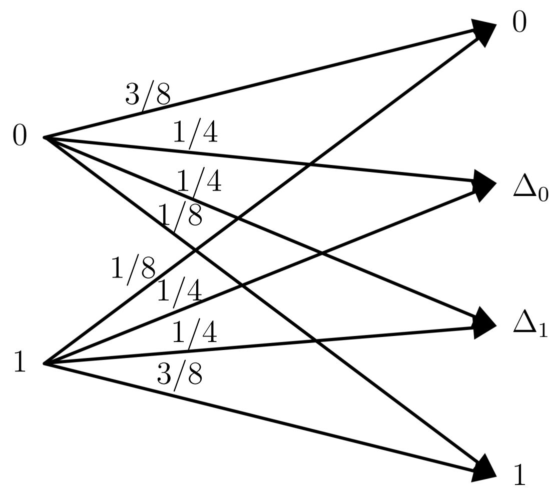

Figure 3.1: A discrete memoryless channel.

(5+5=10p)

Problem 4

The engineer Inga is going on a vacation to an archipelago. She decides to go to an archipelago with four main islands that are connected by four boat lines. There are also one sightseeing tour around one of the islands, see the map in Figure 4.1.

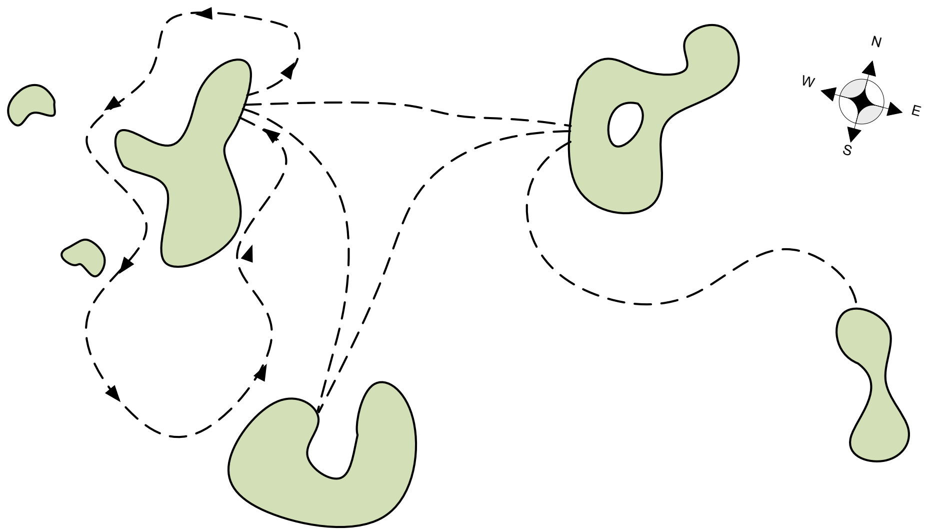

Figure 4.1: A map over the islands and the boat connections.

To avoid unnecessary planing she decides that she will take one boat trip each day. She will randomly with equal probability choose among the trips on the island. All connections between the islands are bi-directional, so it is only the sightseeing tour that only goes one way.

(a) When Inga has been travelling around for a longer time, what is the probability for being at the different islands?

(b) Inga has promised to write home and tell her friends about her trip. How many bits must she, in average, at least write down per trip to tell her route? Assume that she chooses the initialising island according to the distribution in part a.

(5+5=10p)

Problem 5

A string is 1 meter long. It is cut in two parts where one is twice as long as the other. With probability $3/4$ the long part is saved and with probability $1/4$ the short part is saved. The same procedure is repeated with the saved part, and this goes on for a large number of repetitions. How big part is, in average, saved at each cut?
(Hint: Consider the distribution of saved parts for the most common type of outcomes.)

(10p)